

**MODELING AND EVALUATION OF RISK MEASURES FOR THE RESIDUALS OF
FINANCIAL TIME SERIES WITH UNOBSERVED VALUES USING R**

BY

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PG/M.Sc./08/49195

DEPARTMENT OF STATISTICS

UNIVERSITY OF NIGERIA

NSUKKA

AUGUST, 2015

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**BEING A PROJECT SUBMITTED TO THE DEPARTMENT OF STATISTICS,
UNIVERSITY OF NIGERIA, NSUKKA IN FULFILMENT OF THE REQUIREMENT
FOR THE AWARD OF THE DEGREE OF MASTERS OF SCIENCE (M.Sc.) OF THE
UNIVERSITY OF NIGERIA, NSUKKA, NIGERIA.**

SUPERVISOR: PROF F. I. UGWUOWO

AUGUST, 2015

TITLE PAGE

**MODELING AND EVALUATION OF RISK MEASURES FOR THE
RESIDUALS OF FINANCIAL TIME SERIES WITH UNOBSERVED
VALUES USING R**

CERTIFICATION

I here by certify that this project was carried out by me and no part of this work has been presented for a degree or diploma in this or any other university.

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28TH AGUAST, 2015

APPROVAL PAGE

This project has been read and approved as having met the requirement of the Department of Statistics and the School of Post Graduate Studies, University of Nigeria, Nsukka, for the award of a Masters of Science Degree in Statistics.

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DEDICATION

This project is dedicated to my loving wife and to God Almighty.

ACKNOWLEDGEMENT

I am grateful to God, the author and finisher of our faith, for seeing me through this study by His grace. Glory is to God. I use this medium to express my sincere gratitude to my supervisor, Professor F. I. Ugwuowo, for his advice and directives. He is fundamental for the logical conclusion of this research work.

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28TH AUGUST, 2015

ABSTRACT

In this work ARMA+GARCH model is adopted for the daily stock price of First Bank Nigeria, Plc. The methods of weekly average, regression imputation and repetition were used in computing the unobserved values. An alternative method was also used which involves deleting the days with unobserved values. The method of transformation was determined in each of the series and log transformation was adopted for the four series. In the model selection, the model of the repetition had the minimum AIC. The distribution of the residuals was found to be Frechet case of Generalized Extreme Value Distribution (GEVD). This was approximated to the Generalized Parato Distribution (GPD). The parameters of this distribution were used in computation of risk measures. The computed Value at Risk (VaR) has a value of ~~N~~49438.79 and that of the Expected Short Fall (ES) is ~~N~~49291.24 with position of ~~N~~1,000,000.

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CHAPTER ONE

INTRODUCTION

1.0. Preamble

Investment in financial market got to the peak between 2005 and 2007. In 2008, the market witnessed a down turn or setback thereby putting more risk to investment in the capital market. In order to guide against the unexpected, this research work will use an appropriate model to compute risk measures, especially the risk measures associated with investment on stock so as to overcome the uncertainty. The series of stock price is of the financial time series and according to Tsay (2005) both financial theory and its empirical time series contain an element of uncertainty. The choice of the model is of paramount important due to the volatile nature of the financial time series. The current financial crisis that hit most countries is as a result of substantial increase in volatility of stock market.

The financial market is faced with so many factors both within and without that may bring about bull or dull prices hence making the price volatile on daily bases. The financial crisis in Nigeria was caused by some factors as opined by Sere-Ejembi (2008). These factors are the global financial crisis, the subsequent plumbing of the other major indicators to date, the divestment by foreign investors and the panic disposal by local investors.

The time series data of First Bank Nigeria, Plc stock price will be used to bring out a model that will be suitable to such data, that is, financial data. Therefore, the daily returns from the daily price will be computed. The reasons for using the returns of an asset is given by Campbell, Lo, and MacKinlay (1997) as a complete and scale-free summary of the investment opportunity and easier to handle than price series. This is because the returns have more attractive statistical properties (Tsay, 2005).

The model in this study is based on conditional mean structure and conditional variance which measures the uncertainty in the deviation. Therefore, the changes in prices of

a financial time series as the stock series required conditional variance. The daily returns of stocks are observed to have a larger conditional variance during volatile period than a stable period (Cryer and Chan, 2008). Based on this most research carried out in financial time series has been diverted from Auto Regressive Integrated Moving Average (ARIMA) model to the use of Generalized Autoregressive Conditional Heteroscedastic (GARCH) model especially in analyzing and forecasting volatility. Based on the nature of financial time series, especially that of stock price with its volatility and conditional mean structure, an extension of GARCH is required.

This leads to a combination of ARMA and GARCH model, which is written as ARMA (u,v)+GARCH (p,q) . The distribution of the residuals from the fitted model is to be determined. The parameters of the fitted model from the residuals, which has no ARCH effect are then used in getting the Value at Risk (VaR) and Expected Shortfall (ES) (Bystrom, 2005).

1.1. Historical Background of First Bank Nigeria, Plc .

First Bank Nigeria, Plc was incorporated in 1894 in Liverpool as the Bank for British West Africa. The first international branch was opened in Accra, Ghana in 1896 and commenced business in Nigeria as Standard Bank of Nigeria Ltd in 1969. The Bank was listed on the Nigerian Stock Exchange in 1971 and in 2007 floated its first ever hybrid capital offering outside Africa. The bank currently has its branches in all states of the federation with offshore financial subsidiary in the United Kingdom (UK)(<http://firstbanknigeria.com>).

1.2. Stock Exchange Market

The activity in the stock exchange market is an indicator of the national economic growth and development. In fact there is a direct link between the capital market and Foreign Direct Investment (FDI) generation. Stability of capital market indicates stability in the

economy, sources of finance for economic development (public and private), investment avenue for surplus fund and so forth (Oladipupo, 2010)

Nigerian Stock Exchange (NSE)

The Nigerian Stock Exchange (NSE) is the center point of the Capital Market. It was established in 1960 as Lagos Stock Exchange in Lagos and commenced operation in 1961 with nineteen listed companies. The Lagos Stock Exchange metamorphosed to Nigerian Stock Exchange (NSE) in December, 1977. Currently NSE has six branches with trading floor in Kaduna, Port Harcourt, Kano, Onisha, Ibadan, Yola and Abuja. The Nigerian Stock Exchange is a registered company limited by guaranty and is licensed under the Investments and Security Act (ISA). The Regulatory body of NSE is the Securities and Exchange Commission (SEC) of Nigeria.

The NSE is a founding member and executive committee member of the African Securities Exchanges Association (ASEA) and an affiliate member of World Federation of Exchanges (WFE) and the International Organisation of Securities Commissions (IOSCO) (www.nse.com.ng). Nigerian Stock Exchange is an automated exchange and the services provided are; listing and trading services, electronic clearing, settlement and delivery through Central Securities Clearing System (CSCS) Plc, market data dissemination services, market indices and so on. The type of securities traded include Industrial Loan Stocks, Government Bonds and Equity or Ordinary Shares. The NSE has 283 listed companies with a total market capitalization of \$125 billion as at the end of 2007 (www.nse.com.ng).

1.2.1. Stock Market Operation

Nigerian Stock Exchange Markets's major instruments of raising fund are through stocks, equities, bonds and debentures. The market consist of two segments; primary market

and secondary market. The primary market includes offer for subscriptions, rights issues, offer for sales and private placement. While the secondary segment is trading on existing issues which includes over the counter deals where securities are bought and sold after issuance in the primary market. The capital market operators includes Issuing Houses like Merchant Banks and Stock Broking Firms, Stock Brokers, Trustees and Registrars (www.nigeriasite.com).

The CSCS is an associate company to the NSE which also offers custodian services. The market also offers data dissemination services and market indices (www.nse.com.ng).

1.2.2. Stock Behaviour or Movement.

The stock price movement, whether up or down, is very important to investors. There are four modern theories that try to determine price behaviour of stocks and other securities. These are highlighted by Nwude (2012) as the Fundamentalists, the Technicalists (or Chartists), the Random Walkists., and the Efficient Market Hypothesis.

The Fundamentalists rely on history of stock price behaviour coupled with economic and political news to determine why, when and where price of stocks will move in future. The Fundamentalists study causes of movement of the stock price in the past and use them to determine the current movement especially when those events of the past are repeated in the present. The Technicalists believe that the future price can be determined by the pattern and trend of the previous price movement. Therefore, depending on the time horizon, the technical approach can be used based on intra day, daily, weekly, monthly or quarterly prices of stock. The statistical tools used by the Technicalists are stochastic indicators, moving average, ratios, returns and so on.

The third group, the Random Walkists see stock price movement as an event that does not depend on successive events (stock price) making price movement of stock independently

and identically distributed random events. Hence, each stock is assumed to have an intrinsic value based on investors' expectations of the discounted values of future cash flows generated by the stock (Nwude, 2012). The last but not the least is the Efficient Market Hypothesis (EMH) as spearheaded by Fama (1970). This is a broader theory of stock price movements. Market efficiency is determined by allocating scarce resources to their most productive uses. In a security market, efficiency is attained if current security prices reflect all information about each security and quickly adjust to all available information.

