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**EFFECTS OF TWO CONSTRUCTIVIST INSTRUCTIONAL  
MODELS ON STUDENTS' ACHIEVEMENT AND  
RETENTION IN NUMBER AND NUMERATION**

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**DEPARTMENT OF SCIENCE EDUCATION  
UNIVERSITY OF NIGERIA NSUKKA**

**OCTOBER, 2007**

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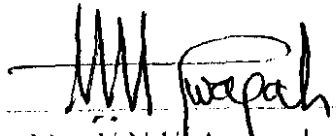
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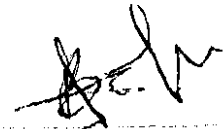
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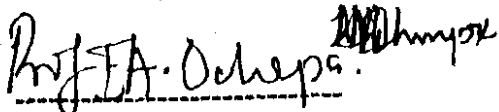
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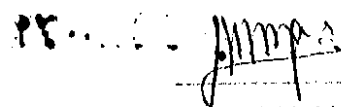
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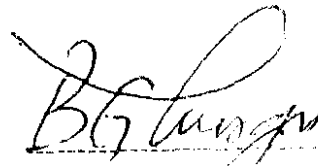
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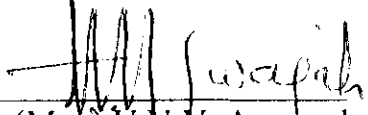


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## CERTIFICATION

OGBONNA, CECILIA CHINYERE, A postgraduate student in the Department of Science Education with the Registration Number PG/Ph.D/03/35254, has satisfactorily completed the requirements for the Degree of Doctor of Philosophy in Mathematics Education. The work embodied in this thesis is original and has not been submitted in part or full for any other diploma or degree of this or any other university.

  
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## **DEDICATION**

This work is dedicated to my husband, Elder Ben I. Ogbonna and my children; Chidinma, Nnanna, Amarachi, Promise and Victor.

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I sincerely and earnestly wish to express my appreciation to my supervisor, Dr. (Mrs.) U.N.V. Agwagah. Her love, encouragement, suggestions, guidance and prompt attention helped to make this work a big success. My profound gratitude also goes to Prof. B.G. Nworgu, Prof. S.A. Ezeudu, Prof. (Mrs.) U. Nzewi for their encouragement. My special appreciation goes to my Head of Department, Dr. (Mrs.) A. A. Nwosu for her pieces of valuable advice, care, concern and words of encouragement. My special thanks go to Dr. E. Nwagu, Dr. K.O. Usman, Dr. B.C. Madu, Dr. U.N. Eze and other lecturers in the Department of Science Education for their sincere concern and encouragement extended to me throughout the period of this study

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**Cecilia C. Ogbonna**

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## ABSTRACT

This study was designed to explore effects of two constructivist instructional models on JS II students' achievement and retention in number and numeration. Four research questions were asked and six hypotheses were formulated to guide the study. The study adopted a quasi – experimental of non – equivalent control group design and was restricted to Umuahia Education Zone of Abia State. Three Co – educational Secondary Schools were drawn for the study using random sampling technique. Out of the three schools selected, one was randomly assigned to Invitation, Exploration, Proposing explanation, Taking action (IEPT) treatment group, one to The Learning Cycle (TLC) treatment group and the third one to the Control Group (CG). A sample of 290 JS two students was involved (130 female and 160 male students). Three equivalent testing instruments were used. These instruments were Pre–Mathematics Achievement Test (PREMAT), Post–Mathematics Achievement Test (POSTMAT) and Delayed Post Test/Mathematics Retention Test (MRT). The three instruments had kendall's W coefficient of concordance of .707, .895 and .693 respectively. All the instruments were subjected to experts' validation. Data collected were analysed based on the research questions and hypotheses. Mean and standard deviation were used to answer all the research questions, while ANCOVA was employed to test the hypotheses at .05 level of probability. The study among other things revealed that students who were taught with IEPT and TLC achieved and retained higher the mathematics contents taught than those taught with CTM. Further more, the main gender on achievement was significant in favour of girls. Again the male and female students improved in their level of mathematics achievement and retention. The findings have serous implications for the mathematics teachers, authors of mathematics textbooks and other stakeholders in mathematics education. Recommendations were therefore made based on the highlighted educational implications of the findings of this study.

## CHAPTER ONE

### INTRODUCTION

#### **Background of the Study**

Mathematics forms one of the major and core subjects that a student has to offer at various levels of the educational system. Its importance to any level of educational pursuit makes it compulsory at the pre-primary, primary, secondary and tertiary levels of educational system. (Federal Republic of Nigeria (FRN), 2004; Adedayo, 2001; Efundajo, 2001; Amoo, 2002). Mathematics as a basic tool for all scientific and technological research, plays an important role in the economic development of a nation. Among other physical sciences, mathematics is the backbone in the national capacity building in science and technology. It helps mankind to enumerate, calculate, measure, collate, group, analyze and relate (Onugwu, 1991). When properly articulated, mathematics is a model for thinking, for developing scientific structures and for drawing conclusions as well as for solving problems.

In spite of the realized importance of mathematics, the place it occupies in the national development and the effort of the stakeholders to encourage students in the study of the subject, students have continued to achieve poorly (Ukeje, 1997). However a number of factors had been empirically found to have contributed to the students' poor achievement in senior school certificate mathematics examination. Some of these factors are rooted to the foundation level of the educational system (the primary level). The primary education is the foundation upon which the rest of the educational system is built (FRN, 2004).

The policy statement further indicated that the primary level is the key to the success or failure of the whole system. Among the major goals of primary education are inculcation of the permanent literacy and numeracy and ability to communicate effectively as well as sound basis for scientific and reflective thinking. This is an indication of the importance attached to mathematics and the need for every child to acquire the concept of number, which is the foundation of mathematics. Ironically, the syndrome of teacher-teach-all subjects at the primary school level has left the proper teaching of mathematics at that level in serious doubt (Adedayo, 2001; Amoo, 2001a). Betiku (2002) observed a poor foundation in primary school mathematics and explained that it is as a result of incompetent mathematics teachers in the school system, psychological fear of the subject and large classes. He maintained that this has contributed largely to the poor performance in mathematics.

Furthermore, other set of factors contributing to students poor achievement in mathematics include lack of students' comprehension reading skills (Agwagah, 1993); students' lack of interest towards mathematics as a school subject (Amazigo, 2000); inadequate use of instructional materials (Adedayo, 2001) attempts by teachers wanting to cover so many topics (areas) within a short time (Efunbajo, 2001); poor environmental background which a student encounters as he leaves home or immediate environment (Buhari, 1994; Amoo, 2000); lack of enough qualified mathematics teachers (Amazigo, 2000). The belief that

mathematics is a male domain is also communicated to young girls by media and parents in many subtle ways, Osafchinti as cited in (Ogbonna, 2004).

Apart from the factors referred above, research findings have shown that instructional model/approach adopted by teachers in presenting instructions has been implicated as a fundamental factor in poor achievement of students in mathematics. Discussing his findings, Simons identified instructional approach as one of the major contributory factors responsible for the ugly trend of poor achievement of students in mathematics in public examinations. Nzewi (2000) asserted that effective teaching makes learning meaningful. She argued that while good teaching helps the learners to learn effectively, poor teaching will lead to poor learning and invariably poor achievement. One may at this point attribute poor mathematics achievement of students to some instructional methods or models adopted by teachers.

Odogwu (1995), in his study observed that many teachers adopt the conventional or traditional approach to the teaching of mathematics. In this approach older methods or ideas are followed instead of adapting to changes and curricular activities rely heavily on textbooks and workbooks. Still in this setting, students are viewed as “blank slates” unto which information is etched by the teacher. More seriously, the teacher seeks the correct answer to validates students’ learning and assessment of students’ learning is viewed as separate from teaching and it occurs almost entirely through testing.

Brooks and Brooks (1985) explained that in conventional setting, many students struggle to understand concepts in isolation, to learn parts without seeing wholes, to make connections where they see disparity and to accept as reality what their perceptions question. For many students, success in school has very little to do with true understanding and much to do with coverage of the curriculum. In many schools, the curriculum is held as absolute, and teachers are reticent to tamper with it even when students are clearly not understanding important concepts. Rather than adapting the curriculum to students' needs, the predominant instructional response in a conventional setting is to view those who have difficulties in understanding the unaltered curriculum as slow or disabled.

However, many instructional strategies and models have been advocated, such as learning by doing, guided inquiry, problem solving and so on. In Nigeria, emphasis is placed on the use of guided discovery instructional method, (Federal Ministry of Education (FME), 1985). This instructional method is activity oriented and involves practical demonstration. Students are guided by materials and leading questions from the teacher to help them discover mathematical concepts. Yet, over the years, the result of this instructional strategy planned towards improving the quality of instruction in mathematics has been disappointing and seems ineffective. Current studies on how students learn science and science related subjects (mathematics) have started revealing new ideas, instructional approaches and models that have proved efficacious. One of such innovative instructional approaches which educators have advocated recently as an appropriate guiding

framework for teaching mathematics to students is the constructivist instructional approach (Cleminson, 1990; Roth, 1990; Cheung and Taylor, 1991; Airasian and Walsh, 1997; Nworgu, 1999).

Central to the constructionist perspective is the premise that a learner constructs meaning from new information and events as a result of an interaction between that individual's alternative concepts and his or her current observations. Some existing knowledge or prior ideas had been described as misconceptions (Novak, 1993), alternative framework (Driver, 1993); Children's' early experience (Adeyegbe, 1989). Students' prior ideas or knowledge therefore is a source of alternative conceptions or perception possessed by them before a formal instruction takes place.

Brooks & Brooks (1987), observed that students of all ages develop refine ideas about phenomena and then tenaciously hold unto these ideas as eternal truths. The writers stressed that even in the face of "authentic" intervention and "hard" data that challenge their views, students typically adhere staunchly to their original notions. Through experiences that might engender contradictions, the framework of these notions weaken, causing students to rethink their perspectives and form new understandings.

Constructivism, therefore, is a set of beliefs about knowing and learning that emphasizes the active role of learners in constructing their own knowledge (Von Glasersfield, 1989). In this view, knowledge is constructed by the learner in an attempt to integrate existing knowledge with new experiences. Continuing, Von



Glaserfield explained that although knowledge construction can be facilitated by instruction, it is not the direct consequence of instruction. Airasian and Walsh (1997), see constructivism as an epistemology of how people learn. It describes how one attains, develops and uses cognitive processes. It is based on the fundamental assumption that people create knowledge from the interaction between their existing knowledge or beliefs and the new ideas or situations they encounter.

Put differently, constructivism is the approach, which holds the view that knowledge are personally constructed and reconstructed by the learner based on his prior knowledge or experiences. Nworgu (1996), explains a constructivist-based method of interaction as that method which accepts the child's ownership of ideas. These aim at assisting the child to restructure or reconstruct his conception on the basis of confrontation with conceptualized evidence. Constructivism therefore is problem-solving oriented, it allows students to explore all work in groups, making meaning of tasks and setting out to solving problems that are perplexing to them.

A typical constructivist classroom as cited in Brooks and Brooks (1993) is a classroom where problem solving, concept development, and construction of learner generated solutions and algorithms are given more importance than memorizing procedures and using them to get right answers. Brooks and Brooks gave five important principles of constructivist pedagogy as:

1. Proposing problems of emerging relevance;
2. Structuring learning around "big ideas" or primary concepts;

3. Seeking and valuing students' point of view.
4. Adapting curriculum to address student's suppositions; and
5. Assessing students' learning in the context of teaching.

Furthermore, Brooks and Brooks explained that in a constructivist classroom, teachers encourage and accept students' autonomy, raw data and primary sources (rather than textbooks) are used in investigation, students' thinking drives the lesson, dialogue, inquiry and puzzlement are valued.

Yager (1991) asserts that constructivist practices require teachers to place students in more central position in the whole instructional programme. This means that students' ideas should form a basis for discussion and investigation in the classroom. The students should be viewed as thinkers with emerging theories about the world. Yager emphasizes that constructivist teachers generally behave in an interactive manner, mediating the environment for students and also seek the students' point of view in order to understand students present conceptions for use in subsequent lessons. Yager went further to explain that in a constructivist classroom, assessment of students learning is interwoven with teaching and this occurs through teacher observation of students at work and through students exhibition, portfolio and primarily, students work in groups.

Tobin (1993), agreed with the above and emphasized that teachers should see themselves as mediators of learning as well as provide constraints for students to channel their thinking to productive direction. Teachers are to interact with students to a greater extent than what is obtained in the traditional or conventional

classroom to ascertain what students know and what they are thinking and that learning should be interactive, co-operative and collaborative. In addition, the constructivist teacher should be interested in the question: "What do you know?" and not "Do you know these materials?" as would be asked by the conventional teacher. In a constructivist classroom, assessment is context bound and this makes multiple paths to the same end equally valid.

Many writers and researchers have proposed different models of instruction in their bids to enhance teaching and learning and in their attempt to describe constructivist instructional models, these writers and researchers have come up with various phases of constructivist instructional models. These models include:

1. The five phase constructivist – based model – The Biological Science Curriculum Study (BSCS, 1993) cited in Nwosu and Nzewi (1998).
2. The Analogy Model of Instruction – Harrison and Treagust as cited in Nzewi (2000).
3. The five step conceptual change Instructional Model (PEDDA), developed by Nworgu based on adaptations from Stofflet and Stoddarts and cited in Nworgu (1996).
4. Negotiation Model of Instruction – Wheatly (1997) as cited in Jegede and Taylor (1998).
5. The four phases constructivist Instructional Model: Invitation, Exploration/Discovery, Proposing explanation and Solution, and Taking action (IEPT) presented by Bybel, R, C. Buchwald, S. Crisman, D. Heil, P.

Kuerbis, C. Massumoto, and J. McInerney (198 (1989) as cited in Ogbonna (2004).

6. The Learning Cycle (TLC) constructivist Instructional Model as published by Akin and Karplus (1962).

A constructivist approach is based on the assumption that the learner is active and purposeful during learning process. Reports by Hameed, Hackling and Cowan (1993); Sofflet and Stodart (1994); Nworgu (1996); Ogbonna (2004) and Madu (2004) have provided theoretical and empirical support for constructivist based instructional models. These researchers have shown that the use of these models in teaching could lead to retention and better achievement.

The IEPT constructivist instructional model, which the researcher intends to use along side with TLC, provides the students the opportunity to recognize the problem through observation. The students make several attempts while persevering to arrive at the solution. Information is then communicated to others and new knowledge is transferred to develop products and ideas. On the other hand, TLC constructivist instructional model highlights the important role of self-regulation in the learning process. Although many interpretations have been given concerning TLC in the literature, TLC to be adopted in this study is the model that described curriculum development and instruction as a three-step cycle, i.e. concept discovery, concept introduction and concept application.

Academic success or failure is closely tied to both recognition and recall and these are attributes of retention. To correctly and effectively apply whatever one

had learnt, retention comes in to play an important role. Ausubel (1969) referred to retention as the process of maintaining the availability of a replica of the acquired new meaning or some part of them. It could be suggested that the amount of the original meaning that would be retained at any point in time is a variable of the quantity and quality at hand. In other words, retention may be explained as the act of retaining, or the state of being retained. For a student to retain, he or she must have good memory.

Memory is explained as the retention of information over time (Santrock, 2005). It is also a capacity to retain an impression of the past experiences. Memory is classified based on duration, nature and retrieval of perceived items. The main stages in information and retrieval of memory from an information processing perspective are:

- Encoding (Processing and combination of received information).
- Storage (creation of a permanent record of encoded information).
- Retrieval (Calling back the stored information in response to some cue for use in process or activity).

From the forgoing, it is obvious that the ability to retrieve an item or information depends so much on what has been retained. Ausubel (1968), asserts that retention may be difficult if the material presented cannot be related to the existing cognitive structure. Ausubel defined cognitive structure of the individual as all the information that the individual has about any particular area of experience. Ausubel went further to explain that when students study new material presented,

relate the new information to what they already know, and organize it into more complete cognitive structure, they are engaging in meaningful reception learning that enhances retention. This implies that any instructional model (approach) which is effective in making students retain concepts in mathematics can as well help students perform impressively in mathematics. Therefore, retention is a crucial construct worth of exploring in this study.

Furthermore, the issue of parity and disparity in the achievement of male and female students in mathematics has formed an important focus of research. This gender disparity in mathematics performance of secondary school students was clearly detected by Alio and Harbor-Petters (2000). During their experimentation with Polya's problems Solving Techniques, they discovered that there existed a significant difference in the achievement of male and female students in favour of males. Study conducted by Ezeugo and Agwagah (2000) revealed that male students performed significantly better than their female counterparts in algebra achievement test used in concept mapping.

Ozofor (1993), found out that significant gender difference does not exist since male and female students he studied performed equally. This view was supported by Ugwu (1998), when she found out that significant gender difference in performance in geometric proof between male and female students she studied did not exist. This result of no significant gender difference is also in compliance with the study conducted by Ogbonna (2004), which indicated that there was no

significant difference between the achievement of male and female students in mathematics using the IEPT constructivist instructional approach.

In the study conducted by Agwagah (1993), female students performed significantly better than their male counterparts in mathematics reading. Similarly, Fadaka (1994) found a high achievement in favour of girls.

In view of these contradicting results, a new investigation seems to be called for, to shed more light on the issue concerning the influence of gender on mathematics achievement. Moreso, since mathematics plays a vital role in technological progress as well as being one of the basic and core subjects taught in Nigerian secondary schools, everybody should have the same opportunity to learn and achieve in mathematics.

However, there is no known evidence to the best of the knowledge of the researcher of investigation into effects of these models at any level of education on JSS two students achievement and retention in number and numeration. Hence, this study intended to compare the effects of IEPT and TLC constructivist instructional models.

### **Statement of the Problem**

The effects of various teaching approaches on students' achievement in mathematics have been studied as indicated in the background to this study. Most of these studies were carried out at the senior secondary school level. They centred on diverse mathematical concepts. Despite these efforts by the researchers, it was noted that students' achievement in the public examination has persistently remain

poor. There is therefore the need to explore more innovative and effective teaching or instructional approaches that would improve the students' poor mathematics achievement.

Interestingly, current studies on how students learn science have revealed constructivist instructional approach as being effective in enhancing students' understanding and improving achievement. In the contrary, literature, so far have not revealed the effects of IEPT and TLC in the teaching of any mathematical concept. Since this is the case, this study therefore, explored the effects of IEPT and TLC constructivist instructional models in mathematics classroom. This is because no matter how attractive educational objectives might be; the usefulness and appropriateness of the content and learning experiences; the curriculum process is assumed to remain defective if adequate instructional strategies are not developed and employed in the task of teaching. If the teacher's teaching strategies are inspiring and progressive, then goals of education are likely to be achieved. On the other hand, if his/her methods fail to communicate adequate knowledge and skill to the students, it is believed that the teacher has failed as a teacher; therefore, objectives of that teaching exercise will hardly be achieved. The issue at hand then was to ascertain the effects of IEPT and TLC constructivist instructional models. Specifically, would using IEPT and TLC constructivist instructional models improve the learning and achievement of JS two students in number and numeration? To what extent would these constructivists instructional models facilitate their retention of the content learnt? Furthermore, would the male and



female students improve in number and numeration from the use of these constructivist models?

### **Purpose of the Study**

The purpose of this study is to determine empirically the effects of IEPT and TLC constructivist instructional models on Junior Secondary School students' achievement and retention in mathematics.

The study specifically seeks to:

1. Determine the effects of IEPT and TLC constructivist instructional models on students' achievement in Mathematics;
2. Determine the effects of IEPT and TLC constructivist instructional models on students' retention in Mathematics;
3. Determine the differential effect of IEPT and TLC on the achievement of male and female students in Mathematics;
4. Determine the differential effects of IEPT and TLC on the retention of male and female students in Mathematics.

### **Significance of the Study**

Research results abound on the teaching of mathematics of which most of them are based on pedagogical theories applicable to the teaching and learning of mathematics. Therefore, the search for any new and relevant methods of teaching mathematics is expected to be based on a particular pedagogical theory. This study was based on Piagetian constructivist theory of learning which emphasizes the teachers' ability to present instruction in such a way that students are actively

involved. The study also took into consideration Bloom's taxonomy of education in sequencing instruction. This was to ensure that the students' interest is made proficient in the three domains of cognitive, affective and psychomotor. The implication of this is that if these teaching techniques may have been found adequate, the teacher should be able to present number and numeration in JS two in such a way and manner that the student will understand it, appreciate it and participate in the solution of problems in proportion, ratio and rate without much difficulty.

Also students learn effectively if given model(s), which suit their ability but do not if there is a mismatch. The mean achievement scores of the experimental group being found high implied that additional innovative teaching strategy has been introduced. This has confirmed the benefit of using constructivist instructional models in teaching number and numeration. Moreover, there are few teachers that can use these instructional models, but a large number of mathematics teachers of the future need to learn to use these models as innovative instructional approaches. The outcome of this study may enable the mathematics teachers to find from day to day, whether the combination of IEPT and TLC will best serve the mathematics need of the students. Since this study was meant to confirm the effects of these instructional models, they would be considered as one of the instructional approaches in education that should be adopted

Since the subjects of this study achieved and retained high the content of the number and numeration taught, it has implication for teacher education institutions,

faculties of education of Nigerian Universities as well as colleges of education. They may incorporate these teaching models into the pre-service training of mathematics teachers. This result would also be of importance to the curriculum writers. They may wish to incorporate the approaches into the mathematics methods textbooks. This they may present as guide to teachers under suggested activities in mathematics curriculum.

Finally, the result of this study provided empirical evidence to policy makers in education and curriculum planners, which of the models, IEPT or TLC that proved more effective or whether the models are of equal effect. This may motivate the government to sponsor more researches on effects of other constructivist instructional models, thereby adding to the frontiers of knowledge.

### **Scope of the Study**

This study was restricted to topics in number and numeration (proportion ratio and rates) as provided in Junior Secondary Two mathematics Curriculum (FME, 1985). Proportion, ratio and rates are considered in JSS two because related concepts such as variations, probability to mention but a few, have been observed as presenting difficulties to students at higher levels. Thus it was necessary to initiate a remedial action on these observed problems early in the beginning of their secondary mathematics education career. Again, the above concepts were considered because they fit into the phases of the constructivist instructional models used and they form one of the basic arithmetic concepts that students should understand. Infact, the inability of students to attain a satisfactory

understanding of the basic arithmetical concepts taught at early stage makes assimilation of other future concepts difficult. This study covered Umuahia Education Zone of Abia State.

The content coverage include:

- (i) solving problems on direct and inverse proportion, using unitary method,
- (ii) finding the ratio of two quantities in the same unit;
- (iii) using the idea of ratio in sharing quantities;
- (iv) solving some word problems on rate.

### **Research Questions**

The following research questions guided the study:

- (1) what is the difference in the mean achievement and standard deviation scores of students taught mathematics using IEPT and TLC constructivist instructional models and those taught using the conventional method?
- (2) what is the difference in the mean retention and standard deviation scores of students taught mathematics using IEPT and TLC constructivist instructional models and those taught using the conventional method?
- (3) What are the differences in the mean achievement scores of male and female students taught mathematics using IEPT and TLC constructivist instructional models?
- (4) What are the differences in the mean retention scores of male and female students taught mathematics using IEPT and TLC constructivist instructional models?

## Hypotheses

The following null hypotheses (HOs) were formed to guide the study and were tested at .05 level of significance.

HO<sub>1</sub>: There is no significant difference in the mean achievement and standard deviation scores of students taught mathematics using IEPT & TLC constructivist instructional models and those taught using conventional method.

HO<sub>2</sub>: There is no significant difference in the mean retention scores of students taught mathematics using IEPT & TLC constructivist instructional models and the mean retention scores of students taught using conventional method.

HO<sub>3</sub>: There is no significance difference in the mean achievement scores of male and female students taught mathematics using IEPT and TLC constructivist instructional model.

HO<sub>4</sub>: There is no significant difference in the mean retention scores of male and female students taught mathematics using IEPT and TLC constructivist instructional models.

HO<sub>5</sub>: There is no significant interaction effect of IEPT and TLC constructivist instructional models and gender as measured by the mathematics achievement test.

HO<sub>6</sub>: There is no significant interaction effect of IEPT and TLC constructivist instructional models and gender as measured by the mathematics retention test.

## CHAPTER TWO

### REVIEW OF LITERATURE

In this chapter, studies related to the present investigation were reviewed under two major broad headings and the summary. The major headings are:

#### **A. THEORETICAL/CONCEPTUAL FRAMEWORK**

#### **B. EMPIRICAL STUDIES**

Literature review under theoretical framework includes:

- (i) Educational theories of learning and mathematics instruction
- (ii) Current status of the teaching and learning of mathematics
- (iii) Concept and Perspectives of constructivism
- (iv) Constructivist models.
- (v) Attributes of constructivist teachers.
- (vi) Concept of learning, forgetting and retention.
- (vii) Gender differences in mathematics

Empirical studies discuss:

- (i) Studies on achievement in mathematics and some approaches that have been adopted.
- (ii) Studies on constructivist models/Approaches.
- (iii) Gender differences and academic achievement.

Summary of literature

#### **Theoretical/Conceptual Framework**

##### **Educational theories of Learning and Mathematics Instruction**

In the development of a curriculum for school mathematics, the nature of mathematics as a discipline and psychological theories of learning must be considered. Teachers need to know about the teaching – learning process. They should be aware of effective teaching and successful classroom practices which provide guidelines that will help them choose instructional approaches that work

with learners. Hence, the substantial change in school mathematics curricula throughout the whole world since the early 1960s. There has been the need for greater number of mathematically minded persons, especially in the present increasing technological advancement of nations. This need has necessitated new knowledge of how children should learn mathematical ideas and concepts. Development in mathematics has resulted in the introduction of new contents and researches into the process of learning and this has equally led to new approaches to the teaching of mathematics.