1.3. Market Risk Management

In any investment there is risk associated to it, meaning that for any business to be successful the investor must consider the risk involved. Sometimes the risks are measured with a quantity (probability). Therefore, the investment in capital market is not exempted from this associated risk because of the volatile nature of the price movements. Market risk can be defined as the risk of losing the value of a financial institution's proprietary trading holdings in equity, debt, commodity instruments as a result of fluctuations in market prices (Blaauw, 2009). There are other ways market risk can occur, for instance; management of client money and using capital market instruments as collateral for loan.

A lot of ideas have evolved on market risk that led to having a conventional approach, that is, the Basel I and II Market Risk Framework. The Basel I was concluded and adopted as amended in 1998 which provides methods of determining market risk capital requirement from simple, intermediate and advanced (internal model) approaches. The Basel II was implemented globally in 2008 which proffer additional requirement as oversight role, disclosure, management of counterparty risk in trading portfolio and so on (Blaauw, 2009).

The factors that affect market risk are exchange rate, commodity price, interest rate and equity prices (FBN Annual Report and Account, 2010). The most commonly used market risks are Value at Risk (VaR) and Expected Shortfall (ES).

1.4. Objectives of the Study

The objectives of this study are:

- i. To fit an appropriate model on the returns of daily stock price
- ii. To forecast the returns and volatility of daily stock price
- iii. To determine the appropriate model of the residuals
- iv. To determine risk measures associated to investment in stock.

CHAPTER TWO

2.0 Review of Related Literature

There have been many works on time series and the nature of the series determines the model to be focused on. The series in this research is a financial time series, that is, the daily stock price of First Bank of Nigeria Plc (FBN). In the series, there are some days without recorded price due to non operation by the Stock Exchange Market which become a concern. The Nigerian Stock Exchange Market does not open its floor on week days of public holidays and during civil disturbances or crisis, thus causing the unobserved values.

Many researchers such as Fung (2006) focused on different methods of estimating unobserved values. These include state space modeling, time series decomposition least square approximation and numerical interpolation in estimating unobserved values. In the work of Velicer and Colby (2005) four different missing-data methods; deletion, mean substitution, mean of adjacent observations and maximum likelihood estimation were used to estimate four measures; level, error variance, degree of autocorrelation and slope. The result indicated that the maximum likelihood estimate gave the best of four measures considered from the simulated data. This research work will estimate the unobserved values of the daily stock price of First Bank Nigeria, Plc from 1998 to 2007 before modeling. This is a diversion of the least square principle of Ferreiro, (1987) which the model is identified before using the fitted model to estimate the unobserved values

Some researchers go for outright omission of the periods with missing values and mostly silent on that aspect as seen in (Hamadu and Mojekwu, 2010), an investigation of Nigerian insurance stock option prices. Contrary to the outright omission, Honaker and King (2010) adopted the use of modern methods of computing missing value because deleting the days with missing values can bring about biased and inefficiency. Three methods were

adopted and they are multiple imputation method, analysis to incorporate knowledge from area experts via priors on individual missing cells values and developing of an algorithm that substantially expands the range of computationally feasible data types and sizes. In other to see the extent of the effect of out right deletion, the series from the outright deletion need to be compared with series of estimated unobserved values.

Shukur and Lee (2015) in the study of wind speed in Iraq and Malaysia made a comparison of the classical and AR-ANN methods of imputation. The classical methods include linear, nearest neighbor, and state space. Their work focused on computing the missing values before analysing the time series of wind speed except in Auto Regressive (AR) where deleting method of dealing with missing values was adopted. The time series was non linear which led to the use of the hybrid artificial neural network (ANN) method that is capable of imputing the unobserved values in wind speed data. While Auto Regressive (AR) model is used for determining the structure of the input layer for the ANN. In the analysis using ARIMA, the AR-ANN performed better than linear, nearest neighbor and state space. They had already considered AR, even though with deleting approach, before the use of ARIMA to test the effectiveness.

In some instances the days without dealings in stock market is overtaken by considering weekly or monthly stock price averages instead of daily prices. Chow and Lawler(2003) used weekly data in studying the time series of Shangai and New York stock price indices. Ajie and Nenbee (2010) used yearly data of stock price in analyzing the monetary policy and stock price in Nigeria. But the effect of compressing the data which may lead to loss of information can not be ruled out.

Another challenge of time series analysis is on model specification but an in depth knowledge of the behavior of the series is of immense assistance in modeling the series. In case of non-stationary model, the first and higher differences are stationary (Box and Jenkins,

2008). In Box - Jenkins modeling of the Nigerian stock prices (Etuk, Uchendu and Udo, 2012) used first differencing for stationarity because the differencing exhibited no trend and degree of variability is invariant with time. The best model for the Nigerian stock price was considered to be ARIMA (2, 1, 3) which was validated by different tests. The volatility nature of such financial series was ignored by this study for not introducing any of the Generalised Autoregressive Conditional Heteroscedastic (GARCH) family models,

In order to remove the effect of non stationarity (Ajie and Nenbee, 2010) used logarithm to transform the yearly stock price. Dwivedia and Rao (2011) applied discrete Fourier transform to test for second-order stationarity by constructing Portmanteau type test statistic for testing stationarity of the time series. This was found by them to be of a good power with some simulations.

Other areas of importance are choosing a tentative model for time series (model specification), estimation of the tentative model parameters and checking for the fitness of the model for purpose of forecasting. This process can be described as model building strategy. Cryer and Chan (2008) in the book *Time Series Analysis with Application in R* opined that with few iterations of the model building strategy, the hope is to choose the best possible model for a given series. The book described the methods of building strategy as being so common among authors who referred to it as Box and Jenkins method. This is because the method was made popular by Box and Jenkins (1976) text.

Hamadu and Mojekwu (2010) in their work used Expert Model Mining System (EMMS) which incorporates Exponential Smoothing, Autoregressive Integrated Moving Average (ARMA) and Seasonal Autoregressive Integrated Moving Average (SARIMA). It was discovered that the best suitable of the models for most stock option price is that of exponential smoothing based on mean square error and coefficient of determination. While Iqbal and Hamid (2000) considers the use of Vector Autoregressive (VAR) analysis to

examine the relationship between stock price and operating performance a firm. The stock price of ESOP firm was used and the analysis showed that there is a relationship between stock price and subsequent performance of ESOP firm, especially when the stock price fall the firm improves its performance.

Alnaa and Ahiakpor (2011) in their study of inflation in Ghana used Autoregressive Integrated moving Averages (ARIMA) with Box-Jenkins's approach. It was discovered that the monthly inflation rate in Ghana is integrated of order one with Autoregressive of order six and moving average of order 6, that is, ARIMA (6, 1, 6). The prediction using this model shows that inflation was highest in the months of March, April and May. Wacławski (1981) also used ARIMA in comparing the forecasted value and actual value of CPU workloads for open systems. There was no significant difference between the forecasted and actual values of the workloads at 5 % level of significant. ARIMA can also be applied to a non financial data as in the work of Earnest et al (2005) who used ARIMA on the number of beds occupied during a SARS outbreak in Singapore. Some of the covariate considered was the daily number of people screened and the daily number of people admitted. The simplest parsimonious order of the model was the ARIMA (1, 0, 3).

Financial time series such as daily stock price is highly volatile. For instance, the mean of the daily returns of the New York Stock Exchange from (NYSE) from February 2, 1984 to December 31, 1991 appears to be stable (approximately zero) but with volatility clustering, that is, highly volatile (Shumway and Stoffer, 2011). They concluded that the variability of the data changes over time. This problem led to development of new models to handle the volatility as the Autoregressive Conditionally Heteroscedastic (ARCH) model by Engle (1982) who used the model, ARCH (4), in estimating the means and variances of inflation in United Kingdom. He made a comparison of ARCH (4) model with popular used Ordinary Least Square model and discovered the usefulness of ARCH model in improving

the performance of least square model and for a better forecast of the variance. His work was later extended to Generalized Autoregressive Conditionally Heteroscedastic (GARCH) model by Bollerslev (1986). He studied the inflation of United State from 1948 to 1988 using Generalized Autoregressive Conditionally Heteroscedastic (GARCH) model comparing GARCH (1, 1) and ARCH (8), two fitted models. He discovered that GARCH (1, 1) not only slightly better model but exhibits a more reasonable lag structure.

Bollerslev (1988) also carried out a study on the Correlation Structure for the Generalised Auto Regressive Conditional Heteroscedastic(GARCH) Process. It was found out that the auto correlation of the square error of the GARCH resembles that of Auto Regressive Moving Average. This conclusion was derived from illustrative examples of different orders on data like IBM stock market price and US dollar – Swiss franc exchange rate. It was recommended in the work that the order of ARMA can be used to determine the order of GARCH.

In line with these models Lo (2003) used ARCH (1) and GARCH (1, 1) on S&P 500 Stock Price Index. The final result showed that ARCH(1) could not adequately explain the S&P 500 Stock Price Index from the test of normal probability plot and the Q-statistic. There was an improvement on the model from ARCH (1) to GARCH (1, 1). It was suggested that there will be an improvement in GARCH (2, 1). Another extension to ARCH/GARCH model in order to improve the model is the ARCH in mean (ARCH-M) by Engle, Lilien and Robbins (1987). In his handbook Pynnonen (2014) used ARIMA (p, d, q)-GARCH (p,q)-M with conditional t-distribution for an illustration whether the mean return of SP500 depend on the volatility level. The MA(1)-GARCH(1,1)-M with conditional t-distribution was used. The test revealed that the mean return does not significantly depend on return level. This is because the constant term and volatility term of the mean equation are not significant. Engle, Lilien, and Robins (1987) studied the excess holding yield of six-month over three-month

treasury bills. In the study ARCH-M model was adopted for quarterly data from 1960 to 1984. They discovered that a significant time-varying risk premium as proxies by the logarithm of the conditional variance provided the best fit for the data after experimenting with different functional forms. Also, on the average the term premium was only 0.14 quarterly percent, but it varies in a systematic way through the sample with a noticeable exception of the yield spread. Also, when a function of the conditional variance is included as a regressor, variables which had previously been found successful in forecasting excess returns generally are no longer significant.

Further study on volatility models is the extreme value distribution on financial time series based on minimum and maximum returns. The limiting distribution of the normalized minimum of the returns is taken to follow Generalized Extreme Value Distribution (GEVD) of Jenkinson (Tsay, 2005). Zhao (2010) proposed model was from the assumption that series came from the Generalized Extreme Value (GEV) distribution following AR(1)-GARCH(1, 1) process which describes the conditional variance of extremes. In his work, the one-stage approach was adopted. The GEV-GARCH (proposed model) from observed and simulated data captured the dynamic of conditional variance of extremes and to model the tail behaviour of the underlying variables.

The two-stage approach, using the proposed model to get the residual before using GEV distribution, has the advantage of removing the volatility dependence using a volatility model like a GARCH. This was applied in the study of Bystrom (2005) where AR (1) +GARCH (1, 1) was used to get the residuals in analysing the tail behavior. In the paper, the hourly spot prices on NordPool and extreme value theory (block maxima method) was used to investigate the tails of the price change distribution which the GEVD was of good fit.

Chavez-Demoulin and Embrechts (2009) also carried a review of Extreme Value theorem, its usefulness and misused in computation of credit/ management risk. They

considered the one-dimensional EVT and more-dimensional EVT. Mcneil and Frey (2000) used extreme value theory in estimation of tail related risk measures for heteroscedastic financial time series by applying the block maxima in getting the extreme values. Thereafter, computed the Value at Risk (VaR) and Expected Shortfall (ES) of the returns. Backtesting was used which shows that their procedure was better estimates than methods which ignore the heavy tails of the innovations or the stochastic nature of the volatility.

Another idea to the computation of risk measures is the approximation of Generalised Extreme Value Distribution (GEVD) to Generalised Pareto Distribution (Tsay, 2005). In work of Santos and Alves (2010) this idea was applied in their paper on VaR Prediction with a duration based Peak Over Threshold (POT) method which was used to carry out one-day-ahead VaR. They also used duration based Peak Over Threshold (DPOT) on the forecast which the DPOT showed a better estimate.

In most of the researches, some fundamental issues were ignored, like using a statistical tool to determine the type of transformation, estimation of the unobserved values in time series and outright use of GARCH models without the determination of the presence of ARCH in the series. Some went as far as using ARMA on purely financial time series. These areas will be addressed in this study in order to have efficient and reliable results.

CHAPTER THREE

METHODOLOGY

3.1. Source of Data

The data used in this work was obtained from the Nigerian Stock Exchange Market and the stock considered is that of First Bank Nig. Plc from 1998 to 2007. The daily stock price of the bank was collected based on days of activities (business) in the market. The series is subjected to some computation of unobserved values caused by some factors as civil disturbance and holidays.

3.2. Unobserved Values

The financial time series data of this nature always come with unobserved values. These are as a result of days without trading on the floor of stock exchange market. The problem posed by the unobserved values can not be over emphasised. Therefore, adequate care must be taken in handling the unobserved values to bring about efficiency. The type of series should determine the method to use in dealing with unobserved values. The use of list wise deletion should be discouraged because it can lead to one third deletions of the data and the use of zero for missing data can lead to misrepresentation, (Chao-Ying, and Lee, 2003). The duo observed that Quality of educational research will be enhanced if researchers explicitly acknowledge missing data problems and the conditions under which they occurred.

Some authors and researchers more than a decade ago have made it a routine to estimate (or impute) the unobserved values before the actual data analysis. Even though, new methods of missing value imputation have been in use for some times now but new imputation approaches are still being developed at an increasing rate. However, what is lagging behind is a systematic and objective evaluation of the strengths and weaknesses of

the different approaches when faced with different types of data sets and experimental questions, (Tero, 2009). This project work will look into estimation of missing data before using the data to determine the model. This approach is considered instead of the traditional time series analysis approach commonly directed toward scalar-valued data, and can be represented by traditional Box-Jenkins's autoregressive, moving average, or autoregressive-moving average models, (Fung, 2006).

Four methods of dealing with the unobserved values are used. These are; the average imputation by Velicer and Colby (2005), regression imputation by Ehman (2003), the preceding value or repetition approach and eliminating or deleting of days approach.

1. Regression Imputation – This is the prediction of observed values from other values using regression model and the model is then used to impute values where there is unobserved value.

2. Average Imputation – This is the replacement of the missing value with the arithmetic average.

3. Repetition Approach – This is the process of replacing the missing value with the preceding value.

4. Deleting Approach – This is outright removal of the variable at the point of unobserved value.

3.3. Transformation of a Financial Time Series

The transformation of the series will be based on the Taylor's Power Law, that is, by plotting the log variance against log mean, and estimating the slope (b) of the line. The table below will stand as a guide.

Table 1: Slope and Transformation

Appropriate transformations for given values of $1 - (b/2)$			
b	$1 - b/2$	Transformation	Y'
0	+1.0	no transform	Y^1
1	+0.5	square root	$Y^{0.5}$
2	0.0	logarithmic	$\log[Y]$
3	-0.5	reciprocal square root	$Y^{-0.5}$
4	-1.0	reciprocal	Y^{-1}

<http://influentialpoints.com/Training/transformations-principles-properties-assumptions.htm>

3.4. Stationarity

The two major classification of time series model is whether it is stationary model or non-stationary model. A time series is said to be stationary if the mean value function, μ_t , is constant and does not depend on time t , and the autocovariance function, $\text{cov}(s; t)$, depends on s and t only through their difference $(s-t)$ (Shumway and Stoffer, 2011).

The stationarity or non-stationarity can be determined by using

1. Time Plot – This is a simple method which entails plotting of series against the time on a graph to see the pattern of the movement of the series. If the scattered point shows a trend, then non-stationary model is applicable otherwise is stationary.
2. Sample Auto Correlation Function (ACF) – If the series is non-stationary, the sample ACF shows an approximate linear decay which is often taken as a symptom that the underlying time series is nonstationary. (Cryer and Chan, 2008).
2. Unit Root Test- the The process $\{Y_t\}$ is nonstationary if the coefficient $\tilde{\alpha} = 1$, but it is stationary if $|\tilde{\alpha}| < 1$.

$$Y_t = \alpha Y_{t-1} + X_t$$

for $t = 1, 2, \dots$

The first aspect Dickey and Fuller (1979) is on testing the coefficient of regression equation, by regressing the first difference of the observed time series on lag 1 of the observed series and on the past k lags of the first difference of the observed series. The second approach in the test is Augmented Dickey-Fuller (ADF) test statistic which is the t -statistic of the estimated coefficient from the method of least squares regression (Cryer and Chan, 2008).

3.5. Models

The series, daily stock price, under study is a financial time series and the series will be transformed to its returns. The returns of investment on the stock price is considered by Cryer and Chan (2008) as the difference of the natural logarithm of the price series, that is, $r_t = \log(y_t) - \log(y_{t-1})$, where r_t is the continuously compounded return on the t th day and the y_t is time series. There are other approaches of computing returns, for instance simple returns which is given as $r_t = \frac{y_t}{y_{t-1}} - 1$. The approach depends on the method of transformation of the series, but where transformation is not required the simple returns is used.

Financial time series is highly volatile being an indication of a higher variability with higher level of stock value. Based on this, the Autoregressive Conditional Heteroscedastic (ARCH) model was first proposed by Engle (1982). The ARCH model considers the conditional variance of time series that measures the uncertainty in deviation of the time series from the conditional mean (Cryer and Chan, 2008).

3.5.1. The ARCH(p) Model

It should be noted that before the Autoregressive Conditional Heteroscedastic (ARCH) model there was the Autoregressive Integrated Moving Average (ARIMA). The ARIMA model is concerned about the conditional mean structure of time series with the assumption that the conditional mean is constant. The ARIMA(p, d, q) at time was not able to handle the volatility nature of financial series, where in practice, the (one-step-ahead) conditional variance may vary with the current and past values of the process, making it a random process known as the conditional variance process. The daily returns of stocks are often observed to have larger conditional variance following a period of violent price movement than a relatively stable period (Cryer and Chan, 2008).