The mental discipline theory of learning had a great influence on mathematics teachers (Kennedy & Tipps, 1998). According to this theory, the mind is like a muscle and benefits from exercise as muscles do. Mathematics is used to give the mind exercise, for instance, the lengthy computations are regularly used to train the mind. Thorndike's stimulus-response theory is in line with the mental discipline theory. This theory is based on the belief that learning occurs when a bond or connection is established between a stimulus and appropriate response. Mathematics lessons consist primarily the presentation of many number combinations so that the child could establish strongly bond between combinations and their answers. Practice is therefore emphasized.

As time goes on, a number of researchers and theorists challenged the simple stimulus-response description for human learning. Piaget (1964), Barzun (1992), Piaget and Inhelder (1971) emphasized development of understanding as fundamental in the learning of mathematics. This implies that the early learners

must understand what they are learning if learning are to be permanent. On these premise, Piaget and Inhelder concluded that all individuals pass through stages as they mature intellectually. Piaget therefore identified four stages, through which he believed all individuals should pass. They include: Sensory –motor stage (ages, 0-2), during which infants and children develop concepts primarily through interactions within the physical world.

Pre-operational stage (ages 2-7), during which the children begin to use language to express ideas but the ideas are still dependent on perceptions. Concrete operational stage (ages 7-12), during which children develop many concepts using concrete objects to explore relationships and model abstracts.

Formal operation stage (ages 12 through adulthood), during which children begin to think abstractly. The Piaget's theory of learning has the belief that learners must construct their own meanings rather than being passive receiver of information.

However, the Piagetion theory of learning is very relevant to this study, as its construct elaborated mathematical model of the mental structures, which characterized the concrete and formal operation & stages. These mathematical models threw light on the art of teaching. Thus, there is the need to state the implications of Piagets theory for the art of teaching mathematics as follows.

- (a) Since the child's mental development advances through qualitatively different stages, these stages should be considered when planning the mathematical experiences of the child at any given stage.



- (b) The child should be tested to ensure that he/she has mastered all the prerequisites for mastering his/her concepts before introducing a new concept. If the child is not ready for the concept, provide him/her with experiences which will make him/her ready.
- (c) The pre-adolescent child is fond of making typical errors of thinking which are characteristics of his/her stage of mental development. Mathematics teachers should endeavour to comprehend these errors.
- (d) To encourage mental growth, the experience of seeing things from many varied points of view is necessary. For instance, in Junior secondary two, the teacher should use many different instructional innovative approaches to teach mathematics.
- (e) Mental growth has relationship with the discovery of invariant in the child, such as those attributes associated with each of the stages.

The subjects of this study were at their late concrete and early operational stages. They were actively involved in each of the lessons taught using the JS two mathematics curriculum. Consideration was adequately made with regard to the fact that the child's mental development advances through qualitatively, different stages. However, the theory of Piaget was very relevant to the instructional approaches adopted in this study. Each of the models was based on Piaget's theory of learning mathematics.

### **Current status of the Teaching and Learning of Mathematics**

The domain of mathematics learning and teaching is one of the most orientation in research on learning and instruction (Grouws, 1992). The inculcation of permanent numeracy as first among the stated goals for primary education stresses that need for every child to be mathematically literate. Similarly, the inclusion of mathematics among the core junior secondary school subjects further affirms the importance attached to the learning of mathematics. The position it occupies in the National Policy on Education and its role towards Technological advancement has put mathematics in special place in Primary, Secondary and Tertiary levels of education (Amoo, 2001).

Despite the recognition accorded mathematics at all levels, Amoo lamented that it is unfortunate that most students, especially secondary school students exhibit nonchalant attitudes towards the subject. A study conducted by Nigerian Education Research Council (NERDC, 1997) on performance of students in public examinations over some years has confirmed students poor achievement in physical sciences (mathematics, physics and chemistry) which form the foundation of future work in the much needed Nigerian Technology. Majority of students who register for Senior School Certificate Examination (SSCE) have been observed to come out with either partial or total failure in mathematics year after year. One ponders why this should be the trend.

Learning of mathematics extends beyond learning concepts, procedures, and their applications. It also includes developing a disposition towards mathematics

and seeing mathematics as a powerful way for looking at situations. Disposition refers not simply to attitudes but a tendency to think and to act in positive ways. Students mathematical disposition are manifested in the way they approach tasks, whether with confidence, willingness to explore alternatives, perseverance, and interest; and in their tendency to reflect on their own thinking.

According to Perkins (1991), the notion of disposition also involves, besides ability, inclination and sensitivity. Inclination is the tendency to engage in a given behaviour because of motivation and habits; sensitivity refers to the feeling for, and alertness to, opportunities for implementing the appropriate behaviour. Ability, then, combines both knowledge and skill. The acquisition of a disposition, especially the sensitivity and inclination aspects require extensive experience with the different categories of knowledge and skills in a large variety of situations. As such, the disposition cannot be directly taught, but rather has to develop over an extended period (Greeno, 1991). The question arises, then as to what kinds of learning process are conducive to the attainment of the intended mathematical disposition in students. The negative answer seems to be that this disposition cannot be achieved through learning as it occurs in most classrooms. Indeed, there are national and international literatures with findings indicating that students lack mathematical disposition because they are not equipped with necessary knowledge, skills, beliefs and motivation to approach new mathematical problems and learning tasks in an efficient and successful way (De Conte, 1992). This lack of students' mathematical disposition can largely be accounted for by the prevailing learning

activities in schools which consist mainly of listening, watching and imitating the teacher and textbook (Greeno, 1991). In other words, the dominant view of mathematics education is still the information transmission model, implying that the mathematical knowledge acquired and institutionalized by past generations has to be transmitted as accurately as possible to the next generation (Romeberg and Carpenter, 1986).

Another shortcoming of current mathematics education, which is related to inappropriate view of learning as information absorption is that knowledge is often acquired independently from the social and physical contexts from which it derives its meaning and usefulness. This has become very obvious from a substantive amount of research carried out for some decades now on the influence of cultural and situation factors on mathematics learning, commonly classified under the heading "ethno mathematics and everyday mathematical Cognition" (Nunes, 1992).

The preceding description of the learner as an absorber and consumer of decontextualized mathematical knowledge contrasts sharply with the conception supported by a substantial amount of evidence in the literature showing that learning is an active and constructive process. Learners are not passive recipients of information; rather, they actively construct their mathematical knowledge and skills through interaction with physical and social environment, and through reorganization of their prior mental structures. Furthermore, learners are encouraged to develop hypothesis, to test out their own and other's ideas; to make

connections among “content” areas, to explore issues and problems of personal relevance (either existing or emerging), to work cooperatively with peers and adults in the pursuit of understanding, and to form the disposition to be life long learners.

At this juncture, while focusing on students and the learning situations, it is very glaring that the teacher stands out at the center of the whole business of teaching and learning. The teacher is the key in the success of any educational endeavor at all levels of the system. The world is becoming more complex with increase demand in technological advancement. This more, complex knowledge-based and multi-cultural society creates new expectations for teaching. To help diverse learners master more challenging contents, teachers must go far beyond dispensing information, giving tests and grades. Teachers must know their subject areas deeply and they must understand how students think, if they are to create experiences that actually work to produce effective learning (Darling – Hammond, 1996).

Findings of many research works in teaching and learning of mathematics have shown that many teachers are deficient in the mathematics contents (Igbokwe, 1997). The National Policy on Education, section 56 Federal Government of Nigeria (1998), recognizes the importance of teachers in the achievement of educational goals at any level by stating that “no educational system can rise above the quality of its teachers”. The teacher is the key factor in determining the quality of education given in schools.

Ironically, the report of NERDC on school curriculum (1997), found that there was scarcity and inadequacy of qualified teachers in science and mathematics and that students are found to be scared of science subjects especially mathematics. Numerous literatures have also revealed that the problem of failure of students in mathematics examination has always been attributed to the teacher's failure to use appropriate method of teaching (Adedayo, 2001). Others relate the failure of students to teacher's incompetence or ineffectiveness (Amoo, 2000). The reports go ahead to regret that unqualified teachers are recruited and that even the prospective mathematics teachers are not adequately equipped to teach practical and some of them lack the knowledge of subject matter (Oyedeji, 1998; Adebayo, 2001; Amoo, 2001).

The teacher is overstretched with overloaded and unrealistic curriculum, he faces poor view about the teaching profession, worst still, at the primary level, he is faced with the problem of Teacher "teach-all" policy (Amoo, 2001; Adedayo, 2001). More available statistics have shown that the ratio of mathematics teachers to students in the secondary school is 1:134 (Salau, 1995). This number includes those with ND and HND in engineering who were drafted to teach mathematics. This ratio has been widened especially in the JSS classes with the introduction of Universal Basic Education (UBE) programme.

However, a way out will include following a set of well fashioned constructivist teaching behaviours as reviewed in the literature which provides a useable framework within which teachers can experiment with new innovative

approaches. Based on this premise, the researcher seeks to investigate the effects of some of the constructivist based approaches/models with reference to students achievement and retention in mathematics.

### **Concept and Perspectives of Constructivism**

Historically, “constructivism” is not a new concept. It has its roots in philosophy and has been applied to sociology and Anthropology, as well as cognitive psychology and education (Yager, 1991). As with many educational terms, constructivism has different interpretations based on different perspectives. Airasian and Walsh (1997), see constructivism as an epistemology of how people learn. It describes how one attains, develops and uses cognitive processes. It is based on the fundamental assumptions that people create knowledge from the interaction between their existing knowledge or beliefs and the new ideas or situations they encounter.

Kant further elaborated this idea by asserting that human beings are not passive recipients of information. Learners actively take knowledge, connect it to previously assimilated knowledge and make it theirs by constructing their own interpretation (Check, 1992). Piaget, in his own contribution believed that learning can best be explained as an active experience, between the learner and objects or people in an environment. For Piagetian constructivists, learning does not come from a body of information that can be transplanted to a person’s mind, but rather from a vital constructive interaction between the learner and the environment strongly influenced by prior knowledge (Piaget, 1971).

Lev Vygotsky, sometimes referred to as “social” constructivist believed learning takes place through social interaction. According to Vygotsky, knowledge is built on what individuals in society construct together. Vygotsky was especially interested in the dialogue between individuals in a group, how they converse, question, explain, and negotiate meaning. Fosnot (1996) explains further that learning is regarded as a social activity in which learners are engaged in constructing meaning through activities, discussions and negotiations among peers, students and teachers. Learners individual constructions of meaning occur when their ideas are compared, explored and reinforced in social settings with each student having the opportunity to recognize his or her ideas through talking and listening (Driver, 1990; Solomon, 1991). Through social interaction, learners become aware of other ideas and see confirmation of their personal constructions (Maor and Taylor, 1995).

Put differently, constructivism is the instructional approach which holds the view that knowledge is personally constructed and reconstructed by the learner based on his prior knowledge or experiences. Von Glasersfield (1989), explains constructivism as a set of beliefs about knowing and learning that emphasizes the active role of learners in constructing their own knowledge. In this view, the learner constructs knowledge in an attempt to integrate existing knowledge with new experiences. When the learner is presented with new information, he/she will reformulate his/her existing cognitive structure only if the new information is connected to the knowledge already in memory. The learner must actively



construct knowledge unto his/her existing mental framework for meaningful learning to occur and retention facilitated.

Although researchers in science education, mathematics inclusive have attended to constructivist theories of learning, they differ substantively in the ways that they have interpreted those theories. For instance, Lawson, Abraham, and Renner (1989) emphasized the role of hypothetico – deductive thought in the development of students' scientific knowledge. Posner, strike, Hawson, and Gertzog (1982) emphasized the role of prior knowledge and conceptual conflict. Vygotsky (1978), a social constructivist, explains the importance of the interplay between language and action as students learn in social settings. Social constructivists emphasize forms of language that facilitate students' meaningful construction. These forms, such as open-ended questions, creative writing, students' explanations and classrooms dialogue involves the interactive and reciprocal use of language.

From the social constructivist view, language can be used to stimulate adaptive cognitive activity. Students use language to represent their current understandings, as well as the processes by which they develop such understandings. Bayer (1990), asserts that students understandings evolve through a meaningful negotiation process in which they discuss and test their own ideas and consider the ideas of others. Bayer stresses that in order to interpret students' understandings; a teacher observes students' actions, listens to their verbal responses, and actively constructs representations of their current views. In

essence, what is required is a multimodal process of communication between teachers and students, and among students, that involve both expressive and interpretive communication.

However, a constructivist classroom differs from a classroom based on the traditional (objectivist) model. The traditional classroom, can sometimes resemble a one-person show with a captive but often uninvolved audience. Classes are usually dominated by teacher-centered, direct instruction and often rely heavily on textbooks. The idea is that there is a fixed body of knowledge that the student must come to know. Information and instruction is separated into parts that make up a whole concept. The teachers seek to transfer their thoughts and meanings to the passive students. There is little room for student initiated questions, independent thought or interaction between students. The goal of the learner in this setting is to regurgitate the accepted explanation or methodology expostulated by the teacher (Caprio, 1994).

Furthermore, the traditional or conventional teaching method of teacher as the sole information-giver to passive students appears outdated. Project (1990) charges that the “present curricular in science and mathematics are over-stretched, over-stuffed and undernourished. The curricular emphasizes the learning of answer more than the exploration of questions, memory at the expense of critical thought, bits and pieces of information instead of understanding in context, recitation over argument, reading in lieu of doing. The curricular fails to

encourage students to work together, to share ideas and information freely with each other, or to use modern instruments to extend their intellectual capabilities.

In a constructivist setting, knowledge is not objective, science is viewed as systems with models that describe how the world might be rather than how it is. These models derive their validity not from their accuracy in describing the rest world, but from the accuracy of any predications which might be based on them (Postlethwaitie, 1993). Postlethwaitie stressed that the role of the teacher is to organize information around conceptual clusters of problems, questions and discrepant situations in order to engage the students' interest. Teachers should assist the students in developing new insights and connecting them with their previous learning. Ideas are presented holistically as broad concepts and students are encouraged to ask their own questions, carry out their own experiments, make their own analogies and come to their own conclusions.

Brooks and Brooks (1993) list five important principles of constructivist's pedagogy:

### **Posing problems of emerging relevance to students**

One common criticism of constructivism is that, as a pedagogical framework, it subordinates the curriculum to the interest of the child. Critics contend that the constructivist approach stimulates learning only around concepts in which students have prekindled interest. Such criticisms miss the mark because the statement does not imply that students are free to study whatever they want on any given day. It does mean that the teacher must plan the lesson so that the topic

will be of interest to students. This could be done with a surprising demonstration, an interesting activity, or a good problem. A good problem might include the following criteria:

- It demands that students make a testable predictions;
- It makes use of relatively inexpensive equipment.
- It is complex enough to elicit multiple problem-solving approaches;
- It must benefit from group effort. In addition, students must view the problem as relevant to them.

### **Structuring learning around primary concepts**

Structuring learning around primary concepts is a critical dimension of the constructivist's pedagogy. When designing learning, constructivists teachers organize information around conceptual clusters of problems, questions and discrepant situations because students are most engaged when problem and ideas are presented holistically rather than in separate, isolated parts. Much of traditional education breaks wholes into parts and then focuses separately on each part. But many students are unable to build concepts and skills from parts to whole. These students often stop trying to see the wholes before all the parts are presented to them and focus on the small memorizable aspects of the broad units without ever creating the big picture. Think for instance, of assembling a bicycle, the package contains precise written directions in sequential order, but often, people continually refer to the picture of the bicycle on the box. There is need to see the "whole" before being able to make sense of the parts. Concentrating on the pieces is likely

to result in the misapplication of isolated facts or algorithms because the student cannot “see the forest for the trees”. On the other hand, when concepts are presented as wholes, students seek to make meaning by breaking the wholes into parts they can see and understand. Students initiate this process to make sense of the information; they construct the process and the understanding rather than having it done for them. With learning activities clustered around broad concepts, students can select their own unique problem – solving approaches and use them as springboards for the construction of new understandings.

Structuring learning around bid “ideas” and broad concepts provides multiple entry points for students. Some become engaged through practical responses to problems, some analyze tasks based on models and principles and others interpret ideas through metaphors and analogies from their unique perspectives. The environment and use of broad concepts invite each student to participate irrespective of individual styles, temperaments and dispositions.

### **Seeking and valuing students’ points of view**

Seeking and valuing students’ points of view is essential to constructivist education. The more people study learning process, the more they understand how fundamental this principle is. Students’ points of view are windows into their reasoning. Awareness of students’ points of view helps teachers to challenge students making school experiences both contextual and meaningful. Each student’s point of view is an entry point that sits at the gateway of personalized

education. Teachers who operate without awareness of their students' points of view often doom students to dull, irrelevant experiences, and even failure.

People have at one time or the other been to workshops or meetings in which the presenter has begun the session by asking the participants what they hope to learn or accomplish. Often, people's responses are made into a list on the board. Then the presenter starts the session and never again refers to the list. This might be an example of looking for students' points of view, but it is not definitely an example of valuing them. Valuing students' points of view means not only recognizing them but also addressing them.

### **Adapting the curriculum to address students' suppositions**

Learning is enhanced when the curriculum's cognitive social and emotional demands are accessible to the students. Therefore, some sort of relationship must exist between the demands of the curriculum and the suppositions that each student brings to a curriculum task. If suppositions are explicitly addressed, most students will find lessons bereft of meaning, regardless of how charismatic the teacher or effective the materials might be.

Constructive teachers design lessons that address students' supposition. The design process is informed and enhanced by an understanding of cognitive demands implied by certain curricular tasks. For example, a lesson on manipulation of fractions can invite students into confrontation with previous constructions of part-whole relationships. The adaptation of curricular tasks to address students' suppositions is a function of the cognitive demands implicit in

specific tasks (the curriculum) and the nature of questions posed by students engaged in the task (the suppositions).

### **Assessing students learning in the context of teaching**

Much crucial in the constructivist instructional approach is the principle that discusses assessing students' learning in the context of teaching. Posing narrow questions of which one seeks a singular answer denies teachers the opportunity to peer into students' mind. Creativity and risk taking are not attributes that can be turned on and off. Both need nurturing, encouragement, and support. Creative thinking is not something that can be scheduled during a particular segment of the school day, separated from the rest of the academic programme.

There is widespread belief that the study of mathematics and science seek right answer, while the study of literature and humanity accepts creativity. Students who learn in settings that encourage individual construction of knowledge do not see the content area boundaries so clearly. Learning about our world is inherently interdisciplinary. Solving our world's problems require creative thought. The big prize paid by teachers who emphasize "rightness" is losing the ability to evoke creative student work. The big question of what exactly are "right" and "wrong" answers arises. Rightness and wrongness relate as much to the filtering system used by adults to sort through students' responses as to the students' conceptions of the issues and questions to which they respond.

To the teacher, inaccurate responses often are "wrong". To the students, inaccurate responses often represent the state of their current thinking about topics.

Students' conceptions, rather than indicating "rightness" or "wrongness" should form entry points for the teacher and also places to begin the sort of intervention that would lead to the learner's construction of new understandings and acquisition of new skills. Teachers can offer this intervention by using assessment as a tool in service to the learners, rather than as an accountability device, hence teachers can begin to rethink the dynamics of relationship between teaching and assessment.

Furthermore, there is emphasis on non-judgemental feedback. Thinking about non-judgmental feedback to students provokes a lot of questions about many traditional school practices:

Why do teachers give tests?

Does doing so facilitate learning?

Or does it create an external factor that diverts students' mind away from the intellectual demands of "real" learning onto the emotional concerns of one's comparative rating in the class?

It is difficult to structure learning around non-judgmental feedback because teachers are all so acculturated to using evaluative words, and expressions. "No", "good", "right", and "wrong", are just a few of the words used all over in school. Upon hearing these words, students either continue or alter their thinking, not because of some internal realization but because of an external prompt. Over time, this sort of feedback makes students teacher-dependent.

Ferrandino's journal entry (1991), asked for some examples on non-judgemental feedback. Teachers seeking to offer non-judgemental feedback might



think about responding to student's questions with additional questions to student's assertions with plausible contradictions, to students requests for assistance with request for explanation of their thinking to date, and students' arguments with responses such as "I can see that this is important to you" or "you have convinced me" or "that is something I have not studied very much" or "your idea makes sense to me, what do your classmates think of it? Such reaction places the responsibility on the students for assessing the efficacy of their own efforts and make pleasing the teacher far less important. Hence, students' assessment of the efficacy of their own efforts could only be done when they are exposed to some constructivist-based approaches/models such as IEPT and TLC adopted in this study.

### **Constructivist Models**

Taking into account the view of mathematical learning as a construction of meaning and understanding, and the goal of mathematics education as the acquisition of mathematical disposition involving mastery of different categories of knowledge and skills, a challenging task has to be addressed. It consists of elaborating a coherent framework of research-based principles for a design of powerful teaching environment, that is situations and contexts that can elicit in students the learning activities and processes conducive to the intended mathematical disposition that will enhance and facilitate achievement and retention.

A variety of models and approaches attempting the constructivist-based powerful mathematical learning setting has already been tried out, reflecting the

methodological shift towards the application of teaching experiments in real classrooms and towards the use of a diversity of techniques for data collection and analysis including qualitative and interpretative methods. These models include:

### **The five phase constructivist-based models**

The Biological Science curriculum Study (BSCSS, 1993) cited in Nwosu and Nzewi (1998).

The model's phases are:

- (ii) Engagement – This phase is the problem identification stage.
- (iii) Exploration - This is the experimenting and problem solving stage.
- (iv) Explanation – This is the classification stage.
- (v) Elaboration - This phase is generalization stage.
- (vi) Evaluation – Evaluation is the signal feedback stage.

**The Analogy Model of Instruction** – Harrison and Treagust as cited in Nzewi (2000) described analogy as being characterized by aspects of science discourse in which familiar situation similar to the unfamiliar phenomenon to be explained is used. It is observed by Duit (1991) to include metaphor, analogies, parables and mental and physical models which are common literary devices used in spoken, action and written communication. It is an instructional strategy used in explaining less familiar domain by using a more familiar domain (Nzewi, 2000).

Glynn (1989) formulated a guide called Teaching with Analogy model (TWA). The model consists of six steps which are:

- (i) Introduction of Target Concepts.
- (ii) Recall Analogy Concepts.
- (iii) Identify similar features of concepts
- (iv) Map familiar features.
- (v) Draw conclusions about concept
- (vi) Indicate where analogy breaks down.

**The five step conceptual change Instruction Model (PEDDA)** developed by Nworgu based on adaptation from Stofflet and Stoddarts and cited in Nworgu (1996). This conceptual instructional model has the following five steps:

- (vii) Identification of prior conception held by students.
- (viii) Exploration of the phenomena
- (ix) Discussion of the result of the experiment
- (x) Development of dissatisfaction.
- (xi) Application.

### **Negotiation Model of Instruction**

Constructivist-oriented classroom needs to be active with exchange of information in the form of discussion. Therefore, the use of negotiated learning strategy is more compelling because negotiation as the name implies, means to discuss in order to come in agreement (Jegede, and Taylor, 1998).

Wheatly (1997) as cited in Jegede and Taylor (1998) stated some distinctive features associated with negotiation as follows:

- (i) problem solving requires considerable negotiation of social norms;
- (ii) attention should be focused on negotiating science meaning even as social norms are negotiated;
- (iii) teachers should be engaged in conceptualizing individual and group activity;
- (iv) a successful negotiation is reached when two parties have no further reason to believe their positions are different;
- (v) negotiation is complex and requires the intension to negotiate.

**The four phase constructivist Model (IEPT)**, presented by Bybee et al., (1989).

The phases are:

- (ii) Invitation – recognizing the problem through observation and then decision to tackle such problem.
- (iii) Exploration/Discovery – In this stage, several attempts would be made to solve the problem (trial and error phase) but perseverance is needed to continue.
- (iv) Proposing explanation and solution – when one has arrived at the solution, then, information would be communicated to others, that is, the explanation stage.
- (v) Taking action-This is the application stage where new knowledge is transferred to develop products and produce ideas.

**The learning Cycle (TLC)) constructivist Instructional Model** as published by Akin and Karplus (1962).

This is the model that described curriculum development and instruction as a three-step cycle. The three steps are:

- (i) Concept Discovery – In this section, students are given time to think, plan, investigate, and organize collected information.
- (ii) Concept Introduction – This is the phase or stage where teachers try to promote equilibrium by introducing a new concept or time to account for phenomenon under study.
- (iii) Concept application: This is a stage when student's knowledge and/or skills, application of new concepts and change in thinking are observed. Students at this stage should access their own learning.

A teacher in a constructivist mathematics classroom teaches to facilitate thorough understanding that will help students not to have difficulties transferring, generalizing and constructing an essential understanding of subjects. Blais (1988), in discussion of students learning algebra, speaks to an issue that surfaces in every subject area:

Considered in isolation, conventional instruction appears to be sensible and helpful. But we cannot fairly judge an instructional approach unless we consider what occurs within the novice. The available evidence indicates the novices sabotage good conventional instruction by selecting from it only the minimum necessary to achieve correct, mandated performance. They resist learning anything that is not part of the algorithms they depend on for success. Thus, drawings, estimations, abstractions, connections to simple examples, informal English, learning to read well, and so on, are viewed

as unnecessary embellishments. Novices feel they know what is important despite their not perceiving essence. They do not understand shallowness because they do not experience depth (p. 627).