However, the inadequacy of ARIMA model on financial series brought about the introduction of ARCH (p) model by Engle (1982). The ARCH (p) model as stated by Cryer and Chan (2008) as a regression model with the conditional volatility as the response variable and the past lags of the squared return as the covariates given below. But in Tsay(2005) the past lags of the squared shock of an asset is used as dependent variables in place of past lags of the squared return. The basic idea of ARCH models is that the returns r_t , given in terms of deviation $\tilde{r}_t = r_t - \mu$, of an asset is serially uncorrelated, but dependent, and the dependence of $\tilde{r}_t$ can be described by a simple quadratic function of its lagged values.

$$\begin{aligned}\tilde{r}_t &= \sigma_{t/t-1} \varepsilon_t, \\ \sigma_{t/t-1}^2 &= w + \alpha_1 \tilde{r}_{t-1}^2 + \dots + \alpha_p \tilde{r}_{t-p}^2\end{aligned}$$

Where

$\{\tilde{r}_t\}$ is a return series which is often a serially uncorrelated sequence with zero mean

ε_t is independent and identically distributed (iid) random variables with mean zero and variance 1, otherwise known as innovations or standardized residuals.

$\sigma_{t/t-1}^2$ is the conditional variance of $\tilde{r}_t$

α_i are the coefficients greater than zero

It should be noted that the squared return, $\tilde{r}_t^2$, provides an unbiased estimator of $\sigma_{t/t-1}^2$

For an illustration, the ARCH (1) model is

$$\begin{aligned}\tilde{r}_t &= \sigma_{t/t-1} \epsilon_t, \\ \sigma_{t/t-1}^2 &= w + \alpha_1 \tilde{r}_{t-1}^2\end{aligned}$$

The ARCH(q) model has the following weaknesses;

- i. The square of the returns removed the negative effect of the returns
- ii. The model is restrictive, for instance, the ARCH(1) model must be in the interval [0, 1/3] if the series has a finite fourth moment (Tsay, 2005).
- iii. The model does not expose a new understanding of the source of variations of a financial time series.
- iv. ARCH model respond slowly to large isolated shocks to the return series.

The improvement to ARCH was the introduction of Generalized Autoregressive Conditional Heteroscedastic, GARCH, which is another approach proposed by Bollerslev (1986) and Taylor (1986). This brought about the combination of ARCH order p with GARCH order q in order to overcome the weaknesses of ARCH. This leads to GARCH (p, q) model.

3.5.2. GARCH(p,q) Model

The Generalized Autoregressive Conditional Heteroscedastic model, GARCH(p, q), is given as

$$\begin{aligned}\tilde{r}_t &= \sigma_{t/t-1} \epsilon_t, \\ \sigma_{t/t-1}^2 &= w + \alpha_1 \tilde{r}_{t-1}^2 + \alpha_2 \tilde{r}_{t-2}^2 + \dots + \alpha_p \tilde{r}_{t-p}^2 + \beta_1 \sigma_{t-1/t-2}^2 + \dots + \beta_q \sigma_{t-q/t-q-1}^2\end{aligned}$$

Where

$\beta_i > 0$, are the GARCH parameters

$\alpha_i > 0$, are the ARCH parameters

This model can be expressed in terms of Backshift as

$$(1 - \beta_1 B - \dots - \beta_q B^q) \sigma_{t/t-1}^2 = w + (\alpha_1 B + \dots + \alpha_p B^p) \tilde{r}_t^2$$

(Cryer and Chan, 2008)

$\{\tilde{r}_t\}$ is a return series which is often a serially uncorrelated sequence with zero mean

ϵ_t is independent and identically distributed (iid) random variables with mean zero and variance 1, otherwise known as innovations or standardized residuals.

$\sigma_{t/t-1}^2$ is the conditional variance of $\tilde{r}_t$.

Note that, if r_t is not used in terms of deviation, therefore, $r_t = \mu + \tilde{r}_t$.

The coefficients in a GARCH model are often constrained to be nonnegative, that is, $B_i > 0$ and $\alpha_i > 0$, because conditional variances must be nonnegative. It should be noted that the nonnegative parameter constraints are not necessary for a GARCH model to have nonnegative conditional variances with probability 1 (Nelson and Cao, 1992) and (Tsay and Chan 2006).

Computation of Conditional Variances

The computation of the conditional variances for this work will be illustrated using the GARCH model preferably GARCH (1,1) model.

$$\tilde{r}_t = \sigma_{t/t-1} \epsilon_t$$

$$\sigma_{t/t-1}^2 = w + \alpha_1 \tilde{r}_{t-1}^2 + \beta_1 \sigma_{t-1/t-2}^2$$

Suppose that there are n observations $\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_m$ to compute the conditional variances for

2. To find σ^2 , we need to set the initial value $\sigma_{1/0}^2$. This may be set to the stationary unconditional

variance $\sigma^2 = \frac{w}{1 - \alpha_1 - \beta_1}$ assuming stationarity or simply as $E(\tilde{r}_t^2)$. Thereafter, we can

compute

$$\sigma_{t/t-1}^2 = (1 - \alpha_1 - \beta_1)\sigma^2 + \alpha_1\tilde{r}_{t-1}^2 + \beta_1\sigma_{t-1/t-2}^2$$

(Cryer and Chan, 2000).

3.5.3. Extension to GARCH(p, q)

One of the extensions to GARCH(p,q) is ARMA(u,v)+GARCH(p, q). The ARMA model is considered with GARCH because in real life the assumption of conditional mean being zero may not hold (Cryer and Chan, 2008). In the more general case, the conditional mean structure may be modeled by some ARMA(u,v) model, thereafter the white noise term of the ARMA model is used to model GARCH(p, q) giving a combination of the two models as ARMA+GARCH(p, q).

Let r_t be a time series, then ARMA(u,v)+GARCH(p, q) given by

$$r_t = \phi_0 + \phi_1 r_{t-1} + \dots + \phi_u r_{t-u} + e_t - \theta_1 e_{t-1} - \dots - \theta_v e_{t-v}$$

$$e_t = \sigma_{t/t-1} \varepsilon_t$$

$$\sigma_{t/t-1}^2 = w + \alpha_1 e_{t-1}^2 + \dots + \alpha_p e_{t-p}^2 + \beta_1 \sigma_{t-1/t-2}^2 + \dots + \beta_q \sigma_{t-q/t-q-1}^2$$

If the return is in deviation, then

$$\tilde{r}_t = \phi_1 \tilde{r}_{t-1} + \dots + \phi_u \tilde{r}_{t-u} + e_t - \theta_1 e_{t-1} - \dots - \theta_v e_{t-v}$$

Where $\phi_0 = \mu$ (the mean).

Cryer and Chan (2008) and Tsay(2005).

3.5.4. Test for ARCH Effect

The autoregressive conditional heteroscedastic effect in a financial series should not only be assumed but tested using the following methods.

Ljung–Box statistics

The first aspect of this test is applying the usual Ljung and Box (1978) $Q(m)$ statistic (Tsay, 2005) for the square data or residual as shown below to test for ARCH effect.

$$Q(m) = T(T+2) \sum_{\ell=1}^m \frac{\rho_{\ell}^2}{T-\ell}$$

$Q(m)$ follows approximately chi-square distribution with m (number of correlation) degree of freedom. This test according to Cryer and Chan (2008) can also be referred to as McLeod or Li test. The second aspect is of testing for conditional heteroscedasticity is the Lagrange multiplier test of Engle (1982). This test is equivalent to the usual F statistic for testing in the linear regression.

3.5.5. The GARCH(p,q) Order

The order of GARCH model can be determined the same way as in the ARMA model (Bollerslev, 1988). In identifying pure AR(u) or pure MA(v), the sample Autocorrelation Function (ACF) and partial correlation Function (PACF) are effective tools. However, in a mixed ARMA model, the theoretical ACF and PACF have infinitely many nonzero values, making it difficult to identify mixed models from the sample ACF and PACF (Cryer and Chan, 2008). Therefore, other methods of graphical tools have been proposed to make it easier for the identification of the ARMA orders. These are; the corner method (Becuinj et al., 1980), the Extended Autocorrelation Function (EACF) method and the Smallest Canonical Correlation (SCAN) method (Tsay and Tiao, 1984). The EACF method used have good sampling properties for moderately large sample sizes according to a comparative simulation study done by Chan (1999).

The EACF method depends on the fact that if the AR part of a mixed ARMA model is determined, filtering out the autoregression from the observed time series results in a pure MA process that enjoys the cutoff property in its ACF. The AR coefficients may be estimated by a finite sequence of regressions (Cryer and Chan, 2008).

The EACF is displayed in a two-way table, where the rows correspond to AR order p and the columns to MA order q . In general, determining the order of $ARMA(p, q)$ model, the triangle of ϕ 's will have its upper left vertex at the (p, q) position (Tsay, 2005).

3.5.6. Generalised Extreme Value Distribution (GEVD).

This is the distribution that is for extreme values, say $r_{(1)} = \min\{r_j\}$ and $r_{(n)} = \max\{r_j\}$. Where $r_{(1)}$ and $r_{(n)}$ are iid random variables. The Extreme Value Theory (EVT) concerns itself with limiting the distribution of normalised minima $r_{(1)}$, in either way the minima $r_{(1)}$ can be expressed in terms of its maxima as $r_{(1)} = -r_{(n)}$. This maxima is derived from block maxima method and it's only possible when non-degenerate limiting distributions are in the GEV family. Under the independent assumption, the limiting distribution of the normalized minimum is given by the distribution function of the Generalized Extreme Value Distribution (GEVD) of McNeil et al (2005) as

$$F_{\xi}(x) = \begin{cases} \exp(-(1+\xi x)^{-\frac{1}{\xi}}), & \xi \neq 0 \\ \exp(-\exp(-x)) & \xi = 0 \end{cases}$$

where

$$(1 + \xi x) > 0$$

ξ is the shape parameter .

Hence, a three parameter family is obtained by defining $F_{\xi, \mu, \sigma}(x) := F_{\xi}((x - \mu)/\sigma)$.

Therefore, $F_{\xi, \mu, \sigma}(x)$ is of type $F_{\xi, \mu, \sigma}((x - \mu)/\sigma)$.

Where σ is the scale parameter and μ the location parameter.

This can be written as the Cumulative Distribution Function, $F_u(x)$, according to Fisher-Tippett (1928) theorem which states that as n gets large, the distribution of the tail of x converges to the Generalised Extreme Value Distribution (GEVD) of the maxima given as

$$G_{\xi, \sigma, \mu}(x) = \text{Exp} \left[- \left(1 + \xi \left(\frac{x_t - \alpha_t}{\beta_t} \right) \right)^{-\frac{1}{\xi}} \right] \text{ if } \xi \neq 0$$

Where β_t is the scale parameter and α_t the location parameter.

The shape parameter ξ whose tail- behavior is defined by its value is to be used to determine the family type of the distribution function of the Generalised Extreme Value Distribution (GEVD).

Type I: $\xi = 0$, the Gumbel family having the CDF as

$$F_{\xi}(x) = \exp(-\exp(-x)), \quad -\infty < x < \infty$$

Type II: $\xi > 0$, Frechet family having CDF as

$$F_{\xi}(x) = \begin{cases} \exp(-(1+\xi x)^{-\frac{1}{\xi}}) & \text{if } x < -\frac{1}{\xi} \\ 1, & \text{otherwise} \end{cases}$$

Type III: $\xi < 0$, the Weibull family having the CDF as

$$F_{\xi}(x) = \begin{cases} \exp(-(1+\xi x)^{-\frac{1}{\xi}}) & \text{if } x < -\frac{1}{\xi} \\ 0, & \text{otherwise} \end{cases}$$

The probability density function (pdf) of the distribution function above is

$$F'_{\xi}(x) = f_{\xi}(x) = \begin{cases} (1+\xi x)^{-\frac{1}{\xi}-1} \exp(-(1+\xi x)^{-\frac{1}{\xi}}), & \xi \neq 0 \\ \exp(-(x + \exp(-x))) & \xi = 0 \end{cases}$$

The kurtosis of the distribution is a measure that can be used to determine the tail nature of the distribution (Cryer and Chan, 2008).

Block Maxima Method of Getting Extreme Values

This method entails the series, for instance the returns of a financial data, being divided into m blocks or sup-groups of size n and the highest value, H_k , in each of the m blocks is taken to form a series, H_k , $k=1,2,3,\dots,m$, of maximum values.

The set of extreme values of the returns series is now the sequence (H_k) of block maxima and the distribution of this sequence should be determined. According to Rocco (2011) the main result of Extreme Value Theory (EVT) is that, as m and n grow sufficiently large that is, as $m, n \rightarrow \infty$, under some additional assumption, the limit distribution of (adequately rescaled) block maxima belongs to one out of three different families depending on the behavior upper tail of the series. The fat-tailed case is of major concern to financial applications, exponentially decaying if of normal distribution, or with upper bounded support which is less relevant case for finance.

3.5.7. Generalised Pareto Distribution (GPD)

This is the distribution of exceedances of the time series residuals from a certain number, u , known as the threshold. Balkema and Haan (1974) and Pickands (1975) show that the excess distribution, $F_u(x)$, for a large class of distributions, can be approximated to generalized Pareto distribution (GPD). In other words the Generalised Extreme Value Distribution in this case can be approximated to GPD. McNeil et al (2005) gives the cumulative density function (CDF) of GPD as two parameter distribution function below.

$$G_{\xi, \beta}(x) = \begin{cases} 1 - (1 + \frac{x\xi}{\beta})^{-\frac{1}{\xi}}, & \xi \neq 0 \\ 1 - \exp(-\frac{x}{\beta}), & \xi = 0 \end{cases}$$

Where

ξ is the tail index which is a measure of the shape of the tail

$\beta > 0$ is the scaling parameter

$x \geq 0$ when $\xi \geq 0$ and

$0 \leq x \leq -\frac{\beta}{\xi}$ when $\xi < 0$

The family type of the distribution is known by the value of ξ

$\xi > 0$ Pareto (reparametrized version)

$\xi = 0$ exponential

$\xi < 0$ Pareto type II.

The Peaks-Over-Threshold Method of Selection of Exceedances

The Peaks-Over-Threshold Method (POTM) entails selecting of residuals that are above a certain high threshold, u and model these observations separate from the rest of the distribution, that is the distribution of $Y = X \bar{\cap} u$. The selection of the high (optimum) threshold will be done through mean excess plot. The other methods are Hill estimator, median, Pareto quantile plot and 10% exceedances.

Mean Excess Plot

This is the scatter plot of the mean excesses $e_T(u)$ of a high threshold against the high threshold u_0 which should be linear for $u > u_0$. Given the high threshold u_0 , suppose the excess $r - u_0$ follows a GPD with parameter ξ and $\psi(u)$, where $0 > -\xi > -1$. Then the mean excess over threshold is defined by

$$E(r - u_0 \mid r > u_0) = \frac{\psi(u_0)}{1 - \xi}$$

The mean excess function is defined as

$$E(r - u \mid r > u) = \frac{\psi(u_0) + \xi(u - u_0)}{1 - \xi} \text{ for any } u > u_0.$$

The fact that, for a given ξ , $e(u)$ is a linear function of u , where $u > u_0$, provides a simple method to infer the threshold u_0 for GPD.

Hence, the sample mean excess is defined as

$$e_r(u) = \frac{1}{n_u} \sum_{t=1}^{n_u} (r_{t_i} - u)$$

Where n_u is the number of returns that exceed u and r_{t_i} are the values of the corresponding returns.

(McNeil et al(2005) and Tsay(2005))

3.6. Model Specification

In determining the tentative model for the time series, the Akaike's information Criterion (AIC) is used to determine the order based on the value of AIC at different orders.

The Akaike's information Criterion (AIC) (Akaike, 1973) normalized by sample n is given by

$$AIC = \frac{-2\ln(\text{maximum likelihood}) + 2r}{n}$$

Where r is the number of parameters.

(Box and Jenkins, 2008).

It should be noted that the AIC can be approximated to

$$AIC = \log \hat{\sigma}_r^2 + \frac{n+2r}{n}$$

$$\text{Where } \hat{\sigma}_r^2 = \frac{SSE_r}{n}$$

(Shunway and Stoffer, 2011).

The best model is specified by r that has the minimum AIC.

3.7. Estimation of the Model Parameters

The estimation of the model parameters can be done by the use of maximum likelihood estimation.

1. Maximum Likelihood Estimation ARCH or GARCH Model Parameters.

The conditional pdf under normal innovation ε_t is given as

$$\begin{aligned} f(r_t / r_{t-1}, r_{t-1}, \dots, r_1) &= \frac{f(r_t, r_{t-1}, \dots, r_1)}{f(r_{t-1}, r_{t-1}, \dots, r_1)} \\ &= \frac{1}{\sqrt{2\pi\sigma_{t/t-1}^2}} \exp\left[-\frac{r_t^2}{(2\sigma_{t/t-1}^2)}\right] \end{aligned}$$

Which implies that

$$f(r_n, r_{n-1}, \dots, r_1) = \prod_{n=1}^t f(r_n) = f(r_n / r_{n-1}, \dots, r_1) f(r_{n-1}, \dots, r_1)$$

Iterating the likelihood function and taking logs gives the equation below.

$$L(w, \alpha, \beta) = -\frac{1}{2} \log(2\pi) - \frac{1}{2} \sum_{n=1}^n \left\{ \log(\sigma_{t-1/t-2}^2) + r_t^2 / \sigma_{t/t-1}^2 \right\}$$

The three parameters; w, α and β can be computed by maximizing the log-likelihood function numerically with their variances derived from the diagonal of covariance matrix Γ of the parameters.