This being the case, the IEPT and TLC constructivist models which the researcher adopt in this study are explained thus:

The first phase of IEPT constructivist model, which is invitation involves understanding the mathematics problem and this is viewed as the most essential factor in problem solving. Greco as cited in Alio (1997) stated that understanding mathematics problems involves mathematics comprehension reading skills. This implies the capability to analyze, identify the basic knowledge, relevant data and unknowns in the problem. This, as it is, relates to recognizing the problem through observation and then decision to tackle it.

Fredriken (1984) stressed that success in problem-solving is due to thorough understanding of the problem. Successful understanding at this stage is an important facilitator in problem solving because it is believed to simplify and reduce the search for sequencing of transformation that would lead to the problem solution.

The second phase is the exploration/discovery or creation. This phase involves a search in the long-term memory for a suitable plan or procedure for solution or information that would help in the construction of suitable plan or procedure for solution. The search might lead to recall of the very similar problem that had been solved in the past, in which case, the students might try to use the same strategy and procedure that was successful before, or explore new production

system appropriate to the problem. It should be stressed that procedural knowledge for a solution requires decisions among many possible alternatives only a few of which could lead to the desired solution. Discussing this still, Donald (1991) emphasized that exploration phase involves being able to describe the problem mathematically using equations, tables, charts and making systematic guess and also using logical reasoning.

The third phase centres on proposing explanation and solution. In this phase, the plan devised or constructed in the preceding phases is carried out. The phase could be completed quickly with few errors if the plan devised is well explanatory and it helps the students to arrive at the solution. It should be stressed that if the plan devised is not well explained, if the steps involved are too complex and if the approach to execute the plan is non-systematic, incomplete and wrong solutions would be obtained at the end.

The fourth and final phase of the IEPT constructivist instructional model is the phase when action is taken. This is the application stage where new knowledge is transferred. In this phase, students reflect back on the problem solving process which should not involve mere checking an answer. Kerch and McDonald (1991) writing on the activity to promote reflecting on the problem solving process stated that:

Reflection (application) encompasses more than mere determining whether an answer is reasonable. Reflection in its complete sense, entails looking at the problem in a general way. Classifying the problem by type and determining what would happen to the problem if certain data were changed.

So, the present study explored instructional models grounded in constructivist framework with the view to improving retention and achievement in mathematics. Would IEPT substantially provide this opportunity?

Looking at the learning cycle, the model has a long history in science education, mathematics inclusive. With respect to learning science, Vygotsky theory of interplay between language and action as students learn in a social setting, suggests that social interaction is essential as learners internalize new or difficult understandings, problems and processes and because active and thoughtful language is the vehicle by which learners negotiate the meaning of their experiences, it is important to provide instruction in a format which encourages its use, hence the science curriculum improvement study (SCIS, 1974), learning cycle model for instruction. It is one instructional framework that may allow for active language use and problem solving. The original SCIS learning cycle included three phases labeled exploration, invention, and discovery. Purportedly, exploration stimulates cognitive disequilibrium by involving students with experiences and concrete materials (Lawson and Renner, 1975). During the invention phase, proponents encourage teachers to promote equilibrium by introducing a new concept or time to account for phenomena under study. During discovery, students are expected to self-regulate and come to new understanding by engaging in related activities. Since the development of SCIS, other interpretations of the learning cycle have been developed (Renner and Marek, 1988; Barman, 1989).



Another interpretation of the learning cycle model is the 5E's proposed by The Maryland Virtual High School of Science and Mathematics. This cycle has 5 components:

- (i) engagement: The activities in this section captures the students' attention, stimulates their thinking and help them access prior knowledge.
- (ii) Exploration: In this section, students are given time to think, plan, investigate, and organize collected information.
- (iii) Explanation: Students are now involved in an analysis of their exploration. Their understanding is classified and modified because of reflective activities.
- (iv) Extension: This section gives students the opportunity to expand and solidify their understanding of the concept and/or apply it to a real world situation.
- (v) Evaluation: Evaluation should take place throughout the learning experience. The teacher should observe students' knowledge and/or skills, application of new concepts and a change in thinking. Students should access their own learning.

Another interpretation of the learning cycle is the model published by Akin and Karplus, (1962). It is the most popular description of the model. Highlighting the important role of self-regulation in the learning process, the model describes curriculum development and instruction as a three-step cycle.

In the first phase, the teacher provides an open-ended opportunity for students to interact with purposefully selected materials. The primary goal of this initial lesson is for students to generate questions and hypothesis from working with the materials. This step is known as the “discovery”.

Concept introduction is the second step. Here, the teacher provides concept introduction lessons aimed at focusing the students’ questions, providing related new vocabulary, framing with students their proposed laboratory experiences, and so forth. The third step, “concept application”, completes the cycle after one or more interactions of the discovery – concept introduction sequence. During concept application, students work on new problems with the potential for evoking a fresh look at the concepts previously studied.

This cycle stands in contrast to the ways in which most curriculum syllabi and published materials present learning and the ways in which most teachers were taught to teach. In the traditional model, concept introduction comes first, followed by concept application activities. Discovery, when it occurs, usually takes place after introduction and application and with only the “quicker” students who are able to finish their application tasks before the rest of the class.

Although various models have been reviewed in the literature, IEPT and TLC were adopted in this study because the content areas to be used for the study fit into the phases of the models.

### **Attributes of Constructivist Teachers**

Some teachers regardless of the approaches they have used in the past, view constructivism as the way they have always known people to learn. Most of these teachers believe that they have been prevented from teaching in accord with that knowledge by a combination of rigid curriculums, unsupportive administrators and inadequate preservice and inservice educational experiences.

Becoming a teacher who helps students to search rather than follow is challenging, and, in many ways, frightening. Teachers might resist constructivist pedagogy for understandable reasons that most of them were not educated in these settings nor trained to teach in these ways. The shift, therefore, seems enormous.

The following are the attributes of constructivist teachers.

#### **Constructivist teachers encourage and accept students' autonomy and initiative**

While the philosophies and mission statements of many school purports to want students to be thinking, exploring individuals who generate hypotheses and test them out, the organizational and management structures of most schools militate against these goals. So, if autonomy, initiative, and leadership are to be nurtured, it must be done in individual classroom.

Autonomy and initiative prompt students' pursuit of connections among ideas and concepts. Students who frame questions and then go about answering and analyzing then take responsibility for their own learning and become problem solvers, and perhaps more important, problem finders. These students in pursuit of new understandings are led by their own ideas and informed by the idea of others.

These students ask for, if not demand, the freedom to play with ideas, explore issues and encounter new information. The way a teacher frames an assignment usually determines the degree to which students may be autonomous and display initiative.

**Constructivist teachers use raw data and primary sources along with manipulative, interactive and physical materials**

Concepts, theorems, algorithms, laws and guidelines are abstractions that the human mind generates through interaction with ideas. These abstractions emerge from the world of phenomena such as falling stars, nations at war, decomposing organic matter, and other happenings that describe ones world. The constructivist approach to teaching presents these real-world possibilities to students, and then helps the students generate the abstractions that bind these phenomena together. When teachers present the students the unusual and the commonplace, and ask students to describe the difference, they encourage students to analyze, synthesize, and evaluate. Learning becomes the result of research related to the real problems and this is what schools strive to engender in their students.

**When framing tasks, constructivist teachers use cognitive terminology such as “clarity”, “analyze”, “predict”, and “create”**

The words one hears and uses in ones everyday life affects one’s way of thinking and ultimately one’s actions. The teacher who asks students to select a story’s main idea from the list of four possibilities on a multiple-choice test is presenting to the students a very different task than the teacher who asks students to analyze the relationships among three of the story’s characters or predict how the

story might have proceeded had certain events in the story not occurred. Analyzing, interpreting, predicting and synthesizing are mental activities that require students to make connections, delve deeply into texts and contexts, create new understandings.

**Constructivist teachers allow student responses to drive lessons, shift instructional strategies, and alter content**

The above statement does not mean students initial interest, or lack of interest in a topic determines whether the topic gets taught, nor does it mean that whole sections of the curriculum are to be jettisoned if students wish to discuss other issues. However, students' knowledge, experiences and interests occasionally do coalesce around an urgent theme – the “teachable moments”. As educators, teachers have each experienced moments when the students enthusiasm, interest, prior knowledge, and motivation have intersected in ways that made a particular lesson transcendental and enabled teachers to think with pride about that lesson for weeks. Teachers recall the gleam in the students' eyes, their excitement about the tasks and discussions, and their extraordinary ability to attend to the tasks for long periods of time and with great commitment. If teachers were fortunate, they encountered a handful of these experiences each year, and wondered why they did not occur more frequently.

It is unfortunate that much of what teachers seek to teach their students is of little interest to them at that particular point in their lives. Curriculum and syllabi developed by publishers or state level specialists are based on adult notions of what students of different ages need to know. Even when the topics are of interest to the

students, the recommended methodologies for teaching the topics sometimes are not. Little wonder, then, why more of those magnificent moments don't occur.

**Constructivist teachers inquire about students' understandings of concepts before sharing their own understandings of those concepts**

When teachers share their ideas and theories before students have an opportunity to develop their own, students questioning of their own theories is essentially eliminated. Students assume that teachers know more than they do. Consequently, most students stop thinking about a concept or theory once they hear "the correct answer", from the teacher. It is hard for many teachers to withhold theories and ideas for various reasons:

- teachers do often have a "correct answer";
- students themselves are often impatient;
- some teachers adhere to the old saying about knowledge being power. Teacher struggling for control of their classes may use their knowledge as a behaviour management device when they share their ideas;
- time is a serious consideration in many classroom. The curriculum must be covered and teachers' theories and ideas typically bring closure to discussions and move the class on to the new topic. Constructivist teachers, the caveats presented in the preceding paragraphs notwithstanding, withhold their notions and encourage students to develop their own thoughts.

**Constructivist teachers encourage students to engage in dialogue, both with the teacher and with one another**

One very powerful way students come to change or reinforce conceptions is through social discourse. Having an opportunity to present one's ideas, as well as being permitted to hear and reflect on the ideas of others, is an empowering experience. The benefits of discourse with others, particularly with peers, facilitate the meaning making process.

Students-to-student dialogue is the foundation upon which cooperative learning (Slavin, 1990) is structured. Reports state that cooperative learning experiences have promoted interpersonal attraction among initially prejudiced peers (Cooper et al., 1980) and such experiences have promoted inter-ethnic interaction in both instructions and free-time activities (Johnson et al., 1980).

**Constructivist teachers encourage students' inquiry by asking thoughtful, open-ended questions and encouraging students to ask questions of each other**

If teachers want students to value inquiry, they as educators must also value it. If teachers pose questions with the orientation that there is only one correct response, how can students be expected to develop either the interest in or the analytic skills necessary for more diverse modes of inquiry? Schools too often present students with one perspective.

Complex thoughtful questions challenge students to look beyond the apparent, to delve into issues deeply and broadly, and to form their own understandings of events and phenomena. Discourse with one's peer group is a

critical factor in learning and development. Schools need to create settings that foster such interaction.

### **Constructivist teachers seek elaboration of students' initial responses**

Initial responses are just that "initial" responses. Students' first thought about issues are not necessarily their final thoughts nor their best thoughts. Through elaboration, students often reconceptualize and assess their own errors. Occasionally, perhaps often, the adult filter through which teachers hear students' responses fail to capture the students' meanings. Students' elaboration enables adults to understand more clearly how students do and do not think about concepts. Students and teachers often discover how disparate their perspectives sometimes are. It's only through that discovery that individuals can engage in the process of trying to reconcile the two.

### **Constructivist teachers engage students in experiences that might engender contradictions to their initial hypotheses and then encourage discussion**

Cognitive growth occurs when an individual revisits and reformulate a current perspective. Therefore, constructivist teachers engage students in experiences that might engender contradictions to students' current hypotheses. They then encourage discussion of hypotheses and perspectives. Contradictions are constructed by learners. Teachers cannot know what will be perceived as contradictions by students, this is an interpersonal process.

But teachers can and must challenge students' present conceptions, knowing that the challenge only exists if the students perceive a contradiction. Teachers must, therefore use information about the students present conceptions, or point of



view, to help them understand which notions students may accept or reject as contradictory.

### **Constructivist teachers allow wait time after posing questions**

In every classroom, there are students who, for a variety of reasons, are not prepared to respond to questions or their stimuli immediately. They process the world in different ways. Classroom environments that require immediate responses prevent these students from thinking through issues and concepts thoroughly, forcing them, in effect, to become spectators as their quicker peers react. They learn over the time that there is no point in mentally engaging in teacher-posed questions because the questions would have been answered before they have had the opportunity to develop hypotheses.

Another reason students need wait time is that, the questions posed by teachers are not always the questions heard by the students. The Gatling gun approach to asking and answering questions does not provide opportunity for the teacher to sense the manner in which most of the students have understood the questions.

### **Constructivist teachers provide time for students to construct relationships**

In a certain classroom, students were given magnets to explore. In a short time, almost all of the students had discovered that one end of a magnet attracted the other magnet while the opposite end repelled it. Soon, most of the students discovered that if one of the magnets were turned around, that the magnets that had attracted each now repelled each other. During this activity, some went beyond

these initial relationships and joined forces with their peers to create magnetic “trains” and to create patterns with iron fillings. A great number of relationships, patterns, and theories were generated during this activity and non of them came from the teacher. The teacher structured and mediated the activity and provided the necessary time and material for learning to occur, but the students constructed the relationships themselves.

From the foregoing, these attributes can only be made manifest using constructivist instructional models. Thus, the researcher explored the comparative effects of IEPT and TLC constructivist instructional models on students’ achievement and retention in mathematics.

### **Concept of Learning, Forgetting, and Retention**

When a person engages in practice of training activities and when observation of his performance shows his performance has changed, learning is usually assumed to have occurred, the change in behaviour being the result of a combination of practice operations with practice conditions.

Forgetting is thought as failure to recall experiences when attempting to do so, or to perform an action previously learned. In daily experience with school learning, teachers are sometimes surprised that students forget most of what they have learned after a short lapse of time. The term “forgetfulness” simply means the tendency to forget.

Long term memory (LTM), short time memory (STM) and ICONIC memory (IM) provides sufficient background for understanding the problems of

forgetting, hence, lack of retention. There is an amount of information a subject is capable of holding in iconic, short, or long term memory. It has been found that each of these three areas of memory has a different capacity.

LTM storage – the capacity appears to be nearly infinite. Material that attains LTM does not seem to push previously stored material out of LTM. There is enough room in which to store raw information in LTM while still retaining most of the old information.

STM storage – The capacity is limited to 5 – 9 items of information. STM does have a limited capacity and once this limit is reached, the new information begins to push the old out of STM.

IM storage – IM capacity is less than 5. After a very brief presentation, a subject usually recalls only 3 or 4 items.

Sometimes, information capacity of the memory system is being exceeded. However, two theories have emerged to account for mechanism underlying information loss in the memory.

- “decay” theory which states that forgetting represents a “fading” of information over time;
- “interference” theory which states that it is competitions among stored “traces” that accounts for forgetting.

The decay theory was held by Thorndike (1913). Thorndike believed that the bond between stimulus and its response “weakened” over time simply because

it was not used. The explanation assumes that learning leaves a “trace” in the brain i.e. the memory trace which involves some sort of physical change that was not present prior to learning. With the passage of time, normal metabolic processes of the brain cause a fading or decay of the memory so that the traces of the material once learned gradually, disintegrate and eventually disappear altogether. Studies have shown that decay is quite prevalent in STM much more so than in LTM. Cermack (1970), concluded that probably an interaction of decay and interference contributes to the rate of retention loss in STM.

The interference theory of forgetting maintains that it is not so much the passage of time that determines the course of forgetting but what we do in the interval between learning and recall, new learning may interfere with prior learning. The theory that new learning may interfere with the old is known as retroactive inhibition. Proactive inhibition, on the other hand is based on the principles that maintains that prior learning may interfere with the learning and recall of new materials.

Explaining further, the trace change theory explains that the “memory trace” is a hypothetical construct, it is not something we can point to in the brain. It refers to whatever representation of an experience that persists in the nervous system. When memory trace “fades” or that something else happens to it, what emerges when attempt is made to recall learned material is different from experience that was originally registered.

Furthermore, forgetting has been explained as a retrieval failure. The process of retrieval involves locating learned material when needed. According to retrieval failure theory, forgetting is very often a temporary rather than a permanent phenomenon. It is not like losing something, but rather is more like being unable to locate and find it. When cues that were available at the time of learning are not available at the time of recall, retention suffers.

Still talking about forgetting, consolidation theory provides a recent explanation of forgetting. It emphasizes the importance of undisturbed period for memory traces to become durable and permanent. If newly formed traces are disturbed and no time is allowed for consolidation, they will be wiped out because memory traces, like cement takes time to harden. For traces to be consolidated, there should be proper encoding. Encoding is referred to as the process of analyzing new information in order to determine where in ones organization system it belongs. Since organization has been shown to be essential to both learning and retention, it is obvious that the faster and the more efficiently a person can organize incoming information, the more he will be able to process and retain it. In order to perform this rapid organizational process, it is imperative that the person be able to analyze the attributes of whatever he has to process, encode, retain and retrieve. Analysis involves identification of several dimensions of the material or item, such as physical attributes, conceptual attributes and so on. This type of analysis determines how information is to be organized in memory and provides cues for retrieval and is referred to as encoding stages.

From the foregoing, there are two aspects of information processing which theorists in memory are aware of:

- The first is that retention appears to depend on the level and precision of encoding that has been achieved by an item as well as the amount of time and the extent of interference that has occurred since its presentation.
- The second is that probability of recall will parallel the extent of encoding in that the longer the encoding time necessary to achieve a particular encoding level, the longer the time needed for that encoding to disappear.

It can be concluded with emphasis that educators should bear in mind that given that information processing is an active ongoing system of hierarchical encoding, differential storage, and retrieval searches, it follows that such a complex interdependent system must take time to develop, and that impairment at any stage might damage the entire system. Also, memory does have an organization and the future encoding of any item depending upon this organization, since the individual will always seek to fit the new item into the already established organization for retention and easy recall. The organization of material in some meaningful fashion is another technique to improve memory, hence retention.

Some techniques that increase retention include:

**Self – recitation:** When a material is being studied, self-recitation increases retention of the material. Studies have shown that reciting materials to oneself

increases retention better than simply reading and re-reading the material. One can read a material once and spend five-sixths of the time asking oneself questions about the material read. Active recall or self-recitation would be better than re-reading several times.

The self-recitation method in ordinary learning forces the learner to define and select the to-be-remembered (TBR). Also recitation represents practice in the retrieval of information in the form likely to be required later on. Time spent in active recall with the material out of sight, is time well spent.

**Highlight of material:** Meaningfulness of material aids or increases retention. Noble (1953), using three serial lists of 12 items each, constructed in such a way that list 1 had a meaningful value of 1.28, list 2 had a meaningful value of 4.42 and list 3, 7.88 found a significance between list difference in the number of trials it took for each criterion of one perfect trial. The group, which received the list with the highest meaningfulness value learned faster, and the group, which received the list with the lowest meaningfulness value, learned slowest. This experiences demonstrated that the higher the meaningfulness of the material, the faster the learning.

**Over learning:** Another technique that increases retention is overlearning and this means learning something well beyond the point of bare recall. That is, learning in which repetition or practice has proceeded beyond the point necessary for the retention or recall required. Such over learning may however, be necessary for the retention or recall required and in view of factors necessary to affect recall which

are bound to enter encoding processes at the various levels. Experimental evidence shows that a moderate amount of overlearning aids retention. Therefore, if one were to retain new learning over a considerable period of time, one would be wise to over learn the material beyond bare mastery of it. At this juncture, it is evident that retention has a strong tie with achievement and that inappropriate instruction approach leads to lack of understanding of concepts and this also leads to forgetting and poor retention. Therefore, to facilitate retention of mathematical concepts, innovative instructional approaches such as IEPT and TLC need be explored, hence the need for the study.

### **Gender differences in mathematics**

Gender issues in education have formed an important focus of research for some years now. In mathematics and mathematics related fields (sciences), there tend to be more males than females. For instance, among the ancient mathematicians, the celebrated names included Euclid, Erasthones, Pythagoras, Pascal and others, all of whom were men. One begins to ponder on the possible causes of such an imbalance (Olagunju, 1996).

Gender differences existed in school curriculum for boys and girls. For instance, girls were made to do subjects like needle work, home management, nutrition, domestic science, housewifery and so on, (Lassa, 1995). This implies that these subjects would prepare the girls for their future roles as mothers and housewives. Within this framework, girls seemed to fail to think beyond becoming wives and mothers, which is societal expectations of them.



Tyler as cited in Okeke (1999), attributed gender parity and disparity to natural, societal, cultural and psychological reasons among others. Socially, gender inequality is well pronounced in the characterization of school milieu, exemplifying the masculine nature of science. Again, school text and curriculum materials carry passive imagery of women in various examples: The illustrations and pronouns which are used often portray females as passive in nature, (Cohn and Cohn 1990). Moreover, the differential curriculum encourages stereotypical choice of subjects. For instance, Home Economics/Technical drawing, Literature/Mathematics and Physics. Thus, indirectly the school provides a platform for perpetrating sex identity. The writers went ahead to explain that in school activities, the expectations from boys and girls reinforce sex roles further. Girls are encouraged to study feminine subjects like languages, Home Economics and Literature, which will prepare them for the expected adult role while boys are encouraged to study science and mathematics.

On cultural dimension, certain careers are unfeminine and incompatible with marital demands. For instance, majority of science related careers have inbuilt inflexibility in working schedules and require those involved to be out of their homes most of the time. These types of jobs are incompatible with feminine responsibility to meet dual role demand of home and work. as a result, girls with potentials for science and mathematical skills are discouraged from pursuing them, (Erinosho 1994).

Furthermore, the psychological dimension has to do with attitude, interest and self-concept. In their upbringing, females are nurtured to develop capacity for emotion, concern and feeling of nature with minimal manipulation of the physical objects in the environment. This makes them not to develop attribute which are masculine in nature. In terms of self concept, girls probably have poor perception of their worth because of various negative experiences which they are exposed to.

### **Empirical Studies**

#### **Achievement in mathematics: Some methods/Approaches that have been adopted**

A number of empirical researches that have interesting relevance to the present study on students' achievement and retention in mathematics have been conducted, and some methods or approaches adopted.

Researches on teaching methods and approaches include Alio, (1997); Okeke, (1999); Iji, (2003); Ogwuche, (2002), to mention but a few. Alio investigated the effects of Polya's problem solving strategy in secondary school students' achievement and interest in mathematic. This study was carried out in Enugu North and South Local Government Area of Enugu State. The main purpose of the study was to examine the effects of Polya's problem solving strategy (POPSST) on students' achievement and interest in mathematics using quasi-experimental design with a sample of three hundred and twenty SS 2 students (160 males and 160 females). Two instruments were used for data collection.

These were:

- (i) A mathematics achievement test (MAT)
- (ii) An interest scale (IS)

The research questions were answered using mean and standard deviation while the null hypothesis were tested using Analysis of Covariance (ANCOVA) at the alpha level of .05.

The findings of the study revealed that using POPSST in teaching problem solving in mathematics to senior secondary school students enhanced achievement and increased their interest in the subject as well. The result also showed among other things that female students develop more interest in mathematics problem solving than the male students after being taught by the strategy. It was then recommended that Polya's problem solving strategy be adopted to teach mathematics problem solving at primary and secondary school levels.

The above study is relevant to the present study since it sought to find achievement of students relative to instructional approach and gender. From the review of the above study, POPSST enhanced achievement of senior secondary school students. Thus, the present study seeks to determine the effect of two constructivist instructional models on junior secondary school students' achievement and retention in mathematics with the view to seeing their obvious implications.

Furthermore, Okeke (1997) worked on gender and school location as Factors of Students Difficulty in Secondary School Geometry. This study was carried out in Nsukka and Obollo Afor Education zone of Enugu State with the

main purpose being to explore gender and school location-related differences with respect to difficulties in geometry among students. The study adopted an analytic survey design with a sample of one thousand (1000) senior secondary three (SS 3) students, made up of 492 males and 508 females. The instrument used for the study was Test On Secondary School Geometry (TOSSG) and was developed by the researcher. Research questions were descriptively answered using percentages, means and standard deviation, while hypotheses were tested at .05 level of significance using Z-test statistic.

The result of the analysis showed that the performance of the students was generally low. The poor performance of students was more in such aspects as construction and locus, geometric proofs, to mention but a few. The analysis also showed that gender and school locations are significant predictors of students' difficulties in geometry. Boys experienced less difficulty than girls while urban students experienced less difficulty than their rural counter parts. It was then recommended that geometry be given more prominence in the mathematics program of colleges of education so as to improve the quality of geometry that mathematics teachers will possess. It was also recommended that MAN in the interim should organize refresher courses for mathematics teachers to enable them update their knowledge.

This study is relevant to the present study because it sought to establish the achievement of male and female students relative to method. The present study seeks to find if the result will be the same or otherwise.

In another study, Iji (2003) explored the effects of Logo and Basic programmes on Achievement and Retention in Geometry of Junior Secondary School Students. The study took place at Ahoda Education Zone of Rivers State. The main purpose of the study was to determine the efficacy of the use of Logo and Basic programme methods in teaching Junior Secondary geometry in Nigeria. The design of the study was quasi – experimental in nature and it took a sample of 285 JSS 1 students drawn from three out of six of the coeducational schools that have computers in Ahoda Education Zone. Two instruments were used for the study. These were:

- i The Geometry Achievement Test (GAT)
- ii Geometry Retention Test (GRT)

Data analysis techniques employed for the study were mean, standard deviation and ANCOVA.