The likelihood function of a GARCH model can well be illustrated with GARCH(1,1) model in which the conditional variances can be computed recursively by the formula

$$\sigma_{t/t-1}^2 = w + \alpha_1 r_{t-1}^2 + \beta_1 \sigma_{t-1/t-2}^2$$

for $t \geq 2$, with the initial value, set under the stationarity assumption as the stationary

$$\text{unconditional variance } \sigma^2 = \frac{w}{(1-\alpha-\beta)}.$$

For GARCH(1,1) model, the Γ is 3 x 3 matrix which is approximately equal to the inverse matrix of the 3x3 matrix whose (i, j) th element according to Cryer and Chan (2008) is equals

$$\frac{1}{2} \sum \frac{1}{\sigma_{t/t-1}^4} \frac{d\sigma_{t/t-1}^2}{d\theta_i} \frac{d\sigma_{t/t-1}^2}{d\theta_j}$$

Then using GARCH(1,1) above the partial derivative $\frac{d\sigma_{t/t-1}^2}{d\theta_i}$ and $\frac{d\sigma_{t/t-1}^2}{d\theta_j}$ is derived for

$\theta_i = \alpha$ and $\theta_j = \beta$ and solving recursively to get the parameters. The likelihood function is called the Gaussian likelihood, and the estimators maximizing the Gaussian likelihood are called the quasi-maximum likelihood estimators (QMLEs)(Cryer and Chan, 2008).

However, the maximum likelihood estimation procedures of normal innovation can be applied to the log likelihood function of the data in a similar manner if the conditional distribution $f(r_t | r_{t-1}, \dots, r_1; \theta)$ is not normal (Tsay, 2005). Moreover, the approach of maximum Likelihood Estimation of GARCH model parameters can be extended to ARMA+GARCH model parameters estimation as in the work of Francq And ZakoiAn (2004).

2. Maximum Likelihood Estimation of GEVD Parameters

Assuming that the sequence $\{r_{n,i}\}$ follows the generalized extreme value distribution, then the pdf of $r_{n,i}$, the minimum of r in the i th sub sample of size n , can be gotten from the pdf of x by substituting $x = ((r_{n,i} - \mu)/\sigma)$ in pdf of GEVD as follows

$$f_{\xi}^{\xi}(r_{n,i}) = \begin{cases} \frac{1}{\sigma_n} (1 + \xi_n ((r_{n,i} - \mu_n) / \sigma_n))^{\frac{1}{\xi_n} - 1} \exp(- (1 + \xi_n ((r_{n,i} - \mu_n) / \sigma_n))^{\frac{1}{\xi_n}}), & \xi_n \neq 0 \\ \frac{1}{\sigma_n} \exp(-(((r_{n,i} - \mu_n) / \sigma_n) + \exp(-((r_{n,i} - \mu_n) / \sigma_n)))) & \xi_n = 0 \end{cases}$$

where it is understood that $1 + \xi_n ((r_{n,i} - \mu_n) / \sigma_n) > 0$ if $\xi_n \neq 0$. The subscript n is added to the shape parameter ξ because its estimate depends on the choice of n . Under the independence assumption, the likelihood function of the subperiod minima is

$$L(r_{n,1}, \dots, r_{n,g} / r_n, \xi_n, \sigma_n, \mu_n) = \prod_{i=1}^g f(r_{n,i})$$

Hereafter, the nonlinear estimation procedures can be used to obtain maximum likelihood estimates of ξ_n, μ_n and σ_n . (Tsay, 2005).

3.8. Model Diagnostic

1. Jarque-Bera Test

Let $x_1, x_2, x_3, \dots$ be iid data which follow normal distribution, then the Jarque-bera test is defined as

$$JB = \frac{ng_1^2}{6} + \frac{ng_2^2}{24}$$

where g_1 is the sample skewness and g_2 is the sample kurtosis.

This test is based on the fact that a normal distribution has zero skewness and zero kurtosis. The Jarque-Bera Test is approximately distributed as chi-square with 2 degrees of freedom under the normality assumption (Cryer and Chan, 2008).

2. Shapiro-Wilk Test

This is the computation of the correlation coefficient (W) between residuals and corresponding normal quantiles in order to determine normality based on the degree of

relationship. The lower the degree of relation between the residuals and the corresponding normal quantile the more evidence to reject normality (Cryer and Chan, 2008).

3. Ljung-Box Test

This is another test of normality of the residuals which was introduced by Box and Pierce (1970) and modified by Ljung and Box (1978) which is also approximately chi-square with lag(K) number of coefficient as degree of freedom v.

The Modified Ljung-Box statistics is defined as

$$Q = \left(\frac{r_1^2}{n-1} + \frac{r_2^2}{n-2} + \dots + \frac{r_k^2}{n-k} \right)$$

Where r is sample auto-correlation coefficient of the residuals.

(Cryer and Chan, 2008).

3.9. Forecasting

The discussion on forecasting is to be presented in two fold of ARIMA and GARCH. These form the core of our forecasting model.

ARMA

Let the forecast origin be h and the available information by F_h . The 1-step ahead forecast, r_{h+1} , can

be represented by the equation below from the ARMA model as

$$r_{h+1} = r_h(1) = E(r_{h+1} / F_h) = \phi_0 + \sum_{i=1}^p \phi_i r_{h+1-i} - \sum_{i=1}^q \theta_i a_{h+1-i}$$

Generally for l - step ahead forecast

$$r_{h+l} = r_h(l) = E(r_{h+l} / F_h) = \phi_0 + \sum_{i=1}^p \phi_i r_h(l-i) - \sum_{i=1}^q \theta_i a_h(l-i)$$

(Cryer and Chan, 2008).

GARCH

In forecasting the l -step-ahead conditional variance $\sigma_h^2(l)$ for GARCH(p,q) is the same approach as that of ARMA model. Consider GARCH(1,1) model with conditional variance or volatility equation

$$\sigma_{t/t-1}^2 = w + \alpha_1 e_{t-1}^2 + \beta_1 \sigma_{t-1/t-2}^2$$

$$e_t = \sigma_{t/t-1} \varepsilon_t$$

For 1-step ahead forecast of the conditional variance or volatility assuming h is the forecast origin is

$$\sigma_{h+1}^2 = w + \alpha_1 e_h^2 + \beta_1 \sigma_h^2$$

Where e_h and e_h^2 are known at the time index h , then the 1-step forecast equation written as

$$\sigma_h(1) = w + \alpha_1 e_h^2 + \beta_1 \sigma_h^2$$

But for multiple ahead forecast and taking $e_t^2 = \sigma_t^2 \varepsilon_t^2$, the forecast equation from conditional variance equation above can be written as

$$\sigma_{t+1}^2 = w + (\alpha_1 + \beta_1) e_t^2 + \alpha_1 \sigma_t^2 (\varepsilon_t^2 - 1)$$

when $t=h+1$ the equation is

$$\sigma_{h+2}^2 = w + (\alpha_1 + \beta_1) e_{h+1}^2 + \alpha_1 \sigma_{h+1}^2 (\varepsilon_{h+1}^2 - 1)$$

Hence, 2-step ahead volatility forecast at the forecast origin h satisfies the equation

$$\sigma_h^2(2) = w + (\alpha_1 + \beta_1) e_h^2(1) \text{ for } E(\varepsilon_{h+1}^2 - 1/h) = 0$$

In general the L step ahead forecast is given as

$$\sigma_h^2(l) = w + (\alpha_1 + \beta_1) e_h^2(l-1), \quad l > 1$$

By repeated substitution the derived equation is

$$\sigma_h^2(l) = \frac{w[1 - (\alpha_1 + \beta_1)^{l-1}]}{1 - \alpha_1 - \beta_1} + (\alpha_1 + \beta_1)^{l-1} \sigma_h^2(1)$$

Therefore,

$$\sigma_h^2(l) \rightarrow \frac{w}{1 - \alpha_1 - \beta_1} \text{ as } l \rightarrow \infty$$

Provided

$$\sigma_h^2(l) = (\alpha_1 + \beta_1) \langle 1$$

It should be noted that the starting value σ_0^2 is fixed at either zero or the unconditional variance of e_t .

Where unconditional variance $e_t = \frac{w}{1 - \alpha_1 - \beta_1}$

It should be noted that as the forecast horizon increases to infinity, the multistep ahead volatility forecasts of a GARCH(1,1) model converge to the unconditional variance of e_t provided that $\text{Var}(e_t)$ exists. (Tsay, 2005)

3.10. Risk Measures

The type of risk measures that will be considered in this research is that of market risk which involves the computation of Value at Risk (VaR) and Expected Shortfall (ES).

3.10.1. Value at Risk (VaR)

This is the estimate of an amount by which an institution's position in a risk category could fall gradually due to general market movements during a given holding period. It can also be referred to as a measure of minimum loss of a financial position during a particular period of time for a given (small) probability.

Considering the following;

- i. Let t be the time index
- ii. Let l be the periods of the financial position

iii. Let $DV(l)$ be the change in value of the assets in the financial position from time

t to $t + l$

iv. Denote the CDF of $DV(l)$ by $F_l(x)$

The VaR of a long position over the time horizon l with probability p under a probabilistic approach is given as

$$p = [DV(l) \leq VaR] = F_l(VaR)$$

For $DV(l) < 0$ (the holder of a long financial position suffers a loss), the VaR above typically assumes a negative value when p is small signifying a loss. From the definition, the probability that the holder would encounter a loss greater than or equal to VaR over the time horizon l is p . Therefore, the probability $(1 - p)$ is the potential loss encountered by the holder of the financial position over the time horizon l is less than or equal to VaR.

For $DV(l) < 0$ (the holder of a short position suffers a loss when the value of the asset increases), hence VaR is defined as

$$p = [DV(l) \geq VaR] = 1 - [DV(l) \leq VaR] = 1 - F_l(VaR)$$

The VaR is concerned with tail behavior of the $F_l(x)$. The left tail of $F_l(x)$ is important for a long position (adopted for this study) while the right tail of $F_l(x)$ is for a short position.

If $F_l(x)$ is known, then VaR is the same as p th quantile x_p .

$$VaR = x_p$$

$$x_p = \inf\{x / F_l(x) \geq p\} \text{ for } 0 < p < 1.$$

Where $\inf$ denotes the smallest real number satisfying $F_l(x) \geq 0$

(Tsay, 2005)

Peak Over Threshold(POT) Approach to VaR Calculation

There are two approaches; Homogeneous case (parameters are fixed over time) and Non-homogeneous case (parameters are time-varying, according to some explanatory variables). This approach was proposed by Davison and Smith (1990) and Smith (1989)

Case I (Homogeneous).

$$VaR = \begin{cases} \beta + \frac{\alpha}{k} \{1 - [-D (\ln(1-p))^k] \} & \text{if } k \neq 0 \\ \beta + \alpha \{-D (\ln(1-p))\} & \text{if } k = 0 \end{cases}$$

where D is the baseline time interval, mostly the number of days trading in a year, used in estimation. In the United States, one typically uses $D = 252$.

Case II (Non-Homogeneous).

$$VaR_q = u + \frac{\beta}{\xi} \{1 - [\frac{T}{N_u(1-q)}]^\xi\}$$

Where

u is the threshold

T is the sample size

N_u is the number of exceedances and

β and ξ are the scale and shape parameters of the GPD

(Tsay, 2005).

This work will consider a two parameter case of non-homogeneous.

3.10.2. Expected Shortfall (ES)

This is expected loss given that the VaR is exceeded otherwise referred to as Expected Tail Loss (ETL). The expected shortfall (ES) is defined as

$$ES_q = E(L / L > VaR_q)$$

Generalised Pareto Distribution GPD)

Using the properties of GPD the ES is defined as

$$ES_q = \frac{VaR_q}{1+\xi} + \frac{\beta + \xi u}{1+\xi}$$

Where ξ and β are the parameters of GPD.

(Tsay, 2005).

3.11. R Statistical Software

R is a free software environment for statistical computing and graphics. It compiles and runs on a wide variety of platforms, windows and MacOS. It is an official part of the Free Software Foundation's GNU project with the web <https://www.r-project.org>. The modeling in this project will be done with the use of R software for statistical computing, R-2.15.3, (2013). The packages used are TSA (Chan, 2010), fGarch (Wuertz et al, 2013) and evir (Pfaff and McNeil, 2012). The codes are in Appendix B.

CHAPTER FOUR

MODELING

4.1. Time Plot of the Series after Imputing Unobserved Values and Deleting Days of Unobserved Values

The data collected from the Nigerian Exchange Market on the daily stock price of First Bank Nigeria, Plc from 1998 to 2007 is presented in Appendix A1. There are four series that will be used in the time plot. The series were gotten from the different approaches of dealing with unobserved values. Before the estimations of the unobserved values were carried out, the data was separated into weekly series. This was done because each of the week unobserved values were computed using the data of that same week. The four series are; FBN1 - weekly average imputation, FBN2 - weekly regression imputation, FBN3 - preceding value imputation and FBN4 - deletion of days of unobserved values.

The time plots of the four series are displayed below from Figure 1 to 4.

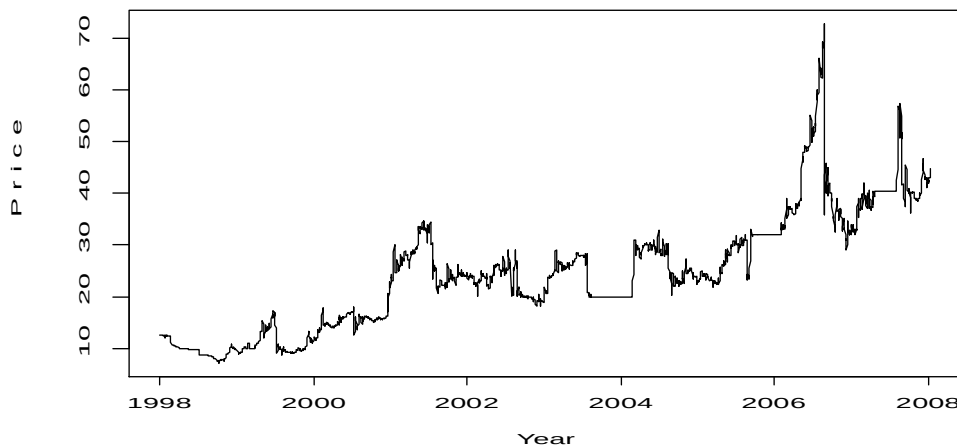


Figure 1: Time plot of Daily Stock Price of First Bank Nigeria, Plc (FBN1)

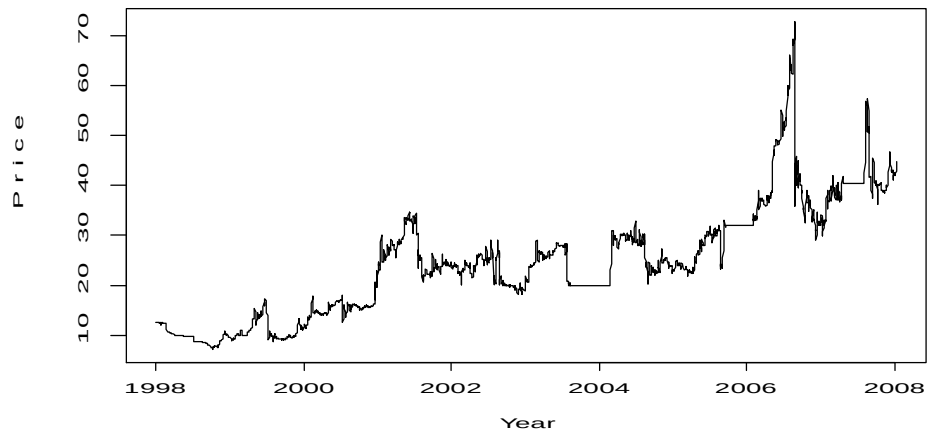


Figure 2: Time plot of Daily Stock Price of First Bank Nigeria, Plc (FBN2)

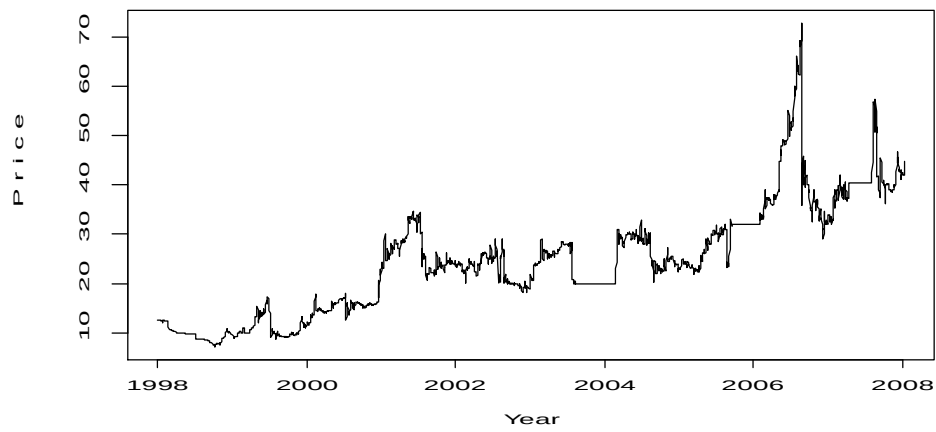


Figure 3: Time plot of Daily Stock Price of First Bank Nigeria, Plc (FBN3)

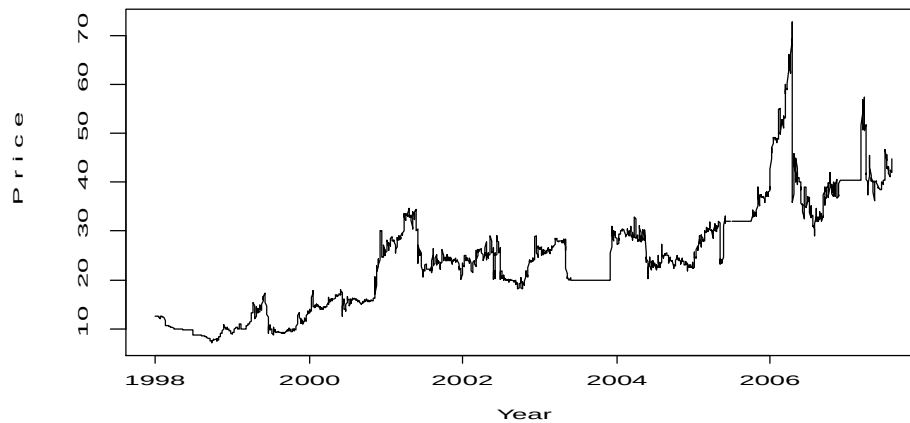


Figure 4: Time plot of Daily Stock Price of First Bank Nigeria, Plc (FBN4)

The time plot in Figure 1 to 4 indicates an upward trend and in considering this the four series are non stationary. Therefore, a non stationary model, the Autoregressive Integrated Moving Average (ARIMA) would have been suggested but this is a financial time series which is highly volatile. This is an indication of a higher variability with higher level of stock value.