The result revealed that students taught with LPM and BPM achieved higher than those taught with CPM. Also, the low achievers improved in the level of their geometry achievement among others. It was recommended among other things that since the methods were relatively new; they should be incorporated in the mathematics curriculum for the pre-service teachers programme. This will help them to learn and use the LPM and BPM in their teaching.

The above reviewed study is similar to the present study in the sense that both of them are rooted to relatively new approaches. Again, the two studies involve two treatment groups and one control group. The two are also related in

comparing achievement and retention of male and female students. Will the present study produce similar results as in the above reviewed study?

In another development, Ogwuche (2002) investigated Age and Sex as correlates of logical reasoning and mathematics Achievement in Ratio and Proportion tasks. This study was done in Zone C Education Zone of Benue State. The main purpose of the work was to correlate logical reasoning with pupils' achievement in mathematical ratio and proportion tasks. The study adopted correlational research design. It specifically sought to establish the relationship that exist between logical reasoning and mathematics achievement in ratio and proportion between primary six pupils and JSS 1 students. Ogwuche took 488 all primary six pupils and JSS 1 students from 16 schools in the area of the study.

Two instruments were used for the study:

- i Test of Logical Thinking (TOLT)
- ii Mathematics Achievement Test on ratio and proportion.

In the study, it was found out that in TOLT, the male students performed better than their female counterparts; that there was significant difference between the achievement of male and female students in mathematics achievement test on ratio and proportion. Specifically, male students performed significantly better than their female counterparts. It was then recommended among other things that workshops, seminars and conferences be organized to enable teachers implement the findings of the study.

The above study is similar to some extent to the present study. In the first instance, the content areas and the subjects used for the study are almost equivalent. The above reviewed study differs from the present one in terms of design. Also, the study was not rooted in constructivist framework. However, the present study wishes to explore constructivist instructional models with the view to determining whether the result will be in line with the above findings of significant difference in achievement of male and female students or not.

As the search for variety of instructional approaches that could facilitate and enhance achievement and retention continues, it is evident that the mathematics educators are concerned with positive change in the method of instruction. It is based on this, therefore, that this study is aimed at complementing the earlier on efforts. Will the result of this present study help to positively change the teacher's method of presenting instructions for the realization of the much-desired changes?

### **Studies on Constructivist Models/Approaches**

In this section, some relevant empirical studies on constructivist instructional models/approaches were reviewed. For instance, Lagoke, Jegede and Oyebanji (1997) have demonstrated the use of Analogy models in facilitating conceptual change. Lagoke et al were interested in science concept attainment through the use of environmental analogies. The sample for the study was 248 Senior Secondary School II students selected from two schools in Zaria township of Kaduna State. The experimental group was instructed using the environmental analogies in ALTF design based on the modified teaching – with – analogy (TWA)

models together with the enriched analogical linkages derived from environment familiar to the students. The control group was instructed using the expository method, which had not enriched analogical linkages. The result indicated that those taught with analogies performed better than those taught without analogies.

In a similar study, Iloputaife (2001) investigated the effects of analogy and conceptual – change instructional models on physics achievement of students taught simple electric circuit in Enugu State Senior Secondary Schools. The study adopted a non – equivalent pretest posttest quasi – experimental design. Percentages mean and standard deviation scores were used to answer the research questions while analysis of covariance (ANCOVA) was used to test the hypotheses at the alpha level of .05. The study among other things revealed that 93 percent of students used for the study who have alternative conceptions before formal physics instructions took place shifted from their initial conception positions to the standard scientific views after the treatment. The study also revealed that analogy and conceptual change instructional models have almost identical pattern in facilitating and enhancing conceptual change and achievement in physics.

The study cited above is related to the present study in terms of design. It also involved two treatment groups and one control. The study is also rooted in the constructivist approach. However, the differences lie in the fact that it was conducted in the area of physics while the present study is in mathematics. More seriously, the instrument was on multiple-choice questions and this does not agree with the ideology of constructivist instructional approaches of structuring learning



around big ideas or broad concepts that provide multiple entry points for students thereby making multiple paths to the same end equally valid. Since the defect in the structuring of the instrument might have affected its reliability, thus the findings, one then wonders what the findings will be in this present study.

In another development, Mandor (2002) investigated the effect of constructivist based – model of instruction on acquisition of science process skills among Junior Secondary School students. The study was carried out using a non – equivalent, pretest posttest quasi – experimental design. Three hundred and twenty two (322) JSS II integrated science students from six schools in Calabar Education Zone of Cross River State participated in the study. Percentages, mean and standard deviation were used to answer the research questions while analysis of covariance (ANCOVA) was used to test the null hypotheses at .05 level of significance. The result showed that constructivist – based model of instruction enhanced students acquisition of science process skills better than the conventional method.

The two studies are similar in the type of research design and sample for the study. They however differ in treatment levels and sampling techniques. Also, unlike Mandor (2002) this present study explored students' retention relative to instructional models.

In another study, Madu (2004) explored the efficacy of a constructivist instructional model PEDDA on students' conceptual change and retention in physics. The study adopted the non – equivalent control group design using two

hundred and four (204) SS II physics students in Nsukka Urban of Enugu State. The main purpose of this study was to determine empirically the effect of constructivist based instructional model PEDDA relative to student's conceptual change and retention in current electricity. The major instrument that was used for pretest, posttest and delayed posttest (retention test) was Physics Concept Test (PCT). Data were analysed quantitatively for overall change using mean, standard deviation and analysis of covariance (ANCOVA) while the qualitative analysis was carried out for specific trace change using frequency and percentages.

The result revealed among other things, that the PEDDA model facilitated conceptual change and enhanced retention of physics concepts and gender influenced the students' conceptual shift from alternative/no conception to scientific conceptions. It was recommended among other things that physics teachers be trained in the effective use of PEDDA as conceptual change pedagogy, and that physics teachers should design activities that will challenge students' systematic conceptions.

The two studies are similar in the type of design. They differ in scope, sample and sampling techniques. Unlike Madu (2004), this present study seeks to determine effects of two constructivist instructional models relative to retention.

Still reviewing studies on constructivist models, Ogbonna (2004), investigated the effects of constructivist instructional approach on Senior Secondary School Students achievement and interest in mathematics. The study adopted a quasi – experimental design referred to as pretest posttest control group

design, using Umuahia Education Zone of Abia State. One hundred and thirty SS 1 students participated in the study. Two different instruments were used for data collection. These were:

- i Mathematics Achievement Test (MAT)
- ii Quadratic Equation Interest Scale (QEIS)

Mean and standard deviation were used to answer the research questions while the null hypotheses were tested at .05 level of significance using analysis of covariance (ANCOVA).

The findings of the study revealed the mean achievement scores of male and female students taught mathematics using IEPT constructivist instructional approach was higher than those taught using conventional method. The study also revealed that instructional approach is a significant factor in enhancing students' interest in mathematics. It was recommended that mathematics teachers should be encouraged to change their orientation in terms of teaching and learning by adopting this approach in order to improve their students' performance. Furthermore, it was also recommended among other things that mathematics educators and curriculum planners should structure learning in such a way that classroom activities do not only motivate interest but also sustains this interest throughout and beyond the lesson periods.

The present study is related to the one reviewed above because both of them are rooted to the same constructivist instructional model. They are also related in the type of design and area of study. They however differ in scope and sample.

Also unlike Ogbonna (2004), the present study intends to ascertain if retention has anything to do with constructivist instructional models.

In summary, it has been observed that constructivist instructional models facilitate and enhance achievement, but there is no evidence of any study of the effect of any two of the models relative to JSS student' achievement and retention in mathematics, hence the present study explored this situation.

### **Gender Differences and Academic Achievement**

In a study, Erinoshio (1994) took a sample of 16,806 students made up of 5,564 boys and 11,242 girls from 25 out of 41 Federal Colleges in Nigeria. This consist of 15 all girls schools and 10 coeducational. She discovered that between 1985 and 1990, the most significant disparity between boys and girls in WAEC results was in mathematics, with 55.5% boys scoring various grades up to pass grade and for the girls, it was 44.5%. It was found that among all science subjects, physics ranked first in terms of achievement among girls with mathematics ranking last. For the boys, mathematics ranked first followed by biology.

The study appears to be sectionalized. For instance, the researcher should have included some state owned schools in the sample. This would have given a wider scope and a more valid generalization especially given the fact that federal government schools have better facilities than state schools. The study was also based on second hand data i.e. WAEC results. All these make the differences between it and the present study which is focused on one aspect of mathematics – number and numeration with a view to differentiating achievement along gender.

In another study, Harbor-Peters (1993), investigated aspects of Further Mathematics curriculum that present difficulties to graduating senior secondary school students. Specifically, the study sought to ascertain content aspects such as Pure Mathematics, Mechanics or Statistics that students find difficult to understand. A purposive sample of 381 SS III students who study the further mathematics option were selected from ten secondary schools in Enugu Education Zone for the study. The instrument used was a 100-structured response, multiple choice achievement/diagnostic test items developed by the researcher from the entire content of the further mathematics. Result of the study indicated that students have difficulty with almost all the topics in further mathematics. The study concluded that since most of the topics had mean achievement below 50%, it is evident that those topics presented difficulties to the students. The reviewed study is different from the present study both in design, scope, sample and sampling techniques and instrument for data collection. Again, unlike Harbor-Peters (1993), the present study sought to see if gender has anything to do with students' achievement in number and numeration.

Still on gender disparity, Harbor-Peters (2001) asserted that gender issues in mathematics had been male - stereotyped since it has been regarded as abstract difficult and has attributes which boys were attracted to. In an attempt to explore the interaction effect of gender and achievement, it will be recalled that Harbor-Peters (1993) designed a study in which male and female teachers taught male and female students. The result of the findings was that male students performed

significantly better than their female counterparts generally. It was also revealed that male students taught by a female teacher performed significantly better than the male students taught by male teachers. On the hand, it was observed that female students taught by male teachers performed significantly better than those taught by a female teacher.

Contrary to the above findings, a study on memory tasks which required recall digits by Tyler as cited in Okeke (1997) found that females were superior to males. In the same vein, Akukwe (1991), using 450 students in Imo State with Continuous Assessment Cumulative Average Score (CASAS) as an instrument, found among other things, that girls achieved higher than boys. Similarly, Robert (1958) stated that sex differences in arithmetic reasoning and spatial relations have been found. Men are superior to women in these areas, while women excel in tasks requiring verbal ability and memory. According to Martin as cited in Okeke (1997), it may be said that since "boys' superior mean score on the deduction items is not due to intelligence, reading comprehension, reading preferences or activities, personality traits such as applicability and initiative, practice effect or knowledge of pertinent principles, it is only reasonable to conclude that the higher scores on the deduction items are due to boys' superior ability to transfer". Will boys and girls experience the same achievement in the present study? This is the focus of the study.

Contributing, Amogu (1993), worked on sex and attitudes as factors in mathematics performance in Junior Secondary School in Katsina Local Government Area of Katsina State. In this study, 4 out of 15 secondary schools were randomly selected to constitute the population of the study. A sample of 240 students (120 males and 120 females) was used. A Teacher Made Test (TMMT) was used as an instrument. The analysis of variance (ANOVA) was used as statistical tool for data analysis. The result of the study indicated that there was no significant difference between the mean performance of male and female students on TMMT. The conclusion was that sex of the students in Junior Secondary School seems not to affect students' achievement in mathematics. Will the present study show similar result since it is related to the above reviewed study in sample.

In support of no significant difference, Ozofor(1993), worked on the effect of teaching methods (Target Task Approach) on students achievement in conditional probability. Sex was a factor in the study. A sample of 240 SS III students was used and ANOVA employed for data analysis. Intact classes were used for the study. The result was of no significant difference between mean performance of male and female students taught using the target task approach. This result might have been as a result of the effects of the experimental treatment on the students. However, it is possible, that some of the findings of the study might have been affected by some validity threats

such as the statistical tool used, for instance, the use of ANOVA as a tool for data analysis. This study used intact classes and as such, analysis of covariance (ANCOVA) would have been used to establish equivalence in the initial differences existing among the students before the commencement of the test.

From the foregoing, one can see that inequality exists in education particularly in science and mathematics. Gender disparity in achievement has been attributed to natural, social, cultural and even psychological factors. Based on the findings of research results, one can infer that the issue of gender on science and mathematics achievement remains inconclusive. One wonders then what the findings of the present study will be.

### **Summary of Review**

This chapter has examined the literature of previous studies, suggestions and recommendations that are relevant to the present study. It gave a brief account of the current status of teaching and learning of mathematics.

The literature revealed that despite all recognition accorded mathematics at all levels of the educational system. Students' achievement has remained unimpressive. This poor achievement, the literature continued



could be summarily attributed to students' factors, curricular factors and teachers' factors to mention but a few.

The review emphasizes teachers' incompetence and use of inappropriate teaching approaches/models as one of the major contributory factors. However, the use of innovative constructivist instructional models have been revealed to have proved efficacious in other sciences and mathematics, hence the present study focused on exploring the effects of some of the constructivist instructional models relative to Junior Secondary School students' achievement and retention in mathematics. In line with the above desire, literature was reviewed on concept and perspective of constructivism, the principles that guide constructivist teachers, some constructivist models and attributes of constructivist teachers. Also reviewed were concepts of learning, forgetting and retention. It was revealed that it is evident, retention has a strong tie with achievement and inappropriate instructional approach leads to lack of understanding of concepts and this invariably leads to forgetting and poor achievement.

Therefore, to facilitate retention of mathematical concepts, innovative instructional models such as IEPT and TLC were explored. This is the focus of this study. In the empirical studies, however, students' achievement in mathematics was reviewed along side approaches/models adopted. It was asserted that since the search for variety of instructional models that could

facilitate achievement and enhance retention continues, it is evident that the mathematics educators are concerned with positive change in the method of instruction. It is based on this, therefore, that this study aimed at complementing the earlier on efforts, hence the review on studies on constructivist instructional models. The review showed that constructivist instructional models facilitate and enhance achievement and retention but there is no evidence of any study on the effect of any two of such models relative to JSS students' achievement and retention in number and numeration, which the present study explored. The review also examined empirical studies on gender and achievement in mathematics. From the studies reviewed, it is clear that the issue of gender and achievement of students in mathematics is not yet closed, as researchers were not conclusive at these points. There is therefore, room for further investigation principally to explore gender related differences with respect to mathematics achievement.

## CHAPTER THREE

### RESEARCH METHOD

This chapter deals with the design of the study, area of the study, the population of the study, sample and sampling techniques, instrument for data collection, validation of the instrument, treatment/experimental procedure, control of extraneous variables, method of data collection/scoring, and method of data analysis.

#### **Design of the Study**

The research design for this study is a quasi - experimental. Specifically, the study is a non - equivalent control group design. The design is considered appropriate for the study because intact classes were used instead of randomly composed samples. The use of intact classes was to ensure non-alteration of regular class periods since secondary school authorities in Abia State do not allow their lesson periods to be altered.

The study made use of two treatment groups and one control group. The design is presented thus:

1	$0_1$	$X_1$	$0_2$	$0_3$	IEPT
2	$0_1$	$X_2$	$0_2$	$0_3$	TLC
3	$0_1$	$X_3$	$0_2$	$0_3$	CG

Where

$0_1$  = Test before treatment

$X_1$  = Treatment using IEPT constructivist instructional model

$X_2$  = Treatment using TLC constructivist instructional model

$X_3$  = No treatment (Control group)

$O_2$  = test after treatment

$O_3$  = Retention test

---- Indicates that the three groups are not equivalent.

### **Area of the Study**

This study was carried out in Umuahia Educational Zone of Abia State. Abia has three Education zones namely; Aba Education Zone, Ohafia Education Zone and Umuahia Education Zone. Through random sampling, Umuahia Education Zone was selected for the study. The zone consist four LGAs namely: Umuahia North, Umuahia south, Ikwuano and Umunneochi. There are 43 secondary schools in the zone, 16 are in Umuahia North, nine are in Umuahia South, nine are in Ikwuano and nine are in Umunneochi LGAs.

### **Population of the study**

The target population of this study was 3699 junior secondary two (JS II) students of 2004/2005 academic session in 36 co-educational public secondary schools in Umuahia Education zone of Abia State. The JS two students were used because they are in their foundation stage of their secondary school contact with the topic 'number and numeration'. The population was limited to only co-education state schools with a good number of male and female students. (See appendix A for Analysis of JS two Erollment within Umuahia Education Zone According to sex, L.G.A and school type as at 2004/2005).

### **Sample and Sampling Technique**

Within Umuahia Education Zone, there are 36 co-educational secondary schools. A purposive sampling technique was employed to sample seven co-educational schools that have the highest population (190 and above) and with a minima difference between the number of male and female students. Out of the seven schools, random sampling technique was employed to select three. To do this, the seven schools were assigned numbers 1-7 and these numbers were written on respective paper slips. Each slip was rolled into a paper ball and the paper balls were well mixed in a container (a basket), then blindly, three of the balls were drawn randomly without replacement. Out of the three schools selected, one was randomly assigned to IEPT treatment group. Out of the three schools selected, one was randomly assigned to LEPT treatment group, one to TLC treatment group and finally, the third school was for the control group. In each school, two intact classes were drawn through simple random sampling technique. Precisely, simple balloting was again used to select the intact classes from the schools.

### **Instrument for Data Collection**

Three research instruments and three lesson plans were developed for the study. These instruments are the Pre Mathematics Achievement Test (PREMAT), this was used for the pre – test. The second one is the Post Mathematics Achievement Test (POSTMAT), this was used for the post – test. The third instrument is the Delayed Post – Test (DELPOSTTEST), this is to be used for the Mathematics Retention Test (MRT). The three instruments are equivalent or

parallel and consist of (10) essay question each. The structure of the questions is different, though the items are testing the same content areas. The items were developed by the researcher from the content areas taught in the lessons. The content areas are:

- 1 Solving problems on direct and inverse proportion using unitary method;
- 2 finding ratio of two quantities in the same unit;
- 3 using the idea of ratio in sharing quantities;
- 4 solving everyday problems involving percentages;
- 5 applying the idea of rates when solving word problems.

To develop the instruments, a test blue print or table of specification was constructed using the five units taught during the period of the study. The table of specification is presented in table 1.

Table of Specification or Test Blue Print on MAT

UNITS	Educational Objectives				Total
	Lower Order Questions		Higher Order Questions		
	Knowl 40%	Comp 20%	Appl 20%	Anal 20%	
1 Solving problems on direct and inverse proportion using unitary method;	-	2	-	-	2
2 Finding ratio of two quantities in the same unit	1	-	-	1	2
3 Using the idea of ratio in sharing quantities	-	-	2	-	2
4 Solving everyday problems involving percentages	1	-	-	1	2
5 Applying the idea of rates when solving word problems	2	-	-	-	2
Total	4	2	2	2	10

The items of the instruments were based on JS II mathematics identified in New General Mathematics, Book 2 by Channon, Smith, Head, Kalejaiye and Macre (1981). These identified units were based on Junior Secondary School mathematics curriculum for class two (Federal Ministry of Education, 1979). The basic guiding principle that was employed in developing the test blue print was the objectives of the content areas studied. The items of the instrument were developed to cover lower order questions on knowledge and comprehension of the cognitive domain and questions involving higher thinking processes covering application and analysis. Since the subjects of this study are JS II students, only the first four levels of the cognitive domain of Bloom's educational taxonomy are applicable.

### **Validation of the Instrument**

The instrument was validated by adopting the following procedures during the study. The test blue print was face validated by three experts in Measurement and Evaluation and two experts in mathematics Education. The achievement test was also subjected to validation by the same experts specified above. The validation exercise was conducted in the following manner: copies of the title of the study, purpose of the study, research questions, hypotheses, the test blue print and the achievement test were sent to the above specified experts. They were requested to do the following:

1. see whether the questions tested the objectives of the lesson;
2. see whether the clarity of questions were ensured and the language appropriate to the class level;

3. check whether the instruments were relevant and appropriate to the age and level of students being tested;
4. add any other useful information which would help to ensure the validity of the instruments.

The lesson plans, achievement test and marking guide/scheme were sent to two secondary school mathematics teachers with MED in Mathematics Education for validation and vetting. They were asked to check the adequacy of the lesson plans with regards to the attainment level of the students, vet the marking guide in addition to the validation exercise enumerated above. The advice of the experts and that of the teachers helped the researchers to delete, modify and select the final set of test items for the study. For instance, there were (15) items for validation in each parallel test initially, some of the items that were grouped as lower order questions were found to be of higher order and vice versa. At the end of the exercise, (10) items for each test were selected for the study.

### **Trial Testing**

A trial testing of the instrument was carried out by the researcher involving a total of 90 JS II students of a co-educational secondary school situated in the same environment where the study was carried out but was not involved for the study. 30 students were used to trial test each of the instruments. The trial testing enabled the researcher to determine the actual time for the test. The times taken by the first, middle and last subjects to complete the test were recorded and the average time found. Eventually, the test lasted for 1 hour and 30 minutes.



### **Reliability of the Instruments**

The necessary reliability coefficients of the PREMAT, POSTMAT and DELPOSTMAT were determined. The 30 scripts of the students used in the trial testing of each of the instruments were photocopied and copies given to three different raters. Scores of these raters for each instruments were correlated to find their coefficient of equivalence using Kendall's W - Test. Kendall's W – Test was considered appropriate because three equivalent test were involved with more than two raters. The PREMAT has a kendall's coefficient of concordance of .707, the POSTMAT has .895 while the DELPOTTEST has .693.

### **Co-ordination of Teachers for the Conduct of the Study**

The regular mathematics teachers of the selected schools for the study were coordinated to assist in the study. This was done for one week before the commencement of the study. The co-ordination exercise was based on:

- (i) The purpose of the study.
- (ii) The content area to be taught.
- (iii) The use of the lesson plans
- (iv) The general conduct of the study.

For the three regular mathematics teachers coordinated, one taught the treatment group 1(IEPT), the second teacher taught the treatment group 2 (TLC) and the third teacher taught the control group (CG). The teachers were advised to observe all the normal classroom procedures such as entry behaviour, set

induction, and so on. The teachers were advised to use the same length of time (four weeks) to teach the contents to the groups.

### **Treatment/Experimental Procedure**

Two levels of treatment conditions were used for this study. These treatment conditions were IEPT and TLC. The IEPT is a four phase constructivist instructional model. The phases are:

1. invitation – Recognizing the problem through observation and the decision to tackle such problem.
2. exploration/Discovery – In this stage, several attempts would be made to solve the problem (trial and error phase), but perseverance is needed to continue.
3. proposing explanation and solution – when one arrives at the solution, then information would be communicated to others, that is, the explanation stage.
4. Taking action – This phase is the application stage where new knowledge is transferred to develop products and produce ideas.

At first, the students are involved in the understanding of the mathematical problem. The students try to identify the basic knowledge, relevant data and unknowns in the problems. The students also try to analyze, at times by making pictorial representation, forming equations. In fact, they try to recognise the problem through observation and then decide to tackle it by going to search in the long-term memory for a suitable plan or procedure for solution. If the plan or

procedure devised is well articulated, solution could be completed quickly with few errors. Finally, new knowledge is transferred during application (see appendix C for IEPT constructivist instructional sample lesson plan).

The other treatment condition, TLC is a three-phase constructivist instructional model published by Akin and Karplus (1962). First, the students are provided with an open ended opportunity to interact with selected materials or problems. The aim of this initial lesson is for students to generate questions while trying to examine and analyze the problem and this leads to “discovery”. Next, the teacher provides “concept introduction” lesson which aims at focusing the students’ questions, guiding them to recall related problems and forming equation and the like from worded problems and so fourth. Finally, comes the “concept application”. During concept application, students will work on new problems with the potentials for evoking a fresh look at the concept previously studied (see TLC constructivist instructional sample lesson plan in Appendix C). Both models (IEPT and TLC) are rooted in constructivist teaching pedagogy. The IEPT treatment is identical to TLC in terms of content, basic instructional objectives and mode of evaluation. The major difference is the instructional activities.

Before the onset of the experiment, the pretest on mathematics achievement (PREMAT) was administered by the coordinated regular mathematics teachers. This was to ascertain the level of achievement of the students. After the pretest, the regular coordinated mathematics teachers started the experiment in their respective schools. The researcher visited the school on regular bases for routine

checks. This was to make sure that the participating mathematics teachers adhered strictly to the lesson plans written by the researcher. The experiment was conducted during the normal school periods following the normal time table of the school. The content areas were taught and covered within the second, third, fourth and fifth weeks. The post-test on achievement which is a parallel test to the pre-test was administered by the same teacher immediately after the four weeks of teaching. Also a delayed post test (retention test) was administered to the three groups to assess the retention level of the students. Data to be collected from the exercise were used to answer the research questions and test the hypotheses stated for the study. The experiment lasted for seven weeks and the students in the experimental and control groups were taught under the same experimental conditions.

### **Control of Extraneous Variables**

The researcher adopted the following measures to ensure that extraneous variables which may introduce bias into the study were controlled.

### **Teacher Variables**

In order to control the errors which might arise as a result of teacher differences, the researcher organized a one week co-ordination exercise for the regular mathematics teachers of the classes that were selected from schools sampled for the study. Separate co-ordination was organized for mathematics teachers in the treatment and control groups. The coordination helped in establishing a common instructional standard among the mathematics teachers. All

topics for the study were treated in details during co-ordination. The researcher used the opportunity of the co-ordination to detect individual problems of the teachers that may introduce errors to the study. The researcher emphasized that everyone involved in the study should adhere strictly to the lesson plans to ensure uniformity.

### **Instructional Situation Variable**

To ensure that instructional situation is the same for all the schools selected for the study, the researcher issued out instructional guides to the teachers in all the groups. Teaching and testing were conducted in all the classes of JSS II in the various schools selected for the study and not just in the intact classes drawn. This was to avoid Hawthorne effect (a situation in which research subjects behaviors is affected not by the treatment per se, but by their knowledge of participation in the study) and Novelty effect (increased interest, motivation, or participation on the part of subject simply because they are doing something different). Pretest, Post-test, and delayed Post-test were administered to all the classes but data for the study were restricted to the intact classes drawn for the study.

### **Intergroup Variable**

Intact classes were used for the study and this implied that initial equivalence may not have been achieved for the research students in the three groups of experiment and control. In order to eliminate the errors of non-equivalence arising from the non-randomization of the subjects, the researcher

used analysis of Covariance (ANCOVA) for data analysis. This corrected the initial differences among the research subjects.

### **Subject Interaction**

Because of the awareness of the possible interaction between the experimental and control groups, the researcher made sure that one group was selected from each school. Again, participating mathematics teachers were strongly instructed not to give comprehensive or elaborate notes, not to give test of any kind, not to give any assignment until the end of the experiment. This was to reduce the errors that will arise from interactions and exchange of ideas among research subjects from the three groups in the same school or home.

### **Method of Data Collection/Scoring**

The pre-mathematics achievement test (PREMAT) was administered to the students in the three groups before the commencement of the experiment and no feedback on the test was given to the students. Scores of the students were recorded and kept aside for use after the experiment by the researcher. At the end of the experiment, the post mathematics achievement test (POSTMAT) were administered to the three groups as well. For each of the groups, data of pretest and post test were recorded separately. The delayed post-test (retention test) was administered after two weeks the post-test has been administered. The PREMAT, POSTMAT and DELPOSTMAT (MRT) were scored out of a maximum mark of 50% and a minimum of zero (0) using the marking guide (See Appendix H, I, J for the marking guides).

**Method of Data Analysis**

Data collected with the aid of the testing instruments were analyzed with respect to the research questions and hypotheses formulated for the study. Means and standard deviations were used in answering the research questions while analysis of covariance (ANCOVA) was used to test the hypotheses formulated for the study at 0.05 level of significance.

## CHAPTER FOUR

### RESULTS

In this chapter, the data collected from the study were specifically analysed and results presented. These results were presented in the following order:

- i. The test of the assumptions for analysis of covariance (ANCOVA) which are:
  - a. The linearity between the dependent variables and covariates.
  - b. The homogeneity of regression.
- ii. Answering the research questions asked as well as testing the hypotheses formulated for the study as follows:
  - a. Effects of methods (models) and gender on students' achievement in JSS2 Mathematics.
  - b. Effects of methods (Models) and gender on students' retention in JSS2 Mathematics.

#### **Preliminary Analysis of the Assumptions of ANCOVA**

The analysis of relationship between covariates and the corresponding dependent variable

HO: There is no significant difference between the covariates and the corresponding dependent variables at  $P \leq .05$ .



**Table 1**

Correlation Coefficient (r) between the covariates and their dependent variables

Covariates	Dependent variables	Treatment		Control	Pooled
		IEPT	TLC		
Pre-Achievement Scores	Post- Achievement Scores	.56	.70	.30	.52
Post Achievement Scores	Delayed Post-test Scores	.67	.75	.42	.61
	N	90	100	100	290
Critical r		.20	.20	.20	.20

Table 1 shows that the value of r between the covariates and their respective dependent variables range from .56 to .75 for treatment groups and .30 to .42 for the control group. The pooled r ranged between .52 to .61. The critical value of r required for significance at .05 level is .20. Since .20 is less than all the observed values of r which is .61, the null hypothesis of no significant relationship between the covariates and their respective dependent variables is rejected. This implies that each of the covariates shows a significant relationship with the corresponding dependent variables or that the covariates are linearly related with their respective dependent measures. Therefore, the data for the study satisfy the linearity assumption for the use of ANCOVA.

#### **The Homogeneity - of - regression**

HO: The difference between the population regression coefficients of the treatment and control groups is not significant ( $P < .05$ ) i.e.  $\beta_1 = \beta_2$

**Table 2**

Analysis for the test of Homogeneity of regression assumption

Variables	SS <sub>w</sub>	SS <sub>hreg</sub>	F Ratio	F Prob
Achievement	9643.5919	15730.8138	.4415	.5075 NS
Retention	8474.7677	13570.4890	.8407	.3604 NS

NS = Not significant.

Table 2 above shows that the F probability for population regression coefficient for the experimental groups and the control group are .5075 and .3604 for achievement and retention scores respectively. These are greater than already set alpha  $\alpha$  level of .05 level of significance for dependent variables. This implies that the null hypothesis of no significant difference between the treatment and control groups is not rejected. This means that the population regression coefficients for the treatment and the control groups are homogeneous for each of the two dependent variables. Consequently, the second condition for the use of ANCOVA is also met. Hence, the justification for the use of ANCOVA in the analysis for achievement and retention scores.

### **Research Question 1**

How do the mean achievement and standard deviation scores of students taught mathematics using IEPT and TLC constructivist instructional models differ from the mean and standard deviation scores of students taught using conventional method?

**Table 3**

Mean Achievement and Standard Deviation Scores of the Control and Experimental Groups

Methods	Pre-Mat		Post Mat		No of Students
	Mean	SD	Mean	SD	
Control Group CMT					
CMT	10.4200	1.0126	23.4600	5.745	100
Exp. Groups IEPT					
IEPT	13.3111	4.6559	29.4222	7.4290	90
TLC	13.7000	4.6958	30.2800	6.7421	100

Table 3 shows the mean scores and standard deviations of the students in the control and experimental groups pretest and posttest. In table 3, the mean PREMAT score for the control group (CTM) is 10.42 with a standard deviation of 1.01, while those of the experimental groups IEPT and TLC are 13.31 and 13.70 respectively. Their standard deviations are 4.65 and 4.69 in the same order. Also, the mean POSTMAT for CTM is 23.46 with the standard deviation of 5.71. The experimental groups have mean POSTMAT of 29.42 and 30.28 respectively. Their standard deviations are 7.42 and 6.74 respectively.

### **Research Question 2**

How do the mean retention and standard deviation scores of students taught mathematics using IEPT and TLC constructivist instructional models differ from the mean retention and standard deviations of those taught using conventional method?

The answer to this question is presented in table 4.

**Table 4**

Mean Retention and Standard Deviation Scores of the Control and the Experimental Groups

Methods	POSTMAT		DEL POSTMAT		No. of Students
	Mean	SD	Mean	SD	
<b>Control Group</b>					
CMT	23.46	5.71	25.78	4.10	100
<b>Experimental Groups</b>					
IEPT	29.42	7.43	33.14	7.05	90
TLC	30.28	6.74	32.51	6.18	100

Table 4 shows that for the control group, the POSTMAT means score is 23.46 with the standard deviation of 5.71. The experimental groups have the mean POSTMAT scores of 29.42 and 30.28 respectively.

Their standard deviations are 7.43 and 6.74 in that order. Again, the mean retention score of the control group is 25.78 with the standard deviation of 4.10 while those of the experimental groups of IEPT and TLC are 33.14 and 32.51 respectively. They have standard deviations of 4.10 and 6.18 in that order.

### **Research Question 3**

What are the differences in the mean achievement scores of male and female students taught mathematics using IEPT and TLC constructivist instructional models?

The answer to this question is presented in table 5.

**Table 5**

Mean Achievement Scores of Male and Female Students in the Experimental Groups

Methods	Sex	PREMAT		POSTMAT		No. of Students
		Mean	SD	Mean	SD	
IEPT	Male	12.26	4.16	27.67	6.94	43
	Female	14.28	4.91	31.02	7.57	47
TLC	Male	13.01	4.11	28.71	6.09	69
	Female	15.23	5.72	33.77	6.91	31

Table 5 shows the mean achievement of male and female students in the experimental groups PREMAT and POSTMAT. From the table, the PREMAT and POSTMAT mean achievement scores of male students in IEPT is 12.26 and 27.67 respectively while the females scored a mark of 14.26 in PREMAT and 31.02 in the POSTMAT. For male and female students in the TLC group, it could be seen that the males had a mean score of 13.01 in the PREMAT, and 28.71 in the POSTMAT. The females on the other hand were observed to have a mean mark of 15.23 in the PREMAT and 33.77 in the POSTMAT.

#### **Research Question 4**

What are the differences in the mean retention scores of male and female students taught mathematics using IEPT and TLC constructivist instructional models?

The answer to this question is presented in Table 6.

**Table 6**

Mean Retention Scores of Male and Female Students in the Experimental Groups of IEPT and TLC

Methods	Sex	POSTMAT	DEL POSTMAT	No. of Students
IEPT	Male	27.67	31.77	43
	Female	31.02	34.40	47
TLC	Male	28.71	31.22	69
	Female	33.77	35.39	31

From table 6, it could be observed that males in the IEPT group had a mean retention score of 27.67 in the POSTMAT and 31.77 in the DELPOSTMAT. From the same table, the female students in the same IEPT group made a mean mark of 31.02 in the POSTMAT and 34.40 in the DELPOSTMAT.

On the other hand, still from table 6, the male students in the TLC group scored a mean of 28.71 in the POSTMAT while in the DELPOSTMAT; they had a mean score of 31.22. The females on their own part scored a mean of 33.77 in the POSTMAT and 35.39 in the DELPOSTMAT. The difference in mean of male and female students in the IEPT POSTMAT is 3.35 in favour of females. In their DELPOSTMAT the difference in the mean score is 2.64, also in favour of the females. Furthermore, for the TLC group, the difference in the mean of male and female students in the POSTMAT is 5.06 in favour of females while for DELPOSTMAT; the difference in the mean score is 4.17, again in favour of the female students.

### Hypothesis 1

There is no significant difference in the mean achievement and standard deviation scores of students taught mathematics using IEPT & TLC constructivist instructional models and those taught using conventional method. This hypothesis was tested at  $P \leq .05$  probability level using analysis of covariance. The ANCOVA result for this hypothesis 1 is shown in table 7.

**Table 7**

Two-way Analysis of Covariance of the Control and Experimental Group Students on Mathematics Achievement Test Due to Method and Gender

Source of Variation	Sum of Squares	DF	Mean Squares	F	Sig of F.
Covariates	7776.920	1	7776.920	303.642	.000
PREMAT	7776.920	1	7776.920	303.642	.000
Main Effects	483.544	1	483.544	18.880	.000
Method	483.544	1	483.554	18.880*	.000 S
Gender	114.837	1	114.837	4.276*	.040 S
2-Way Interaction	1.865	1	1.865	.074	.785
Method x Gender	1.865		1.865	.074**	.785NS
Explained	7897.217	2	3948.608	147.024	.000
Residual	7681.081	286	26.857		
Total	15578.298	288	54.091		

\* = Significant at .05 level of probability

\*\* = Not Significant

Table 7 indicates that the value of the significance of F on achievement is .000 as against the already set alpha level of .05 at 1 degree of freedom. The null hypothesis of no significant achievement scores is therefore rejected. The

implication of this is that the experimental groups significantly achieved higher than the control group in the said JSS 2 mathematics contents.

However, it is necessary to ascertain which of the methods that caused the difference. To determine this, Scheffe test was carried out and the result is presented in table 8.

**Table 8**

Result of Scheffe Test for Post-Test Mean Achievement Scores of the Treatment and Control Groups

		<b>Group Mean (<math>\bar{x}</math>)</b>	<b>1-Control 23.46</b>	<b>2-IEPT 29.42</b>	<b>3-TLC 30.28</b>
Group	Mean ( $\bar{x}$ )				
1- Control	23.46			*	*
2 – IEPT	29.42				
3-TLC	30.28				

\* Denotes pairs of groups significantly different at .05 level. The result in table 8 shows that the difference in the post treatment mean scores of the treatment groups (IEPT and TLC) and the control group was significant at .05 level. The students who were exposed to IEPT constructivist instructional model had an overall mean achievement score of 29.42 as against the overall mean achievement score of 23.46 for the students in the control group. Students exposed to TLC constructivist instructional model an overall mean achievement score of 30.28 as against the overall achievement score of 23.46 for the students in the control group. The result also showed that there was no significant difference in the overall mean achievement scores of students exposed to IEPT and



TLC. This therefore implies that both IEPT treatment and TLC treatment are equally effective.

### Hypothesis 2

There is no significant difference in the mean retention scores of students taught mathematics using IEPT & TLC constructivist instructional models when and the mean retention scores of students taught using the conventional method.

**Table 9**

Two-way Analysis of Covariance of the Control and Experimental Group Students on Mathematics Retention Test Due to Methods and Gender

Source of Variation	Sum of Squares	DF	Mean Squares	F	Sig of F.
Covariates	6537.251	1	6537.251	311.367	.000
PREMAT	6537.251	1	6537.251	311.367	.000
Main Effects	972.654	1	972.654	46.337	.000
Methods	972.654	1	972.654	46.337*	.000 S
Gender.000	18.184	1	18.184	.749**	.388
2-Way Interaction	27.709	1	27.709	1.336	.249
Method x Gender	27.709		27.709	1.336*	.249 NS
Explained	6559.518	2	3279.759	135.033	.000
Residual	6946.055	286	24289		
Total	13506.055	288	46.896		

\* = Significant at .05 level of probability

\*\* = Not significant

Table 9 shows that the value of significance of F on mean retention scores is .000 as against the already set alpha level of .05 for 1 df. Since the value for significance of F is smaller than the set alpha value of .05, the null hypothesis of no

significant difference in the mean retention scores of the experimental and control groups is therefore rejected. This implies that the experimental groups significantly retained higher than the control group in the Mathematics contents taught.

However, it may be necessary to determine which of the methods that caused the difference. To determine this, Scheffe test was again used. The result of the Scheffe test is presented in table 10.

**Table 10**

Results of Scheffe Test for Delayed Post-Test Mean Retention Scores of the Treatment and Control Groups

		<b>Group</b>	<b>1-Control</b>	<b>2-IEPT</b>	<b>3-TLC</b>
		<b>Mean (<math>\bar{x}</math>)</b>	<b>23.46</b>	<b>29.42</b>	<b>30.28</b>
<b>Group</b>	<b>Mean (<math>\bar{x}</math>)</b>				
1- Control	25.78			*	*
2 – IEPT	33.14				
3-TLC	32.51				

\* Denotes pairs of groups significantly different at .05 level.

The result in table 10 shows that the difference in the mean delayed post treatment scores of the treatment groups of IEPT and TLC, and the control group was significant at .05 level. The students treated with IEPT constructivist instructional model had an overall mean retention score of 33.14 as against the overall mean retention score of 25.78 for students in the control group. Students exposed to TLC constructivist instructional model had an overall mean retention

score of 32.51 as against the overall retention score of 25.78 for students in the control group. The result also showed that there is no significant difference in the overall mean retention scores of students exposed to IEPT and TLC. This therefore implies that both IEPT and TLC treatments are of equal effects.

### **Hypothesis 3**

There is no significant difference in the mean achievement scores of male and female students taught mathematics using IEPT and TLC constructivist instructional models.

The ANCOVA result presented in table 7 indicates that the significance of F of mean achievement scores for sex is .040. This value is of course less than the already set .05 level of significance for 1 degree of freedom (DF). The implication of this finding is that the null hypothesis of no significant difference is rejected. This means that the mean achievement scores of male and female students in PREMAT and POST MAT are not equal, although both male and female students achieved significantly in the mathematics contents taught.

### **Hypothesis 4**

There is no significant difference in the mean retention scores of male and female students taught mathematics using IEPT and TLC constructivist instructional models.

From table 9, it could be seen that the value of the significance of F of the mean retention scores of male and female students is .388. This is greater than the already set alpha value of .05 level of significance for 1 df. The implication of this

is that the null hypothesis of no significant difference in the mean retention scores of male and female students is upheld. This means that the mean retention scores of male and female students in the POSTMAT and DELPOSTMAT though not equal (in the favour of the female students) is not statistically significant.

### **Hypothesis 5**

There is no significant interaction effect of IEPT and TLC constructivist instructional models and gender as measured by the post mathematics achievement test.

Table 7 reveals the ANCOVA result for the 2-way interactions. The result shows that the significance of F of the 2-way interaction (Method x Gender) is .785 as against .05 level of significance already set prior to the experiment. Since the significance of F value is greater than the already set alpha value of .05, the null hypothesis of no significant difference is upheld. This means that the interaction effect between methods and gender as measured by their mean mathematics achievement test scores is not statistically significant.

### **Hypothesis 6**

There is no significant interaction effect of IEPT and TLC constructivist instructional models and gender as measured by mathematics retention.

From table 9, it could be observed that the value of significance of F is .249 as against the already set alpha value of .05. Since the significant interaction between methods and gender is greater than the set alpha level of .05, the null hypothesis of no significant interaction between methods and gender is therefore upheld. This

means that gender does not appear to combine with instructional models to affect students' retention in the understanding of the mathematics contents taught.

### **Summary of Findings**

Based on the results of the analysis of data presented in this chapter, the following major findings are made:

1. There are differences in the mean achievement scores of the experimental groups (pretest 13.31; posttest 29.42 for IEPT & pretest 13.70; posttest 30.28 for TLC) and the control group (pretest 10.42; posttest 23.46) based on the mathematics contents taught.
2. The experimental groups (Post MAT 29.42; MRT 33.14 for IEPT & Post MAT 30.28, MRT 21.51 for TLC) retained higher the JSS two mathematics contents taught than the control group (Post MAT 23.46; MRT 25.78)
3. Male and female students in TLC group achieved higher than their male and female counterparts in the IEPT group, though the difference in the mean was not significant.
4. Male and female students improved significantly on the mathematics content taught during the study.
5. Male and female mean retention score in JSS two mathematics taught was statistically significant

6. The experimental females retain higher the mathematics contents taught than their male counterparts, although their mean retention scores was not statistically significant.
7. The interaction effect of methods and gender on students' mean achievement scores was not significant.
8. The interaction effect of methods and gender on students' mean retention scores was not statistically significant.

## CHAPTER FIVE

### DISCUSSIONS, CONCLUSIONS, IMPLICATIONS AND RECOMMENDATION

In this chapter, findings of the study are discussed. Also conclusions were drawn and educational implications discussed. Again, limitations of the study were highlighted and recommendations made in the face of the observable implications of the study. Finally, suggestions for further research and summary of the entire work are presented.

#### **Discussion of Findings**

The discussions are made under the following headings

- Students' achievement in mathematics due to the instructional methods/models adopted.
- Students' retention in Mathematics due to the instructional models adopted.
- Students' achievement in Mathematics due to gender.
- Interaction effects of instructional models and gender on mathematics achievement and retention of students group.

#### **Students Achievement in Mathematics Due to the Instructional Methods/Models Adopted**

In table 3, the mean achievement scores of the experimental groups in POSTMAT are 29.42 for IEPT and 30.28 for TLC. These are higher than the POSTMAT mean achievement scores of the control group (conventional teaching

method) which is 23.46. This indicates that the use of IEPT and TLC in teaching the mathematics content improved the subjects' achievement.

Also, Table 3 showed that achievement among the experimental and control group students are statistically significant. This is in favour of the experimental groups. In other words, the mean post achievement scores of the students in the experimental groups were higher than that of the mean post achievement scores of students in the control group. This was further confirmed by the result on table 7, which revealed that method was a significant factor on students' achievement in mathematics. Hence, students who were taught mathematics using IEPT and TLC constructivist instructional models achieved higher than those taught using the conventional model (CTM). This implies that instructional method/model used in teaching mathematics can produce differential effects on students with respect to mathematics achievement.

These findings seem to support the findings of other previous researches such as (Ozofor, 1993; Alio, 1997; Lagoke et al, 1997; and Ogbonna, 2004), where experimental treatments proved better than the non-treatment control group. This finding equally agrees with other previous similar study (Aiyedun, 2000), who confirmed that appropriate methods lead to students improvement in mathematics achievement.

The result of this study may have been due to the introduction of the innovative instructional models of which the phases involved active participation of the student. The control group on the other hand may have carried over the



societal syndrome of seeing mathematics as a difficult subject, and therefore, maintain such attitude in their early secondary education. They may as well not have seen any thing different in their teachers' methods of teaching mathematics at their present level of education and the previous level they have passed. This supposition may have been based on the level of commitment and enthusiasm noted in terms of responses to question, general participation in various small groups by the experimental group students.

Furthermore, the result of Scheffe test in table 8 showed that IEPT and TLC constructivist instructional models produced equal effects in the main achievement of students in mathematics contents taught during the study. This implies that the experimental groups benefited equally from the study. The implication of this result, put differently, is that each of the models (IEPT and TLC) is as effective as the other in enhancing students' achievement in the mathematics contents taught.

#### **Students' Retention in Mathematics Due to the Instructional Models Adopted**

The result in table 4 showed that the students in the experimental groups obtained mean POSTMAT scores of 29.44 and 30.28 for IEPT ad TLC respectively. These are higher than that of control group (CTM), which is 23,46. The mean DELPOSTMAT Score of control group students is 25.78, while those in the experimental groups (IEPT and TLC) are 33.14 and 3251 This showed that the experimental groups retained more of the mathematics content taught than the control group.

This was further confirmed by ANCOVA result of table 9, which shows that the experimental groups significantly retained higher than the control group in the mathematics contents taught. This finding is at variance with Obodo (1990) who established no significant difference among three models of teaching algebra in junior secondary two. It also differs from Eze (1992) who found no significant difference in students' retention.

However, this result is in agreement with Madu (2004), who found method to be significant in enhancing retention. Furthermore, the outcome of the Scheffe test in table 10 showed clearly that IEPT and TLC constructivist instructional models produced equal effects in the retention of students' relative the Mathematics contents taught. One may therefore conclude that each of these methods is as effective as the other in improving students' retention.

#### **Student's Achievement in Mathematics Due to Gender**

In table 5, the result indicated that the difference in the mean achievement scores of female and male students in the IEPT POSTMAT is 3.35. On the other hand, the difference in the mean achievement scores of the male and female students in the TLC PREMAT is 2.21 while the difference in POSTMAT is 4.46. This shows that both male and female students in both IEPT and TLC improved in their mean achievement scores based on the mathematics contents taught during the cause of the study. However table 7 showed that gender is a significant variance in students' achievement in mathematics and this has been statistically significant in this study in favour of the female students.

This result is in agreement with the findings in some studies (Akukwe, 1991; Okeke, 1997; Ogwuche, 2002), which indicated that there was significant difference between the achievement of male and female students in mathematics. This significance could be as a result of the effects of the two constructivist instructional models of IEPT and TLC of which TLC proved more efficacious than IEPT, although the difference was not statistically significant.

Furthermore, the findings of this study are in serious compliance with the findings of (Alio, 1997; Mkpa, 1997; Okonkwo, 1997) who found the girls to be superior to boys in the mathematics achievement.

However, the result of this present study is contrary to the findings of some earlier on studies, which revealed no significant effect of male and female students in mathematics (Ozofor, 1993; Olagunju, 1996; Aiyedun, 2000; Ogbonna, 2004).

#### **Students' Retention in Mathematics Due to Gender**

From table 6, the result indicated that the difference in the mean of male and female students in IEPT POSTMAT is 3.35 while in the DELPOSTMAT, the difference in their mean mark is 2.64, all in favour of the female students. For the TLC group, the difference in the mean score between male and female students in the POSTMAT is 5.06 and in the DELPOSTMAT, it is 4.17, all again in favour of the female students. From the same table 6, it could be observed that both male and female students in the two groups of IEPT and TLC improved in their mean retention scores as regards the mathematics contents covered during the

study. However, table 9 showed that gender is not significant in retention of students based on the mathematics contents taught during the study.

The result of this study disagrees with Iji (2003) who found out gender to be statistically significant in students' retention in mathematics. The result of no significant difference may be principally attributed to the effects of IEPT and TLC since both of them are rooted in the constructivist principles of teaching and learning. Finally, the major remarkable notice is that the male and female students improved significantly in their mean retention scores as was observed from the difference that existed between the POSTMAT and DELPOSTMAT.

#### **Interaction Effects of Instructional Models and Gender on Mathematics Achievement and Retention of Students**

From table 7, the result of the Analysis of covariance test of interaction revealed that, for the 2-way-interaction, the significance of F is 0.785 as against the already set alpha significant level of .05. Following the normal decision of upholding the null hypothesis if the set alpha ( $\alpha$ ) is smaller than the significance of F, the researcher concludes that there is no significant interaction between instructional models and gender on students' achievement in the mathematics contents taught during the study. The findings of this study agrees with Ogbonna (2004) who found no significant interaction effect of instructional model and gender on student's mathematics achievement.

On the other hand, from table 9, the result of the Analysis of covariance test of interaction showed that, for the 2-way-interaction, the significance of F value is .249 since this is greater than the significant level already set at .05, the conclusion

is that there is no significant interaction between instructional models and gender on student's retention. The implication of these findings is that achievement and retention for male and female students is consistent.

## **Conclusion**

Based on the result of the analysis of the data in this study, the following conclusions have been drawn.

- i. The use of IEPT and TLC constructivist instructional models enhanced significantly students' achievement in JSS two mathematics. Students taught with the conventional method achieved less.
- ii. The students taught with IEPT and TLC retained higher the mathematics contents taught than their counterparts taught using CTM. That is, the difference in mean retention score of the experimental groups was higher and statistically significant.
- iii. The difference between the mean gain achievement scores of the female and male students was statistically significant.
- iv. The use of IEPT and TLC constructivist instructional models resulted to the male and female students improving upon their mathematics achievement.
- v. The difference between the mean gain retention scores of male and female students was statistically not significant.

- vi. Female students achieved and retained higher the mathematics content of the study.
- vii. Although there is interaction effect between method and gender on achievement and retention, this was not statistically significant.
- viii. In general, the method have positive influence on the subjects of this study as indicated in their mean performances.

### **Implications of the Study**

The conclusions drawn based on the findings of this study have some obvious implications for the mathematics classroom. The results of this study may have provided empirical evidence in the comparative effects of IEPT and TLC constructivist instructional models in the teaching of mathematics the fact that IEPT and TLC were observed to have facilitated and enhanced students achievement in mathematics implied that mathematics curriculum planners and authors of mathematics textbooks might not hesitate the inclusion of these models.

Considering the fact that both male and female students improved greatly on their mean achievement and retention scores in the mathematics contents taught, the teacher training institution might have to play the role of including IEPT and TLC constructivist instructional models in their mathematics education method classes. This might help in no small measures to minimize the mass failure of students being currently experienced all over the nation. Since these methods have proved to be very effective, more especially the TLC model, the mathematics teacher will have the choice of which and when to explore any of the methods.

Having seen the comparative effects of the IEPT and TLC constructivist instructional models, it is evident that there is need for MAN, Ministry of Education and all other stakeholders in mathematics to organize workshops, seminars, conferences where the concepts of teaching mathematics with IEPT and TLC constructivist instructional models will be promoted.

### **Limitations of the Study**

The conclusion and generalizability of the results of the study is constrained by the following limitations:

- i. The sample for the study was limited when compared with the population of students in JS II in the area of study. This constituted a limitation to this study.
- ii. The students' ability levels and socio-economic status prior to treatment were not controlled in this study. This might have posed a limitation to this study too.
- iii. Even though the researcher did train the secondary school mathematics teachers used for this study whose academic qualifications are the same, other variables like teacher personality, temperaments, classroom environment, school type and location might have affected the result of the study.

## Recommendations

The findings of this study and the implications to education have necessitated the following recommendations:

1. Since IEPT and TLC models appear relatively new, they should be incorporated in the mathematics curriculum for the pre-service teachers programme. This will help the teachers to learn and use these models in the teaching of mathematics.
2. Also workshops should be organized for in-service mathematics teachers to enable them learn how to use IEPT and TLC constructivist instructional models. This will help them to be able to incorporate the models in their teaching.
3. Again ministries of education, state secondary school education boards and professional bodies such as Mathematical Association of Nigeria (MAN), National Mathematical Centre Abuja and Curriculum Organization of Nigeria (CON) should be involved in promoting these models of teaching as innovation in the teaching of mathematics. This they could do by organizing conferences, seminars and workshops for the serving teachers on the teaching of mathematics using IEPT and TLC. These professional bodies should further sponsor researches on effects of these models on other mathematics concepts.



4. Since textbooks are the major source of information and knowledge for the teachers and students, authors and publishers of mathematics textbooks could incorporate the IEPT and TLC phases in their worked examples in order to offer students the opportunity of learning even unguided.

### **Suggestions for further Studies**

The following suggestions for further studies are put forward:

1. Efforts should be made to conduct similar research at the senior secondary school levels using other mathematics concepts.
2. Other constructivist instructional models may be tried out to ascertain their comparative effects in teaching mathematics.
3. Study should also be conducted to examine the effects of IEPT and TLC constructivist instructional models on school location, and gender relative to students' achievement and retention in mathematics.

### **Summary of the Study**

This study was born out of the continuous poor achievement of students in mathematics as was reviewed in the literature. Among other factors responsible for the ugly trend was teachers' instructional approach. This study was therefore conducted to ascertain the effects of IEPT and TLC constructivist instructional models on JS II students' achievement and

retention in number and numeration with the view to finding out whether the result will assist in helping students to improve upon their mathematics achievement and retention.

Four research questions were asked and six hypotheses were formulated to guide the study. The study was restricted to the mathematics theme of Number and Numeration with JS II mathematics curriculum used as a guide. A total of 290 students, 160 of which were males and 130 were females from three coeducational secondary schools in Umuahia Education Zone of Abia State were used for the study. A quasi-experimental design of non-equivalent control group was adopted.

Out of the three selected schools, one school was randomly assigned to experimental group 1 (IEPT), the other to experimental group 2 (TLC) and finally, the last to control group (CTM). In each of the schools, two intact classes were randomly drawn. Three teachers were used, one each for the three schools.

Three testing instruments and three lessons plans were used. The testing instruments include mathematics Achievement test (PREMAT), Post Mathematic Achievement Test (POSTMAT) and delayed Post Mathematics Achievement Test (DELPOSTMAT). The reliability indices of these instruments were established using Kendall's W Test. The PREMAT, the POSTMAT, and the DELPOSTMAT had Kendall's coefficient of

concordance of .707, .895 and .693 respectively. Experts validated the instruments accordingly.

The data generated from the study were analysed using mean, standard deviation, analysis of covariance and t-test statistic. The result showed that students taught with IEPT and TLC achieved higher than those taught with CTM. Also that male and female students improved in their mathematics achievement and retention among others. These results have serious implications for mathematics teachers, author of mathematics textbooks, teachers training institutions and other stakeholders in mathematics education. Recommendations were therefore made based on the highlighted implications. Limitations of the study were articulated while suggestions for further studies were made.

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## APPENDIX A

ANALYSIS OF JSSII ENROLMENT WITHIN UMUAHIA EDUCATION ZONE  
ACCORDING TO SEX, L.G.A. AND SCHOOL TYPE AS AT 2004/2005

S/N	Local Government Areas	No. of boys	No. of Girls	Total
	Umuahia North L.G.A.			
	Boys Schools			
1	Government College Umuahia	602	-	602
	Girls Schools:			
2	Girl's Sec. School, Umuahia	-	480	480
3	Girls' High School, Umuahia	-	420	420
4	Amuzukwu Girls' Sec. School	-	358	358
5	Afugiri Girls' Sec. School.	-	210	210
	Coeducational:			
6	Williams Mem. Sec. School Umuahia	226	76	300
7	Okaiuga Nkwogwu Sec. Sch.	26	37	63
8	Ohuhu Comm. Sec. School	90	41	131
9	Sec. Tech. School Afara	122	76	198
10	Ibeku High School, Umuahia	240	300	540
11	Comm. High Sch. Isieke	147	43	190
12	Ossah Comm. High School	107	103	210
13	Mbom Comm. Sec. School	4	23	57
14	Sec. Tech. School, Ofeme	33	39	72

15	Comm.. Sec. Sch. Isingwu	76	61	137
16	Orieamenyi Comm. Sec. Sch.	75	18	93
	<b>Total</b>	<b>1778</b>	<b>2283</b>	<b>4061</b>
	Umuahia South L.G.A.			
	Boys' Schools:			
1	Holy Ghost Sec. Tech. Sch. Um	94	-	94
	Coeducational:			
2	Comm. Sec. School Nsirimo	21	26	47
3	Nsirimo Sec. School	21	20	41
4	Sec. Tech. School Umuanwanwa	15	17	32
5	Ubakala Sec. School.	151	145	296
6	Evangel High Sch. Old Umuahia	145	112	257
7	Olokoru High School	90	109	199
8	Umuokpara Sec. School	48	83	131
9	Amakamma Comm. Sec. School	69	71	140
	<b>Total</b>	<b>654</b>	<b>583</b>	<b>1237</b>
	Ikwuano L.G.A.			
1	Oboro Sec. School, Oboro	145	175	320
2	Ikwuano Sec. School, Oboro	61	51	112
3	Ibere Comp. Sec. School	48	50	98
4	Azuiyi Oloko Sec. School	15	16	31
5	Comm.. sec. school Ntalwku	20	11	31



6	Awo na-Ebo Sec. Tech. Sch.	69	78	147
7	Senior Science Sch. Ariam	-	-	-
8	Comm. Grammer Sch. Olokoru	47	60	107
9	Ambassador's College Ibere	-	-	-
	<b>Total</b>	<b>405</b>	<b>441</b>	<b>846</b>
	Umunneochi L.G.A.			
	Girls' Schools:			
1	Ngodo Girls' Sec. School	-	54	54
	Coeducational			
2	Nneato Sec. School	70	88	158
3	Isuochi Sec. School	50	40	90
4	Lokpanta Sec. School	40	42	82
5	Umuaku sec. School	28	30	58
6	Comp. Sec. Sch. Umuchieze	57	85	142
7	Obinulo Sec. Tech School	31	9	40
8	Leru Sec. School	25	50	75
9	Mbala Comm. Sec. School	32	42	74
	<b>Total</b>	<b>333</b>	<b>440</b>	<b>773</b>
	Summary of Analysis			
	L.G.A.			
1	Umuahia North	1778	2283	4061
2	Umuahia South	654	583	1237

3	Ikwuano	405	441	846
4	Umunneochi	3170	3747	6917
	<b>Total</b>	<b>3170</b>	<b>3747</b>	<b>6917</b>

**APPENDIX B****LESSON PLAN ON PROPORTION, RATIO AND RATE USING  
CONVENTIONAL INSTRUCTIONAL APPROACH**

**Subject: Mathematics**

**Class: JSS II**

**Date:**

**Time 40 Minutes per lesson**

**Average age of the students: 13 years**

**Topics: Proportion, Ratio and Rate.**

**Content:**

- (i) Proportion
- (ii) Ratio
- (iii) Percentages
- (iv) Rate

**Objectives: By the end of the lessons, students should be able to:**

- (i) solve problems on direct and inverse proportion using unitary method;
- (ii) find the ratio of two quantities in the same units;
- (iii) use the idea of ratio in sharing quantities;
- (iv) solve everyday problems involving percentages;
- (v) apply the idea of rate when solving word problems.

**Entry Behaviour:**

Students are supposed to be familiar with the concept of fraction. They should be familiar with different types of fractions and how to find their equivalence. Students should be able to express numbers as powers of 10.

Content Development (CD)	Teacher's Activities	Students' Activities	Strategies	Remarks
Set Induction	<p>The teacher arouses the interest of the students by revising the following problems with them:</p> <p>(i) Mention the three types of fractions</p> <p>(ii) Identify the following types of fractions: (a) <math>\frac{1}{4}</math> (b) <math>3\frac{1}{7}</math> (c) <math>\frac{5}{2}</math></p> <p>(iii) <math>\frac{2}{5} = \frac{\square}{20}</math></p> <p>(iv) Express 1000 in powers of 10</p> <p>(v) Change 75% to a fraction in its lowest term.</p>	<p>The students listen to the teachers explanations. They also answer the teacher's questions.</p>	<p>Examples and explanations</p>	

	The teacher explains and reminds the students of the basic fundamental concepts and principles.			
CD I Proportion	<p>The teacher explains that the lesson is on proportion using the unitary method.</p> <p>Example (1)</p> <p>A worker gets N900 for 10 days of work. Find the amount for</p> <p>(a) 3 days (b) 24 days (c) x days</p> <p>The teacher at this juncture explains that they are of find money and because of this, money should come last in every line of working.</p>	<p>The students listen alternatively to the teacher's explanations.</p> <p>They put down the examples in their exercise books. The students ask and answer teacher's questions.</p>	<p>Examples, illustrations, explanations and questioning</p>	

	<p>For 10 days the worker gets N900</p> <p>For 1 day the worker gets <math>N900 \div 10</math></p> <p style="padding-left: 40px;">= ₦ 90.</p> <p>(a) For 3 days the worker gets <math>3 \times \text{₦} 90</math></p> <p style="padding-left: 40px;">= ₦ 270.</p> <p>(b) For 24 days, the worker gets <math>24 \times \text{₦} 90</math></p> <p style="padding-left: 40px;"><math>90 = \text{₦} 2160.</math></p> <p>(c) For <math>x</math> days, the worker gets</p> <p style="padding-left: 40px;"><math>X \times \text{₦} 90 = \text{₦} 90x.</math></p> <p>The teacher asks the students to note the following:</p> <ol style="list-style-type: none"> <li>1. Each line of working is a complete sentence.</li> </ol>			
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	<p>2. The quantity to be found comes last in each sentence.</p> <p>3. The first sentence states the given facts.</p> <p>4. The second sentence gives the pay for 1 day, a unit.</p> <p>This is an example of direct proportion. The less time worked (3 days), the less money paid (₦270). The more time worked (24 days), the more money paid (₦ 2160).</p> <p>Example 2:</p> <p>Seven workers dig a piece of ground in 10 days. How long would five workers take?</p> <p>We are to find time. Time comes last in every</p>			
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	<p>line of working.</p> <p>7 workers take 10 days</p> <p>1 worker takes <math>10 \times 7</math> days</p> <p style="padding-left: 40px;">= 70 days.</p> <p>5 workers take <math>70 \div 5</math> days</p> <p style="padding-left: 40px;">= 14 days.</p> <p>The teacher asks the students to notice the following:</p> <ol style="list-style-type: none"><li>1. We assume that all the workers work at the same rate.</li><li>2. The second sentence gives the time for one unit, a single worker.</li></ol> <p>The teacher explains that this is an example</p>			
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	<p>of an inverse proportion. Fewer workers (5) take a longer time (14 days). More workers (7) take a shorter time (10 days).</p> <p><b>Example (3)</b></p> <p>Five people took 8 days to plant 1,200 trees. How long will it take ten people to plant the same number of trees?</p> <p>5 people take 8 days</p> <p>1 person takes <math>8 \times 5</math> days = 40 days.</p> <p><math>\therefore</math> 10 people take <math>40 \div 10</math> days = 4 days.</p> <p>The teacher at this juncture explains that the number of trees was not used. She</p>			
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	<p>emphasizes that when solving problems by the unitary method, the students should always:</p> <ol style="list-style-type: none"> <li>1. Write in sentences with the quantity to be found at the end;</li> <li>2. Decide whether the problem is an example of direct or inverse proportion;</li> <li>3. find the rate for 1 unit before answering the question. This is where the unitary method gets its name from.</li> </ol>			
CD II Ratio	<p>The teacher illustrates that assuming two chairs cost N600 and <del>N</del>800. The ratio of their prices is 600:800, "six hundred to eight hundred".</p>	<p>The students listen attentively to the teachers' explanations. They ask</p>	<p>Use of examples, illustrations,</p>	

	<p>The teacher explains that ratios behave in the same way as fractions.</p> <p>For example, <math>\frac{600}{800} = \frac{300}{400} = \frac{1200}{1600} = \frac{3}{4}</math></p> <p>and <math>600:800 = 300:400 = 1200:1600 = 3:4</math>.</p> <p>Thus, both parts of a ratio may be multiplied or divided by the same number. It usual to express ratio as whole numbers in their lowest terms.</p> <p><b>Example 4</b></p> <p>Express the ratio of 8cm to 3.5 cm as simply as possible.</p>	<p>questions and also answer the teachers' questions.</p>	<p>explanations and questioning.</p>	
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$$8 \text{ cm to } 3.5 \text{ cm} = 8:3.5$$

$$= 8 \times 2 : 3.5 \times 2$$

$$= 16:7$$

Notice that we do not give units in a ratio

### Example 5

Express the ratio 96k to ₦1.20 as simply as possible.

The teacher explains that both sums of money should be in kobo.

$$\frac{96}{\text{₦}1.20} = \frac{96k}{120k} = \frac{96}{120}$$

$$\frac{96 \div 24}{120 \div 24} = \frac{4}{5}$$

The teacher at this juncture asks the students to notice that quantities must be in the same units before they can be given as a ratio.

**Example 6**

Fill the gap in the ratio  $2:7 = \quad :28$

Let the missing number be a

Then  $2:7 = a:28$

$$\text{Or } \frac{2}{7} = \frac{a}{28}$$

$$\text{Thus } a = \frac{2 \times 28}{7}$$

$$= 8$$

$\therefore$  The missing number is 8.

	<p>Sharing,</p> <p>The teacher explains that ratios are often used when sharing quantities.</p> <p><b>Example 7</b></p> <p>Two students share 35 oranges in the ratio 2:3.</p> <p>How many oranges does each student get?</p> <p>Number of oranges to be shared = 35</p> <p>Ratio = 2:3</p> <p>Sum of ratio = <math>2 + 3 = 5</math></p> <p>The first student gets = <math>\frac{2}{5} \times \frac{35}{1}</math></p>			
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$$= 14 \text{ oranges}$$

∴ The first student gets 14 oranges.

The second student gets:  $\frac{3}{5} \times \frac{35}{1}^7$

$$= 21 \text{ oranges}$$

∴ The first student gets 21 oranges.

### Example 8

If four watches cost N1920, find the cost of nine watches.

Cost of 9 watches: cost of 4 watches

$$= 9:4$$



but, cost of 4 watches = ~~N~~1920

$$\text{cost of 1 watch} = \frac{\text{N}1920}{4}$$

$$\begin{aligned}\therefore \text{cost of 9 watches} &= \text{N}\left(\frac{1920}{4} \times 9\right) \\ &= 9 \times \text{N}480 \\ &= \text{N}4320\end{aligned}$$

The teacher at this point explains that this is an example of direct ratio, or direct proportion.

### Example 9

If 450g of jam costs N180, how much does

1  $\frac{1}{2}$  kg cost?

Unit must be the same.

	<p>Working in grammes,</p> <p>Cost of 1500g: cost of 450g</p> <p>1500:450</p> <p>10: 3</p> <p>cost of 450g = ₦180</p> <p>cost of <math>1\frac{1}{2}</math>kg = <math>\frac{10}{3}</math> ₦180</p> <p style="padding-left: 100px;">= ₦600</p> <p><b>Percentages</b></p> <p><b>Example 10</b></p> <p>Find 15% of 2.8kg</p> <p>15% of 2.8kg</p> $\frac{15}{100} \times 2.8\text{kg}$	<p>The students ask questions at any point they don't understand.</p> <p>The students ask questions at any point they don't understand.</p>	<p>Use of examples, illustrations and explanations</p> <p>Use of examples, illustrations</p>	
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	<p><math>15 \times 0.028 \text{ kg}</math> <math>= 0.42 \text{ kg or } 420\text{g}.</math></p> <p><b>Example 11</b></p> <p>Express 3.3m as a percentage of 7.5 m. the teacher explains that the quantities should be expressed as a fraction:</p> $\frac{3.3\text{m}}{7.5\text{m}} = \frac{33}{75}$ <p>Express the fraction as percentage.</p> $\frac{33}{7.5} \times 100$ $= \frac{33 \times 4}{3} \%$ $= 44\%$		and explanations.	
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	<p><math>\therefore 3.3\text{m}</math> is 44% of 7.5m</p> <p><b>Example (12)</b></p> <p>N646 is 85% of a sum of money. Find the sum of money.</p> <p>Using unitary method,</p> <p>85% of the money = N646</p> <p>1% of the money = <math>\frac{N646}{85}</math></p> <p>100% of the money = <math>\frac{N646}{85} \times 100</math></p> <p>= N760.</p>			
<b>CD III</b> <b>RATE</b>	The teacher introduced the topic by writing 45km/hr, N800/day, 9km/litre as examples of			

	<p>rates. The first rate tells the distance gone in 1 hour. The second tells how much money is made in 1 day. The third tells the distance traveled on 1 litre.</p> <p><b>Example 13</b></p> <p>A car goes 160 km in 2 hours. What is its rate in km/hr.</p> <p>In 2 hrs, the car travels 160 km</p> <p>In 1 hr the car travels <math>\frac{160km}{2}</math></p> <p style="text-align: center;">= 80 km</p> <p>∴ The rate of the car is 80km.hr.</p>	<p>The students ask and answer the teacher's questions.</p>		
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**Example 14**

A worker gets N2, 400 for 5 days' work.

What is the rate of pay per day?

In 5 days, the worker gets N2400

In 1 day the worker gets  $\frac{2400}{5} = \text{N}480$ .

The worker's rate of pay is N480/day.

**Example 15**

A car used 10 litres of petrol to travel 74km.

Express its petrol consumption as a rate in km per liter.

On 10 litres, the car travels 74 km

	<p>On 1 litre, the car travels <math>\frac{74}{10} km</math></p> $= 7.4km$ <p><math>\therefore</math> Its petrol consumption is 7.4 km/litre.</p> <p><b>Example 16</b></p> <p>The price of an article is reduced from N400 to N360. Express the reduction as a rate of kobo in the Naira.</p> <p>Reduction = N400 – N360 = N40.</p> <p>On N400 the reduction is N40.</p> <p>On N1 the reduction is <math>\frac{N40}{400}</math></p>	<p>The students listen attentively and also copy down all the teacher's worked examples. They also ask and answer the teacher's questions.</p>	<p>Use of questions, illustration and explanation</p>	
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<b>Summary</b>	$= N \frac{1}{10}$ $= 10k \text{ in the Naira.}$ <p>Notice that in every example, the unitary method is used to find rate.</p> <p>The teacher summarizes the lessons on the chalkboard thus:</p> <ol style="list-style-type: none"><li>1. Problems in proportion can be solved by the unitary method:<ol style="list-style-type: none"><li>(a) Write a sentence with the quantity to be found at the end.</li><li>(b) Decide whether the problem is an example of direct or inverse proportion.</li></ol></li></ol>			
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	<p>proportion.</p> <p>(c) Find the rate for 1 unit</p> <p>(d) Then find the required quantity by multiplying (for direct proportion) or dividing (for inverse proportions).</p> <p>2. A ratio is said as “three to ten” and written 3:10.</p> <p>3. Ratio behaves like fractions. For example,</p> $\frac{3}{10} = \frac{30}{100} = \dots$ <p>Similarly, 3:10 = 30:100 and so on.</p> <p>4. it is usual to express ratio in their lowest whole-number terms.</p> <p>5. The unitary method can be used to solve</p>			
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	<p>problems involving ratios. However, it is necessary to be clear about direct and inverse ratios.</p> <p>6. The principles of ratios and the unitary method can be used when solving problems with percentages.</p> <p>7. A rate is a ratio that involves a relation between two different quantitative (e.g. km per hour, Naira per month). Again, the unitary method is very helpful when solving problems with rate.</p>			
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## APPENDIX C

### LESSON PLAN ON PROPORTION, RATIO AND RATE USING IEPT CONSTRUCTIVIST APPROACH

**Subject:** mathematics

**Class:** JSS II

**Date:**

**Time:** 40 minutes

**Average Age of students:** 13 years

**Topics:** Proportion, Ratio and Rate

**Content:**

- (i) Proportion
- (ii) Ratio
- (iii) Percentages
- (iv) Rate

**Objectives:** By the end of the lessons, students should be able to:

- (i) solve problems on direct and inverse proportion using unitary method.
- (ii) Find the ratio of two quantities in the same units.
- (iii) Use the idea of ratio in sharing quantities
- (iv) Solve everyday problems involving percentages.
- (v) Apply the idea of rate when solving word problems.

**Entry Behaviour**

Students are supposed to be familiar with the concept of fraction e.g. finding the equivalence of a given fraction. They are supposed to be familiar with the different types of fractions and their examples. They are supposed to be familiar with the concept of percentages and express numbers as powers of 10.

Content Development (CD)	Teacher's Activities	Students' Activities	Strategies	Remarks
Set Induction	<p>The teacher asks students the following questions to arouse their interest and also to assess their entry point:</p> <p>(i) Mention with examples the different types of fractions.</p> <p>(ii) Identify the following types of fractions:</p> <p>(a) <math>\frac{1}{4}</math> (b) <math>3\frac{1}{7}</math> (c) <math>\frac{3}{2}</math></p> <p>(iii) Simplify <math>\frac{2}{5} = \frac{\square}{20}</math></p> <p>(iv) Express 1000 in powers of 10.</p> <p>(v) Change 75% to a fraction in its</p>	<p>The students try to solve the problems individually and collectively. They answer teacher's questions. They also explain their solution procedures step by step. The students listen to the teacher's explanations and ask questions about points that are not clear to them.</p>	<p>Use of examples, questions, illustrations and explanations.</p>	

	<p>lowest terms. The teacher is to be directed by the student's ability to react to the problems. She explains and reminds the students of some basic facts and fundamental principles.</p>			
<p><b>CDI</b> <b>Proportions</b></p>	<p>The teacher introduces the lesson by telling the students that it took two (2) cubes of sugar to make her cup of tea that morning. Assuming she wanted to make three such cups of tea, how many cubes of sugar would she have required? The teacher goes on with such similar</p>	<p>The students are expected to suggest answers to the teacher's questions. They are expected to ask questions and also answer the teacher's questions. The students also listen attentively to the teacher's explanations and illustrations. They</p>	<p>Use of examples, illustrations, explanation, explanations and tions</p>	

	<p>questions. It takes two yards of cloth to make a school uniform for each JSS II student. How many yards of cloth would be needed for such ten uniforms?</p> <p>After series of examples, the teacher now asks for their observation.</p> <p>At this point, the teacher introduces the</p>	<p>are also expected to give their own examples along the same line of reasoning.</p> <p>The students at this juncture are expected to observe that, for instance, the less the number of cups of tea to make, the less the number of cubes of sugar to be used. Again, the less the number of uniforms to be made, the less the number of yard of cloth to be used.</p>	<p>and questio ning.</p>
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	<p>topic proportion. The teacher explains that the above are examples of direct proportion. The teacher at this juncture emphasizes the concept with the following worked examples thus:</p> <p><b>Example (1)</b></p> <p>A worker gets N900 for 10 days of work.</p> <p>Find the amount for</p> <p>(a) 3 days</p> <p>(b) 24 days</p> <p>(c) x days</p> <p>The teacher asks the students to identify what is being looked in the problem. The</p>	<p>The students are expected to observe</p>		
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	<p>teacher explains that the term being looked for should always come last in every line of working:</p> <p>For 10 days, the workers gets ₦ <input type="checkbox"/></p> <p>For 1 day the worker gets ₦ <math>900 \div 10</math></p> <p style="padding-left: 40px;">₦ <input type="checkbox"/></p> <p>(a) For 3 days the worker gets</p> <p style="padding-left: 40px;"><input type="checkbox"/> x ₦ 90</p> <p style="padding-left: 40px;">= ₦ 270</p> <p>(b) For 24 days the worker gets</p> <p style="padding-left: 40px;"><math>24 \times \text{₦} \text{ <input type="checkbox"/>}</math></p> <p style="padding-left: 40px;">= ₦ 2160</p>	<p>that they are to find money and as such, money should always come last in every line of working.</p> <p>For 10 days, the workers gets ₦900</p> <p>For 1 day the worker gets</p> <p style="padding-left: 40px;">₦ <math>900 \div 10</math></p> <p style="padding-left: 40px;">₦ 90</p> <p>(a) For 3 days the worker gets</p> <p style="padding-left: 40px;"><input type="checkbox"/> x ₦ 90</p> <p style="padding-left: 40px;">= ₦ 270</p> <p>(b) For 24 days the worker gets 24</p> <p style="padding-left: 40px;">x ₦ <input type="checkbox"/></p> <p style="padding-left: 40px;">= ₦ 2160</p>	
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	<p>(c) for x days the worker gets</p> <p><math>x \times ₱90</math></p> <p><math>= ₱ \square</math></p> <p>The teacher asks the students to note the following:</p> <ol style="list-style-type: none"> <li>1. each line of working is a complete sentence</li> <li>2. The quantity to be found comes last in each sentence.</li> <li>3. The first sentence states the given facts.</li> <li>4. The second sentence gives the pay for a day, a unit.</li> </ol>	<p>for x days, the worker gets</p> <p><math>n \times ₱90</math></p> <p><math>= ₱ 90x.</math></p> <p>The students are expected to contribute to these points to be noted.</p> <p>They eagerly answer the teacher's questions.</p>		
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	<p>This is an example of a direct proportion. The less time worked (3 days), the less money paid (N70). The more time worked (24 days), the more money paid (N2160).</p> <p>Example (2)</p> <p>Seven workers dig a piece of ground in 10 days. How long would five workers take?</p> <p>The teacher asks the students to identify the term to be found.</p>			
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<p>The teacher proceeds thus:</p> <p>7 workers take 10 days</p> <p>1 worker takes less or more?</p> <p>→ <math>10 \times 7</math> days</p> <p>= □ days.</p> <p>∴ 5 workers take <math>\frac{70}{5}</math></p> <p>= □ days.</p> <p>The teacher at this point asks the students to notice:</p> <p>(1) We assume that all the workers work at the same rate.</p> <p>(2) The second sentence gives time for</p>	<p>The students are expected to observe that “time” is the term to be found and so it should come last in every line of working.</p> <p>7 workers take 10 days</p> <p>1 worker takes more days</p> <p>→ <math>10 \times 7</math> days</p> <p>= 70 days.</p> <p>∴ 5 workers take <math>\frac{70}{5}</math></p> <p>= 14 days.</p>
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	<p>one unit, a single worker.</p> <p>The teacher then explains that this is an example of inverse proportion. Fewer workers (5) take a larger time (14 days). More workers (7), take a shorter time (10 days). The teacher emphasizes that proportions are not always direct and warns that students should be careful to identify types.</p> <p><b>Example 3</b></p> <p>Five people took 8 days to plant 1,200 trees. How long will it take ten people to</p>	<p>The students listen to the teacher attentively. They are expected to observe the differences between the direct and inverse proportions. They are also expected to ask questions on any point that is not clear to them.</p>		
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	<p>always:</p> <ol style="list-style-type: none"> <li>1. write the sentences with quantity to be found at the end;</li> <li>2. decide whether the problem is an example of direct or inverse proportion;</li> <li>3. find the rate for 1 unit before solving the problem.</li> </ol> <p>She stresses that this is where unitary method get its name from.</p>	<p>The students are expected to observe that the number of trees acts as a distractor and should not be allowed to bring confusion in the solution of such problems. The students are expected to ask and answer teacher's questions at interval.</p>		
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<p><b>CD II</b></p> <p><b>Ratio</b></p>	<p>The teacher illustrates by assuming the cost of two chairs to be ₦ 600 and ₦ 800 respectively. The ratio of their prices is 600:800, “six hundred to eight hundred”. The teacher explains that ratio behave in the same way as factions. For instance:</p> $\frac{600}{800} = \frac{300}{400} = \frac{1200}{1600} = \frac{3}{4} \text{ and}$ $600: 800 = 300: 400 = 1200:1600 = 3: 4.$ <p>Thus, both parts of a ratio may be multiplied or divided by the same number. She emphasizes that it is usual to express ratios as whole numbers in their lowest terms.</p> <p><b>Example (4)</b></p> <p>Express the ratio of 8 cm to 3.5cm as simply as possible.</p>	<p>The students listen attentively to the teacher’s explanations. They ask questions on any aspect of the explanation that is not clear to them. They also answer the teacher’s questions.</p>	<p>Use of examples, illustrations and questioning.</p>
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<p><b>CD III</b></p> <p><b>Rate</b></p>	<p>The teacher introduces the lesson by explaining to the children that rate is a ratio that involves a relation between two different quantities.</p> <p>She writes 45km/hr, ₦800/day, 96km/litre as examples of rates.</p> <p><b>Example 13</b></p> <p>A car goes 160 km in 2 hours. What is its rate in km/hr?</p>	<p>The students listen attentively to the teachers explanations.</p> <p>The students are guided to observe that the first rate tells the distance gone in 1 hour. The second tells how much money is made in 1 day. The third tells the distance traveled on 1 litre.</p>		
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	<p>In 2 hrs, the car travels <math>\square</math> km</p> <p>In 1 hr, the car travels <math>\frac{160}{\square}</math> km</p> <p>= <math>\square</math> km.</p> <p><math>\therefore</math> The rate of the car is</p> <p><math>\square</math> km/hr.</p> <p><b>Example 14</b></p> <p>A worker gets ₦ 2,400 for 5 days work.</p> <p>What is the rate of pay per day?</p> <p>In <math>\square</math> days, the worker gets ₦ 2400</p> <p>In 1 day the worker gets <math>\frac{\square}{5}</math></p> <p>= ₦ <math>\square</math></p>	<p>In 2 hrs, the car travels 160 km</p> <p>In 1 hr, the car travels <math>\frac{160}{2}</math> km</p> <p>= 80 km.</p> <p><math>\therefore</math> The rate of the car is</p> <p>80 km/hr.</p> <p>In 5 days, the worker gets ₦ 2400</p> <p>In 1 day the worker gets <math>\frac{2400}{5}</math></p> <p>= ₦ 480</p>		
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	<p>∴ The workers pay is ₦ []/day</p> <p><b>Example 15</b></p> <p>A car uses 10 litres of petrol to travel 74 km. Express its petrol consumption as a rate in km per litre.</p> <p>On [] litres the car travels 74 km</p> <p>On 1 litre the car travels <math>\frac{74}{\square}</math> km</p> <p>= [] km</p> <p>∴ Its petrol consumption is 7.4km/[]</p>	<p>∴ The workers pay is ₦480/day</p> <p>On 10 litres the car travels 74 km</p> <p>On 1 litre the car travels <math>\frac{74}{10}</math> km</p> <p>= 7.4 km</p> <p>∴ Its petrol consumption is 7.4km/litre</p>		
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	<p><b>Example 16</b></p> <p>The price of an article is reduced from ₦ 400 to ₦ 300. Express the reduction as a rate of kobo in the naira.</p> <p>The teacher proceeds as follows:</p> <p>Reduction = <math>\frac{\text{₦ } \square}{\text{₦ } \square} - \frac{\text{₦ } \square}{\text{₦ } \square}</math></p> <p style="padding-left: 40px;">= <math>\frac{\text{₦ } 40}{\text{₦ } 400}</math></p> <p>on ₦ 400 reduction is <math>\frac{\text{₦ } \square}{\text{₦ } \square}</math></p> <p>on ₦ 1 reduction is <math>\frac{\text{₦ } \square}{400}</math></p> <p style="padding-left: 40px;">= <math>\frac{\text{₦ } \square}{10}</math></p> <p style="padding-left: 40px;">= <math>\square</math>k</p>	<p>The students are expected to suggest that the reduction be done first and then the unitary method should be used in solving the problem.</p> <p>Reduction = <math>\frac{\text{₦ } 400}{\text{₦ } 360}</math></p> <p style="padding-left: 40px;">= <math>\frac{\text{₦ } 40}{\text{₦ } 400}</math></p> <p>on ₦ 400 reduction is <math>\frac{\text{₦ } 40}{\text{₦ } 400}</math></p> <p>on ₦ 1 reduction is <math>\frac{\text{₦ } 40}{400}</math></p> <p style="padding-left: 40px;">= <math>\frac{\text{₦ } 1}{10}</math></p> <p style="padding-left: 40px;">= 10k</p>		
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	<p>∴ The reduction is at the rate of 10% in the naira.</p> <p>The teacher summarizes the lessons on the chalkboard as follows:</p> <p>1. Problems in proportion can be solved by the unitary method:</p> <p>(a) Write the sentence with the quantity to be found</p> <p>(b) Decide whether the problem is an example of direct or inverse proportion.</p> <p>(c) Find the rate for 1 unit.</p>	<p>∴ The reduction is at the rate of 10% in the naira.</p> <p>The students are encouraged to ask questions on point not clear to them. They also answer the teacher's own questions.</p> <p>The students are expected to take down this summary note into their own exercise books.</p>	
<p><b>Summary</b></p>			

	<p>(d) Then find the required quantity by multiplying (for direct proportion) or dividing (for inverse proportion).</p> <p>2. A ratio is said as “three to ten” and written 3:10.</p> <p>3. Ratio behave like fractions. For example: <math>\frac{3}{10} = \frac{30}{100} = \dots</math></p> <p>Similarly, 3:10 = 30:100 and so on.</p> <p>4. It is usual to express ratio in their lowest whole number terms.</p> <p>5. the unitary method can be used to</p>		
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	<p>solve problems involving ratios.</p> <p>However, it is necessary to be clear about direct and inverse ratios.</p> <p>6. The principles of ratios and the unitary method can be used when solving problems with percentages.</p> <p>A rate is a ratio that involves a relation between two different quantities (e.g. km per hour, naira per month). Again, the unitary method is very helpful when solving problems with rates.</p>		
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**APPENDIX D****LESSON PLAN ON PROPORTION, RATIO AND RATE USING TLC  
CONSTRUCTIVIST INSTRUCTIONAL APPROACH**

**Subject: Mathematics**

**Class: JSS II**

**Date:**

**Time: 40 minutes**

**Average Age of Students: 13 years**

**Topic: Proportion, Ratio and Rate**

**Content:**

(i) Proportion

(ii) Ratio

(iii) Percentages

(iv) Rate

**Objectives: By the end of the lessons, the students should be able to:**

- (i) solve problems on direct and inverse proportion using unitary method;
- (ii) find the ratio of two quantities in the same units;
- (iii) use the idea of ratio in sharing quantities;
- (iv) solve everyday problems involving percentages;
- (v) apply the idea of rate when solving word problems.

**Entry Behaviour**

Students are supposed to be familiar with the concept of fraction e.g. finding the equivalence of a given fraction. They are supposed to be familiar with different types of fractions and their examples. They are also supposed to be familiar with the concept of percentages and expression of numbers in powers of 10.



Content Development (CD)	Teacher's Activities	Students' Activities	Strategies	Remarks
Set Induction	<p>The teacher starts by asking students the following questions to stimulate their interest and also establish a baseline for the lesson proper:</p> <p>(i) Mention with examples the different types of fractions.</p> <p>(ii) Identify the following types of fractions:</p> <p>(a) <math>\frac{1}{4}</math> (b) <math>5\frac{2}{7}</math> (c) <math>\frac{8}{3}</math></p> <p>(iii) Simplify <math>\frac{3}{10} = \frac{9}{1}</math></p> <p>(iv) Express 1000 in powers of 10</p>	<p>The students are expected to try out the problems singly and collectively.</p> <p>They answer teacher's questions and also ask their own questions. The students should be meant to explain their solution procedures step by step. They also listen to the teacher's explanations eagerly.</p>	Use of questions, examples, illustrations and explanations.	

	<p>(v) Change 25% to a fraction in its lowest term.</p> <p>The teacher is to be directed by the students' reaction to the problems. She explains and reminds students of some basic fundamental facts and principles based on the situations on ground.</p>			
<p><b>CD I</b></p> <p><b>Proportions</b></p>	<p>Materials: weighing balance, metal washers and plastic blocks.</p> <p>The teacher divides the students into small groups depending on the class size and asks each group to chose a temporary leader. The teacher asks</p>	<p>The students in their small groups choose their temporary leaders. They are expected to experiment with the materials provided for them. They are</p>		

<p>each group to use a balance to determine how many plastic blocks equal one metal washer in weight.</p>	<p>expected to discuss and exchange ideas while working towards achieving a common goal of balancing the metal washers with the plastic blocks.</p>	
<p>The teacher asks "How many blocks does it take to balance one washer? If I place one more washer on this side, how many more blocks do you think we would need to balance it?"</p>	<p>Student leader, (after a few seconds of experimenting) answers four. The students is expected to suggest an answer (probably one).</p>	
<p>The teacher asks the students to try out his/her suggestions.</p>	<p>The student is expected to place one more block in the balance tray. It is to</p>	

	<p>The teacher asks "how many blocks does it take to balance one washer?"</p> <p>And how many to balance two washers?</p> <p>If I put one more washer on this side,</p>	<p>be noticed that a balance would not be achieved. The students may look confused and places another block in the tray and then a third. Still no balance. He/she is expected to place one more block in the tray. Balance should be achieved. The student at this juncture probably will smile and looks at the teacher.</p> <p>Counting, the student says eight.</p> <p>The student pondering and looking</p>		
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	<p>how many more blocks will you need to balance it?</p> <p>The teacher asks the students to try it.</p> <p>At this juncture, the teacher draws the attention of the students by telling them she is going to give them a really very hard question thus:</p> <p>If I take four blocks off the balance, how many washers will I need to take off in order to balance it?</p>	<p>quizzically at the teacher is expected to say four.</p> <p>The student after successfully balancing with four blocks, concludes that each washer is the same as four blocks in weight.</p> <p>The student is expected to answer one!</p>	
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	<p>The teacher now proceeds with the following examples in order to consolidate the students discovery</p> <p>Example (1)</p>	<p>At this point the students discover that it takes 4 plastic block to balance a metal washer, 8 blocks to balance 2 metal washers, 20 blocks to balance 5 metal washers and so on. They also discover on the other hand, if 2 metal washers are taken off, 8 plastic block would also need to be taken off in order to maintain a balance.</p>		
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	<p>A worker gets ₦900 for 10 days of work. Find the amount for</p> <p>(a) 3 days</p> <p>(b) 24 days</p> <p>(c) x days</p> <p>The teacher asks the students to identify the term being looked for in the problems. She explains that the term to be looked for should usually come last in every line of working:</p> <p>For 10 days the worker gets ₦[ ]</p> <p>For 1 day the worker gets ₦900 ÷ [ ] = ₦[ ]</p>	<p>The students are expected to observe that they are to find money and as such, money should always come last in every line of working.</p> <p>For 10 days the worker gets ₦900</p> <p>For 1 day the worker gets ₦900 ÷ 10 = ₦90</p>		
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	<p>(a) For 3 days the worker gets</p> $= 3 \times \text{N} \square$ $= \text{N}270$ <p>(b) For 24 days the worker gets</p> $\square \times \text{N}90$ $= \text{N}\square$ <p>(c) For x days the worker gets</p> $\square \times \text{N}90$ $= \square$ <p>The teacher asks the students to note the following:</p> <ol style="list-style-type: none"> <li>1. Each line of working is a complete sentence.</li> </ol>	<p>(a) For 3 days the worker gets</p> $= 3 \times \text{N} 270$ $= \text{N}270$ <p>For 24 days the worker gets</p> $24 \times \text{N} 90$ $= \text{N}2160$ <p>(c) For x days the worker gets</p> $\square \times \text{N}90$ $= \text{N}90x$ <p>The students are expected to contribute to these points to be noted.</p>	
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	<p>2. The quantity to be found comes last in each sentence.</p> <p>3. The first sentence states the given facts.</p> <p>4. The second sentence gives the pay for 1 day, a unit.</p> <p>The teacher asks the students to observe from the above examples that the less time worked (3 days), the less money paid (N279). The more time worked (24 days), the more money paid (N2160). She goes on to explain that this is an example of direct proportion.</p>	<p>They ask and answer the teacher's questions.</p>		
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	<p><b>Example (2)</b></p> <p>Seven workers dig a piece of ground in 10 days. How long would five workers take? The teachers ask the students to identify the term to be found.</p> <p>The teacher proceeds thus:</p> <p>7 workers take 10 days</p> <p>1 worker takes less or more days?</p> <p>→ 10 x 7 days</p> <p>= [] days</p> <p>∴ 5 workers take <math>\frac{70}{\square}</math></p>	<p>The students should observe that “time” is the term to be found and so it should come last in every line of working.</p> <p>7 workers take 10 days</p> <p>1 worker takes less days</p> <p>→ 10 x 7 days</p> <p>= 70 days</p> <p>∴ 5 workers take <math>\frac{70}{5}</math></p>	
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	<p>= <math>\square</math> days</p> <p>The teacher at this point asks the students to notice:</p> <p>(1) We assume that all the workers work at the same rate.</p> <p>(2) The second sentence gives the time for one unit, a single worker.</p> <p>The teacher now explains that this is an example of inverse proportion and</p>	<p>= 14 days</p> <p>The students are expected to react to these points to be noted in case of any doubt. They are also expected to observe that the fewer workers (5) take a larger time (14 days) more workers (7) take a shorter time (10 days). The students are expected to observe that this is in contrast to the first example treated under direct proportion.</p>	
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	<p>emphasizes that proportions are not always direct, hence, she warns that students should be careful to differentiate between types.</p> <p><b>Example 3</b></p> <p>Five people took 8 days to plant 1,200 trees. How long will it take ten people to plant the same number of trees?</p> <p>The teacher asks for the identification of the required term.</p>	<p>The students are expected to observe that time is to be found.</p>		
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	<p>5 people take [ ] days</p> <p>1 person takes 5 [ ] 8 days = 40 days</p> <p><math>\therefore</math> 10 people take 40 [ ] 10 days. = 4 days</p> <p>The teacher now explains that the number of trees acts as a distractor and should not be allowed to bring confusion in the solution of such problems. She stresses that when solving problems by the unitary method, the students should always:</p>	<p>5 people take 8 days</p> <p>1 person takes 5 x 8 days = 40 days</p> <p><math>\therefore</math> 10 people take 40 <math>\div</math> 10 days. = 4 days.</p> <p>The students are expected to observe that the number of trees was not used. The students listen attentively to the teacher's explanations. They ask questions at intervals and also answer the teacher's questions.</p>	
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	<p>1. Write in sentences with the quantity to be found at the end.</p> <p>2. Decide whether the problem is an example of direct or inverse proportion.</p> <p>3. find the rate for 1 unit before solving the problem. She emphasizes that this is from where unitary method gets its name.</p>			
<p><b>CD II</b></p> <p><b>Ratio</b></p>	<p>The teacher asks the students to recall their earlier on example where it takes 12 plastic blocks to balance 3 metal washers. She explains this can be</p>	<p>The students are expected to observe a link between the discussion on ground and the concepts of proportion. They are to generate a lot of questions</p>	<p>Use of examples, demonstrations</p>	

	<p>written as "12 blocks to 3 washers. That is 12:3. The teacher goes on to assume the cost of two chairs to be N600 and N800 respectively. This can also be written as 600:800, six hundred to eight hundred.</p> <p><b>Example 4</b></p> <p>Express the relation 8cm to 3.5cm as simply as possible.</p> <p>8cm to 3.5 cm        = 8: [ ]        = 8 x 2 : 3.5 x [ ]</p>	<p>between themselves, between them and the teacher.</p> <p>8cm to 3.5 cm        = 8: 3.5        = 8 x 2 : 3.5 x 2</p>	<p>trations, discussions and questioning.</p>
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	<p>= 16:[ ]</p> <p>The teacher asks the students to notice that units are not given to ratios.</p>	<p>= 16:7</p>	
<p><b>Example 5</b></p> <p>Express the relation 96k to N1.20 as simply as possible.</p> <p>The teacher asks for the students' observations in terms of the units. The teachers proceed thus:</p> $\frac{96k}{\text{N}1.20} = \frac{96k}{[\ ] 120}$ $\frac{96 \div 24}{120 \div [\ ]}$ $= \frac{[\ ]}{5}$ <p><math>\therefore</math> The relation is 4:5</p>	<p>The students are to observe that both sums of money should be in kobo.</p> $\frac{96k}{\text{N}1.20} = \frac{96k}{120k} = \frac{96}{120}$ $\frac{96 \div 24}{120 \div 24}$ $= \frac{4}{5}$		



<p><b>CD III</b></p> <p><b>Rate</b></p>	<p>The teacher introduces the lesson by writing 45km/hr, ₦ 800/day, 9km/litre.</p> <p><b>Example 3</b></p> <p>A car goes 160 km in 2 hours.</p> <p>What is its rate in km/hr?</p> <p>In 2 hrs the car travels [ ] km</p> <p>In 1 hr the car travels <math>\frac{160km}{2}</math></p>	<p>The students are guided to observe that the first relation tells eh distance gone in one hour. The second tells how much money is made in 1 day. The third tells the distance traveled on 1 litre.</p> <p>The students are expected to recall the fact that unitary method should be used in solving the problem.</p> <p>In 2 hrs the car travels 160 km</p> <p>In 1 hr the car travels <math>\frac{160km}{2}</math></p>		
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= [ ] km.

∴ The rate of the car is [ ] km/hr.

#### Example 14

A worker gets ₦ 2400 for 5 days work.

What is the rate of pay per day?

In [ ] days the worker gets ₦ 2400

In 1 day the worker gets  $\frac{[ ]}{5}$

= ₦ [ ]

∴ The worker's pay is ₦ [ ]/day.

= 80 km.

∴ The rate of the car is 80 km/hr.

In 5 days the worker gets ₦ 2400

In 1 day the worker gets  $\frac{240}{5}$

= ₦ 480

∴ The worker's pay is ₦480/day.

	<p><b>Example 15</b></p> <p>A car uses 10 litres of petrol to travel 74 km. Express its petrol consumption as a rate in km per litre.</p> <p>On [ ] litres the car travels 74 km</p> <p>On 1 litre the car travels <math>\frac{[ ]}{10}</math> km</p> <p>= [ ] km</p> <p><math>\therefore</math> Its petrol consumption is 7.4 km/[ ]</p>	<p>On 10 litres the car travels 74 km</p> <p>On 1 litre the car travels <math>\frac{74}{10}</math> km</p> <p>= 7.4 km</p> <p><math>\therefore</math> Its petrol consumption is 7.4 km/litre</p>		
	<p><b>Example 16</b></p> <p>The price of an article is reduced from ₦ 400 to ₦ 360. Express the reduction as a rate of kobo in the naira.</p>			

	<p>as a rate of kobo in the naira.</p> <p>The teacher proceeds thus:</p> <p>Reduction = ₦ [ ] - ₦ [ ]</p> <p>= ₦ 40</p> <p>on ₦ 400 reduction is ₦ [ ]</p> <p>on ₦ 1 reduction is <math>\frac{40}{[ ]}</math></p> <p>= ₦ [ ]</p> <p>= [ ]k</p>	<p>Reduction = <del>₦</del>400 - <del>₦</del> 360</p> <p>= ₦ 40</p> <p>on ₦ 400 reduction is ₦ 40</p> <p>on ₦ 1 reduction is <math>\frac{40}{400}</math></p> <p>= <del>₦</del> <math>\frac{1}{10}</math></p> <p>= 10k</p>	
	<p>∴ The reduction is at the rate of [ ] in the naira.</p>	<p>∴ The reduction is at the rate of 10k in the naira. The students at this point are expected to observe that the rate is a rate that involves a relation between</p>	

	<p>The teacher stresses that students should notice that in every example, the unitary method is used to find rate.</p>	<p>a rate that involves a relation between two quantities.</p>	
<p><b>Summary</b></p>	<p>The teacher summarizes the lessons on the chalkboard as follows:</p> <ol style="list-style-type: none"> <li>1. Problems in proportion can be solved by the unitary method:             <ol style="list-style-type: none"> <li>(a) Write a sentence with the quantity to be found at the end.</li> <li>(b) Decide whether the problem is an example of direct or inverse</li> </ol> </li> </ol>	<p>The students are expected to take down this summary note into their exercise books.</p>	

	<p>proportion.</p> <p>(c) Find the rate for 1 unit.</p> <p>(d) Then find the required quantity by multiplying (for direct proportion) or dividing (for inverse proportion).</p> <p>2. A ratio is said as “three to ten” and written 3:10</p> <p>3. A ratio behaves like fractions.</p> <p>For example:</p> $\frac{3}{4} = \frac{30}{100} = \dots$ <p>Similarly, 3:10 = 30:100 and so on.</p>			
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	<p>4. It is usual to express ratio in their lowest whole number terms.</p>		
	<p>5. The unitary method can be used to solve problems involving ratios.</p>		
	<p>6. The principles of ratio and unitary method can be used when solving problems with percentage.</p>		
	<p>7. a rate is a ratio that involves a relation between two different quantities (e.g. km per hour, naira per month). Again, the unitary method is very helpful when solving problems with rate</p>		

**APPENDIX E****THE MATHEMATICS ACHIEVEMENT TEST (MAT)  
(PRE TEST)**

Class: JSS II

Subject: Mathematics

Time: 1 hour 30 minutes

Instruction: Answer all questions.

All workings must be shown on your answer scripts clearly.

1. A woman is paid N750 for 5 days of work. Find her pay for 22 days.
2. It takes four people 3 days to dig a small pit. How long would it take three people to do the same work?
3. Express the ratio 3 days is to 3 weeks in its simplest form.
4. The number of boys in a school is 120. If the ratio of boys to girls is 2:3, find the total number of students in the school.
5. If two students share 22 oranges in the ratio 9:2. How many oranges does each student get?
6. A farmer divides 180 cattle among his children in the ratio 4:3:2. How many cattle does each child get?
7. What percentage of 8kg is 400g?
8. In a school of 575 students, if 44% are boys. How many girls are in the school?



9. A lorry goes 320km in 4 days. What is its rate in km/hr.?
10. A car factory made 375 cars in 5 days. Find its production rate in cars per day.

**APPENDIX F****THE MATHEMATICS ACHIEVEMENT TEST (MAT)  
(POST-TEST)**

Class: JSS II

Subject: Mathematics

Time: 1 hour 30 minutes

Instruction: Answer all questions.

All workings must be shown on your answer scripts clearly.

1. A man received N840 for 7 days of works. Find his pay if he works for 44 days.
2. If five men take 4 days to clear a portion of a farm. How long would it take 4 men to do the same work?
3. Express as a ratio 20 minutes to 2 hrs and simplify.
4. The number of girls in a school is 180. If the ratio of girls to boys is 3:2, find the total number of students in the school.
5. Share 28 mangoes among two students in the ratio 5:2. Find the number of mangoes each has.
6. Divide 240 chickens among Poultry farmers in the ratio 5:4:3. How many chickens does each farmer get?
7. Express 400m as a percentage of 8km.
8. A school has 425 students. If 56% are boys. How many girls are in the school

9. A bicycle goes 160km in 4 hrs. Find its rate in km/hr.
10. A machine factory made 434 machines in 7 days. Find its rate of production in machines per day.

**APPENDIX G****THE MATHEMATICS ACHIEVEMENT TEST (MAT)  
(DELAYED POST-TEST)**

Class: JSS II

Subject: Mathematics

Time: 1 hour 30 minutes

Instruction: Answer all questions.

All workings must be shown on your answer scripts clearly.

1. A boy is paid N420 for 3 days of work. What is his pay if he works for 44 days?
2. It takes 7 women 6 days to weed a farm. How long would it take 6 women to do the same weeding?
3. Express 4 days to 4 weeks as a ratio in its simplest form.
4. A school has 240 boys, if the ratio of boys to girls is 4:6. What is the total number of students in the school?
5. Two students share 18 apples in the ratio 7:2. Find the number of apples that each student will get.
6. A woman divides 120 eggs among her children in the ratio 3:2:1. Find the number of eggs that each child gets.
7. What percentage of 10m is 50cm?
8. A school has 425 students, 44% of them are boys. How many are girls?

9. A lorry travels 480km in 6 hrs. What is its rate in km/hr?
10. A factory made 162 bicycles in 3 days. Find its rate of production in bicycles per day.

## APPENDIX H

THE PRE-MATHEMATICS ACHIEVEMENT TEST (PREMAT)  
MARKING GUIDE

1. A woman is paid N750 for 5 days of work. Find her pay for 22 days  
for 5 days the woman is paid N750  $M_1$  for 1 day the woman is paid

$$\frac{N750}{5} \quad M_1 \quad 150$$

∴ For 22 days the woman is paid  $\frac{750 \times 22}{5}$   $M_2$

$$= N3,300 \quad A_1$$

(5marks).

2. It takes four people 3 days to dig a small pit. How long would  
it take three people to do the same work?

For 4 people it takes 3 days  $M_1$

For 1 person it takes  $3 \times 4$  days  $M_1$

∴ For 3 people it takes  $\frac{3 \times 4}{3}$  days  $M_2$

$$3$$

$$= 4 \text{ days} \quad A_1$$

(5marks)

3. Express the ratio 3 days to 3 weeks in its simplest form

1 week = 7 days  $M_1$

3 weeks =  $(7 \times 3)$  days

$$= 21 \text{ days} \quad M_1$$

∴ The ratio 3 days to 21 days

$$= 3 : 21 \quad M_2$$

$$= 1 : 7 \quad A_1$$

(5marks)

4. The number of boys in a school is 120. if the ration of boys to girls is 2 : 3, find the total number of students in the school.

Number of boys in the school = 120

Ratio of boys to girls = 2 : 3

Sum of ratio = 5

$$120 : \square = 2 : 3 \quad M_1$$

Let the unknown (number of girls) = X  $M_1$

Then,

$$120 : X = 2 : 3$$

$$\frac{120}{X} = \frac{2}{3}$$

$$2X = 360$$

$$\therefore X = \frac{360}{2}$$

$$X = 180$$

$$= 180 \text{ girls} \quad M_2$$

∴ The total number of students in the school is  $120 + 180 = 300$   $A_1$

(5marks)

5. If two students share 22 oranges in the ratio 9 : 2, how many oranges does each student get?

Number of oranges to be shared = 22  $M_1$

Ratio = 9 : 2  $M_1$

Sum of ratio = 11  $M_1$

1<sup>st</sup> student gets  $\left(\frac{9}{11} \times \underline{22}\right)$  oranges

= 18 oranges  $A_1$

2<sup>nd</sup> student gets  $\left(\frac{2}{11} \times \underline{22}\right)$  oranges

= 4 oranges  $A_1$  (5marks)

6. A farmer divides 180 cattle among his children in the ratio  $4r : 3 : s_2$ .  
How many cattle does each child get?

$$\text{Number of cattle to be divided} = 180 \quad M_1$$

$$\text{Ratio} = 4 : 3 : 2 \quad M_1$$

$$\text{Sum of ratio} = 9$$

$$\begin{aligned} \text{1}^{\text{st}} \text{ child gets } & \left( \frac{4}{9} \times \frac{180}{1} \right) \text{ cattle} \\ & = 80 \text{ cattle} \quad A_1 \end{aligned}$$

$$\begin{aligned} \text{2}^{\text{nd}} \text{ child gets } & \left( \frac{3}{9} \times \frac{180}{1} \right) \text{ cattle} \\ & = 60 \text{ cattle} \quad A_1 \end{aligned}$$

$$\begin{aligned} \text{3}^{\text{rd}} \text{ child gets } & \left( \frac{2}{9} \times \frac{180}{1} \right) \text{ cattle} \\ & = 40 \text{ cattle} \quad A_1 \end{aligned}$$

(5marks)

7. What percentage of 8kg is 400g?

$$1\text{kg} = 1000\text{g} \quad M_1$$

$$8\text{kg} = 8000\text{g} \quad M_1$$

.. 400g as a percentage of 8kg

$$= \left( \frac{400}{8000} \times \frac{100}{1} \right) \% \quad M_2$$

$$= 5\%$$

.. 400g is 5% of 8kg  $A_1$

(5marks)

8. In a school of 575 students, if 44% are boys. How many girls are in the school.

$$\text{Total number of students} = 575 \quad M_1$$



$$\text{Boys} = 44\% \quad M_1$$

$$\text{Girls} = (100 - 44)\%$$

$$= 56\% \quad M_1 \quad 14 \quad 23$$

$$\text{Number of girls} = \frac{56}{100} \times \frac{575}{1}$$

$$= 322 \text{ girls} \quad A_2$$

(5marks)

9. A lorry goes 320km in 4 days. What is its rate in km/hr?

$$\text{In 4 days the lorry goes 320km} \quad M_1$$

$$\text{In 1 day the lorry goes } \frac{320}{4} \text{ km} \quad M_2$$

$$= 80\text{km} \quad A_1$$

∴ The rate of lorry is 80km/hr  $A_1$

(5marks)

10. A car factory made 375 cars in 5 days.

Find its production rate in cars per day.

$$\text{In 5 days the factory made 375 cars} \quad M_1$$

$$\text{In 1 day the factory made } \frac{375}{5} \text{ cars} \quad M_2$$

$$= 75 \text{ cars} \quad A_1$$

∴ The production rate is 75 cars per day.  $A_1$

(5marks)

## APPENDIX I

TEST POST - MATHEMATICS ACHIEVEMENT TEST (POSTMAT)  
MARKING GUIDE

1. A man received N840 for 7 days of work.  
Find his pay if he works for 44 days.  
For 7 days the man received N840       $M_1$   
For 1 day the man received  $\frac{N840}{7}$        $M_1$
- .. For 44 days the man received  $(\frac{N840}{7} \times \frac{44}{1})$        $M_2$
- $= N5,280$        $A_1$   
(5marks)
2. If five men take 4 days to clear a portion of a farm. How long would it take 4 men to do the same work?  
For 5 people it takes 4 days       $M_1$   
For 1 person it takes  $(4 \times 5)$  days       $M_1$   
.. For 4 people it takes  $(\frac{4 \times 5}{4})$  days       $M_2$
- $= 5$  days.       $A_1$   
(5marks)
3. Express the ratio 20 minutes is to 2hrs in its simplest form.  
1 hour = 60 minutes       $M_1$   
2 hours =  $(60 \times 2)$  minutes  
= 120 minutes       $M_1$   
The ratio 20 minutes to 120 minutes  
= 20 : 120       $M_2$   
= 1 : 6       $A_1$   
(5marks)

4. The number of girls in a school is 180 if the ratio of girls to boys is 3 : 2, find the total number of students in the school.

Number of girls in the school = 180

Ratio of girls to boys = 3 : 2

Sum of ratio = 5

$180 : \square = 3 : 2$   $M_1$

Let the unknown (number of boys) =  $y$   $M_1$

Then,

$180 : y = 3 : 2$

$$\frac{180}{y} = \frac{3}{2}$$

$$3y = 360$$

$$\therefore y = \frac{360}{3}$$

$$= 120$$

$$= 120 \text{ boys} \quad M_2$$

$\therefore$  The total number of students in the school is  $180 + 120 = 300$   $A_1$

(5marks)

5. Share 28 mangoes among two students in the ratio 5 : 2 Find the number of mangoes each has.

Number of mangoes to be shared = 28  $M_1$

Ratio = 5 : 2  $M_1$

Sum of ratio = 7  $M_1$

1<sup>st</sup> student gets  $\left(\frac{5}{7} \times \underline{28}\right)$

$$= 20 \text{ mangoes} \quad A_1$$

2<sup>nd</sup> student gets  $\left(\frac{2}{7} \times \underline{28}\right)$  mangoes

$$= 8 \text{ mangoes} \quad A_1 \quad (5\text{marks})$$

6. Divide 240 chickens among poultry farmers in the ratio 5 : 4 :

3. How many chickens does each farmer get?

Number of chickens to be divided = 240  $M_1$

Ratio = 5 : 4 : 3  $M_1$

Sum of ratio = 12

1<sup>st</sup> farmers share =  $\frac{5}{12} \times \frac{240}{1}$

= 100 chickens  $A_1$

2<sup>nd</sup> farmers share =  $\frac{4}{12} \times \frac{240}{1}$

= 80 chickens  $A_1$

3<sup>rd</sup> farmers share =  $\frac{3}{12} \times \frac{240}{1}$

= 60 chickens  $A_1$

(5marks)

7. Express 400m as a percentage of 8km.

1km = 1000m  $M_1$

8km = 8000m  $M_1$

... 400m as a percentage of 8km

=  $\left(\frac{400}{8000} \times \frac{100}{1}\right)\%$   $M_2$

= 5%

.. 400m is 5% of 8km  $A_1$

(5marks).

8. A school has 425 students. If 50% are girls. How many boys are in the school?

Total number of students = 425  $M_1$

$$\text{Girls} = 56\% \quad M_1$$

$$\text{Boys} = (100 - 56)\%$$

$$= 44\% \quad M_1$$

$$\text{Number of girls} = \frac{44}{100} \times \frac{425}{1}$$

$$= 187 \text{ boys. } A_1$$

(5marks)

9. A bicycle goes 160km in 4hrs. Find its rate in km1hr.

$$\text{In 4 hrs the bicycle goes } 160\text{km} \quad M_1$$

$$\text{In 1 hr the bicycle goes } \frac{160\text{km}}{4} \quad M_2$$

$$= 40\text{km} \quad M_1$$

∴ The rate of the bicycle is 40km 1hr.  $A_1$

(5marks)

10. A machine factory made 434 machines in 7 days. Find its rate of production.

In machines per day

$$\text{In 7 days the factory made } 434 \text{ machines} \quad M_1$$

$$\text{In 1 day the factory made } \frac{434}{7} \quad M_2$$

$$= 62 \text{ machines} \quad M_1$$

∴ The production rate is 62 machines per day.  $A_1$

(5marks).

## APPENDIX J

THE DELAYED POST MATHEMATICS ACHIEVEMENT  
TEST/MATHEMATICS RETENTION TEST (DELPOST TEST/MRT)  
MARKING GUIDE

1. A boy is paid N420 for 3dys of work. What is his pay if he works for 44 days?

For 3 days the boy is paid N420 M<sub>1</sub>

For 1 day the boy is N  $\frac{420}{3}$  M<sub>1</sub>

∴ For 44 days the boy is paid N  $\frac{420}{3} \times \frac{44}{1}$  M<sub>2</sub>

= N6,160 A<sub>1</sub>

(5marks)

2. It takes 7 women 6 days to weed a farm.  
How long would it take 6 women to do the same weeding?

For 7 women it takes 6 days. M<sub>1</sub>

For 1 woman it takes (6x7) days. M<sub>1</sub>

For 6 women it takes  $\frac{(6 \times 7)}{6}$  days M<sub>2</sub>

= 7 days A<sub>1</sub>

(5 marks)

- 3 Express 4 days is to 4 weeks as a ratio in its simplest form.

1 week = 7 days M<sub>1</sub>

4 weeks = (7x4) days

= 28 days M<sub>1</sub>

The ratio 4 days to 28 days

$$= 4:28 \quad M_2$$

$$= 1:7 \quad A_1$$

(5marks)

- 4 A school has 240 boys. If the ratio of boys to girls is 4:6 what is the total Number of students in the school.

$$\text{Number of boys} = 240$$

$$\text{Ratio of boys to girls} = 4:6$$

$$\text{Sum of ratio} = 10$$

$$240 : \square = 4:6 \quad M_1$$

$$\text{Let the unknown (number of girls)} = P \quad M_1$$

Then,

$$240 : P = 4:6$$

$$\frac{240}{P} = \frac{4}{6}$$

$$4P = 1440$$

$$\therefore P = \frac{1440}{4}$$

$$P = 360$$

$$= 360 \text{ girls} \quad M_2$$

$$\therefore \text{The total number of students in the school is } 240 + 360 = 600 \quad A_1$$

(5marks)

5. 2 student share 18 apples in the ratio 7:2. Find the number of apples that each student will get.

$$\text{Number of apples to be shared} = 18 \quad M_1$$

$$\text{Ratio} = 7:2 \quad M_1$$

$$\begin{aligned} \text{1}^{\text{st}} \text{ student gets } & \left( \frac{7}{9} \times \frac{18}{1} \right) \text{ apples} \\ & = 14 \text{ apples} \quad A_1 \end{aligned}$$

$$\begin{aligned} \text{2}^{\text{nd}} \text{ student gets } & \left( \frac{2}{9} \times \frac{18}{1} \right) \text{ apples} \\ & = 4 \text{ apples} \quad A_1 \\ & \text{( 5 marks)} \end{aligned}$$

6. A woman divides 120 eggs among her children in the ratio 3 : 2:1. Find the number of eggs each child gets.

$$\text{Number of eggs to be divided} = 120 \quad M_1$$

$$\text{Ratio} = 3: 2: 1 \quad M_1$$

$$\text{Sum of ratio} = 6$$

$$\begin{aligned} \text{1}^{\text{st}} \text{ child gets } & \left( \frac{3}{6} \times \frac{120}{1} \right) \text{ eggs} \\ & = 60 \text{ eggs} \quad A_1 \end{aligned}$$

$$\begin{aligned} \text{2}^{\text{nd}} \text{ child gets } & \left( \frac{2}{6} \times \frac{120}{1} \right) \text{ eggs} \\ & = 40 \text{ eggs} \quad A_1 \end{aligned}$$

$$\begin{aligned} \text{3}^{\text{rd}} \text{ child gets } & \left( \frac{1}{6} \times \frac{120}{1} \right) \text{ eggs} \\ & = 20 \text{ eggs} \quad A_1 \\ & \text{(5marks)} \end{aligned}$$

7. What percentage of 10 m is 50 cm?

$$1\text{m} = 100 \text{ cm} \quad M_1$$

$$10\text{m} = (100 \times 10) \text{ cm}$$

$$= 1000\text{cm} \quad M_1$$

$$: 50\text{cm as a percentage of } 10\text{m}$$



$$= \frac{50}{1000} \times \frac{100}{1} \% \quad M_2$$

$$= 5 \%$$

: 50 cm is 5% of 10m  $A_1$

(5marks)

8. A school has 425 students. 44 % of them are boys. How many are girls?

Total number of student = 425  $M_1$

Boys = 44 %  $M_1$

Girls = (100 - 44)%

= 56 %  $M_1$

Numbers of girls =  $\frac{56}{100} \times \frac{425}{1}$

= 238 girls  $A_2$

(5 marks)

9. A lorry travels 480km in 6 hrs. What is its rate in km/hr?

In 6hrs the lorry travels 480km  $M_1$

In 1 hr the lorry travels  $\frac{480}{6}$   $M_2$

= 80km  $A_1$

The rate of the lorry is 80 km/hr  $A_1$

(5 marks)

10. A factory made 162 bicycles in 3 days. Find its rate of production in bicycles per day.

In 3 days the factory made 162 bicycles  $M_1$

In 1 day the factory made  $\frac{162}{3}$   $M_2$

= 54 bicycles  $A_1$

: The production rate is 54 bicycles per day.  $A_1$

(5 marks)

## COMPUTATION OF KENDALL'S COEFFICIENT OF CONCORDANCE

## Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum
PRERAT1	30	5.2667	5.7352	.00	20.00
PRERAT2	30	5.2667	5.7231	.00	18.00
PERRAT3	30	7.7000	6.0808	.00	23.00

## Kendall's W Test

## Ranks

	Mean Rank
PRERAT1	1.72
PRERAT2	1.63
PERRAT3	2.65

## Test Statistics

N	30
Kendall's W <sup>a</sup>	.707
Chi-Square	24.404
df	2
Asymp. Sig.	.000

a. Kendall's Coefficient of Concordance

## Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum
POSTRAT1	30	2.8333	4.5340	.00	16.00
POSTRAT2	30	3.3333	5.3002	.00	17.00
POSTRAT3	30	3.9000	5.1014	.00	17.00

## Kendall's W Test

## Ranks

	Mean Rank
POSTRAT1	1.68
POSTRAT2	1.82
POSTRAT3	2.50

## Test Statistics

N	30
Kendall's W <sup>a</sup>	.895
Chi-Square	17.718
df	2
Asymp. Sig.	.000

a. Kendall's Coefficient of Concordance

## Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum
DEL RAT1	30	1.2000	2.7965	.00	14.00
DEL RAT2	30	1.0667	2.3625	.00	11.00
DEL RAT3	30	1.9000	2.0736	.00	5.00

## Kendall's W Test

## Ranks

	Mean Rank
DEL RAT1	1.93
DEL RAT2	1.73
DEL RAT3	2.33

## Test Statistics

N	30
Kendall's W <sup>a</sup>	.693
Chi-Square	11.586
df	2
Asymp. Sig.	.003

a. Kendall's Coefficient of Concordance

APPENDIX L  
RESULTS OF DATA ANALYSES

Case Summaries

GROUP		PRETEST	POSTTEST	DELTEST
EXP1-IEPT	N	90	90	90
	Std. Deviation	4.6559	7.4290	7.0481
	Mean	13.3111	29.4222	33.1444
EXP 2- TLC	N	100	100	100
	Std. Deviation	4.6958	6.7421	6.1767
	Mean	13.7000	30.2800	32.5100
CONTROL	N	100	100	100
	Std. Deviation	1.1026	5.7145	4.1012
	Mean	10.4200	23.4600	25.7800
Total	N	290	290	290
	Std. Deviation	4.1041	7.2939	6.7380
	Mean	12.4483	27.6621	30.3862

Case Summaries

SEX		PRETEST	POSTTEST	DELTEST
Male	N	160	160	160
	Std. Deviation	3.6566	6.5497	5.7964
	Mean	11.9875	26.5875	29.6563
Female	N	130	130	130
	Std. Deviation	4.5460	7.9446	7.6698
	Mean	13.0154	28.9846	31.2846
Total	N	290	290	290
	Std. Deviation	4.1041	7.2939	6.7380
	Mean	12.4483	27.6621	30.3862

## Case Summaries

SEX		PRETEST	POSTTEST	DELTEST
Male	N	160	160	160
	Std. Deviation	3.5566	6.5497	5.7964
	Mean	11.9875	26.5875	29.6563
Female	N	130	130	130
	Std. Deviation	4.5460	7.9446	7.6698
	Mean	13.0154	28.9846	31.2846
Total	N	290	290	290
	Std. Deviation	4.1041	7.2939	6.7380
	Mean	12.4483	27.6621	30.3862

## Case Summaries

GROUP		PRETEST	POSTTEST	DELTEST
EXP-IEPT & TLC	N	190	190	190
	Std. Deviation	4.6686	7.0699	6.5937
	Mean	13.5158	29.8737	32.8105
CONTROL	N	100	100	100
	Std. Deviation	1.1026	5.7145	4.1012
	Mean	10.4200	23.4600	25.7800
Total	N	290	290	290
	Std. Deviation	4.1041	7.2939	6.7380
	Mean	12.4483	27.6621	30.3862

## Case Summaries

GROUP	SEX		PRETEST	POSTTEST	DELTEST
EXP 1- IEPT	Male	N	43	43	43
		Std. Deviation	4.1639	6.9375	6.1444
		Mean	12.2558	27.6744	31.7674
	Female	N	47	47	47
		Std. Deviation	4.9110	7.5742	7.6319
		Mean	14.2766	31.0213	34.4043
	Total	N	90	90	90
		Std. Deviation	4.6559	7.4290	7.0481
		Mean	13.3111	29.4222	33.1444
EXP 2- TLC	Male	N	69	69	69
		Std. Deviation	4.1070	6.0878	5.2071
		Mean	13.0145	28.7101	31.2174
	Female	N	31	31	31
		Std. Deviation	5.5720	6.9075	7.2142
		Mean	15.2258	33.7742	35.3871
	Total	N	100	100	100
		Std. Deviation	4.6958	6.7421	6.1767
		Mean	13.7000	30.2800	32.5100
CONTROL	Male	N	48	48	48
		Std. Deviation	8.440	4.9246	3.9464
		Mean	10.2708	22.5625	25.5208
	Female	N	52	52	52
		Std. Deviation	1.2897	6.2914	4.2633
		Mean	10.5577	24.2885	26.0192
	Total	N	100	100	100
		Std. Deviation	1.1026	5.7145	4.1012
		Mean	10.4200	23.4600	25.7800
Total	Male	N	160	160	160
		Std. Deviation	3.6566	6.5497	5.7964
		Mean	11.9875	26.5875	29.6563
	Female	N	130	130	130
		Std. Deviation	4.5460	7.9446	7.6698
		Mean	13.0154	28.9846	31.2846
	Total	N	290	290	290
		Std. Deviation	4.1041	7.2939	6.7380
		Mean	12.4483	27.6621	30.3862

## Case Summaries

SEX	GROUP		PRETEST	POSTTEST	DELTEST
Male	EXP -IEPT & TLC	N	112	112	112
		Std. Deviation	4.1268	6.4165	5.5649
		Mean	12.7232	28.3125	31.4286
	CONTROL	N	48	48	48
		Std. Deviation	.8440	4.9246	3.9464
		Mean	10.2708	22.5625	25.5208
	Total	N	160	160	160
		Std. Deviation	3.6566	6.5497	5.7964
		Mean	11.9875	26.5875	29.6563
Female	EXP -IEPT & TLC	N	78	78	78
		Std. Deviation	5.1694	7.3960	7.4369
		Mean	14.6538	32.1154	34.7949
	CONTROL	N	52	52	52
		Std. Deviation	1.2897	6.2914	4.2633
		Mean	10.5577	24.2885	26.0192
	Total	N	130	130	130
		Std. Deviation	4.5460	7.9446	7.6698
		Mean	13.0154	28.9846	31.2846
Total	EXP -IEPT & TLC	N	190	190	190
		Std. Deviation	4.6686	7.0699	6.5937
		Mean	13.5158	29.8737	32.8105
	CONTROL	N	100	100	100
		Std. Deviation	1.1026	5.7145	4.1012
		Mean	10.4200	23.4600	25.7800
	Total	N	290	290	290
		Std. Deviation	4.1041	7.2939	6.7380
		Mean	12.4483	27.6621	30.3862