Therefore, a combination Autoregressive Moving Average (ARMA) and Generalized Autoregressive Conditional Heteroscedastic (GARCH) models is suggested. The GARCH considers the conditional variance of time series that measures the uncertainty in deviation of the time series from the conditional mean associated with ARMA. The use of GARCH model requires the presence of Autoregressive Conditional Heteroscedastic (ARCH) effect in the stock series. The test is conducted in the next section.

4.2. Test for ARCH Effect

Apart from relying on the assumptions of volatility nature of financial time series the test for ARCH effect is eminent. The four series will be subjected to a test to determine if there is ARCH effect and the statistic used is McLeod ĩ Li test.

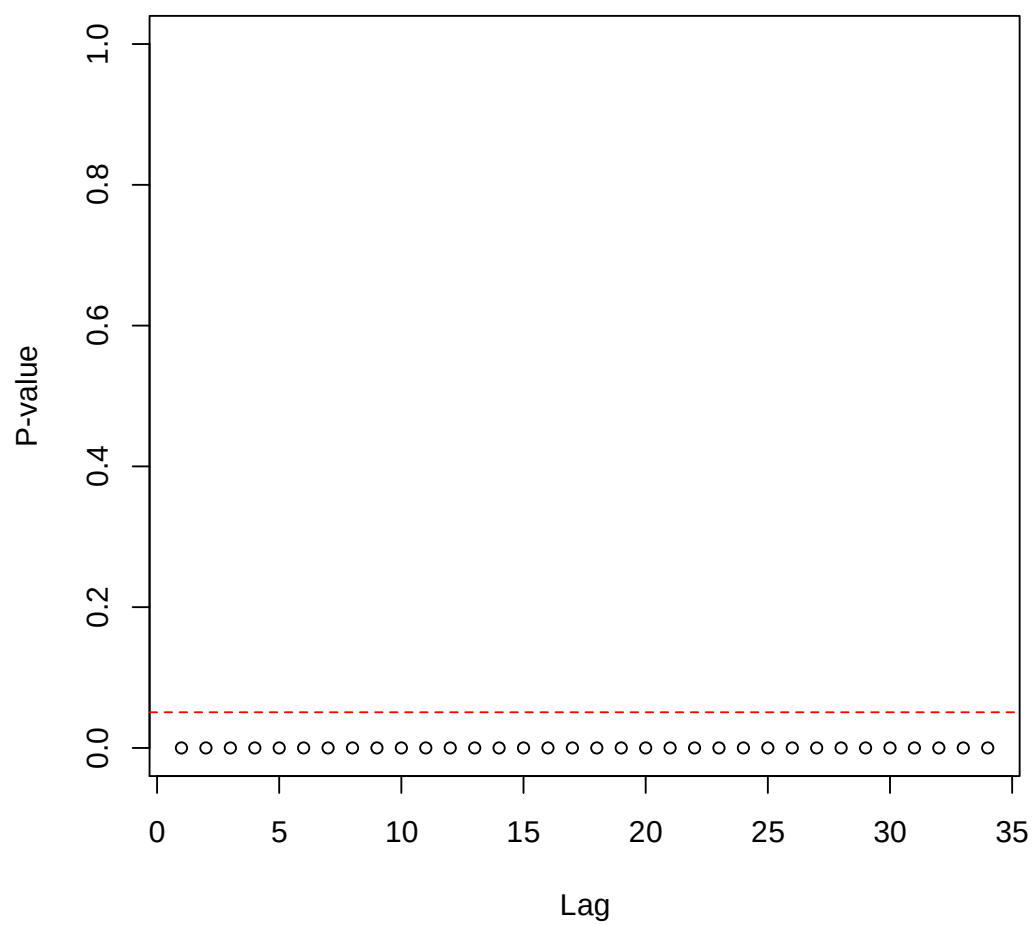


Figure 5: McLeod-Li Test for Daily Stock Price of First Bank Nigeria, Plc (FBN1)

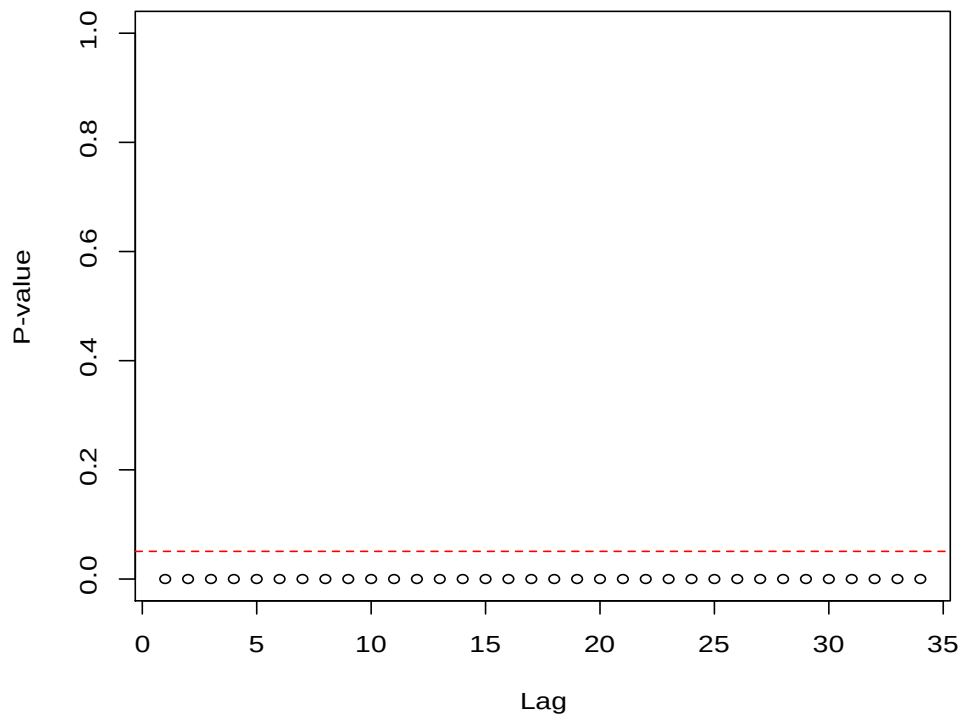


Figure 6: McLeod – Li Test for Daily Stock Price of First Bank Nigeria, Plc (FBN2)

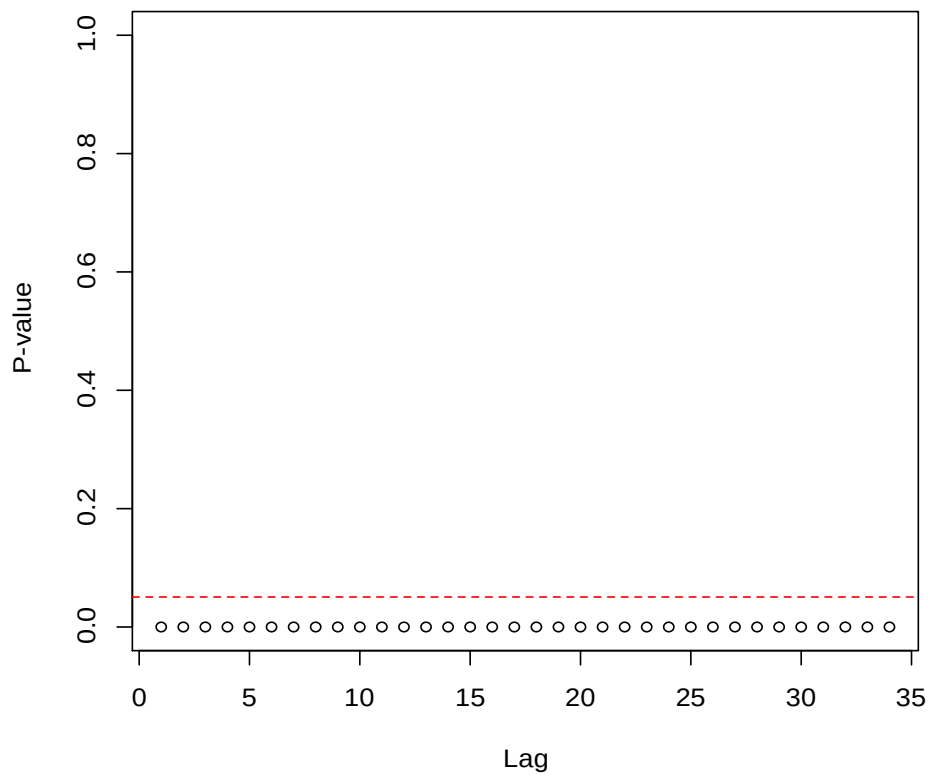


Figure 7: McLeod i Li Test for Daily Stock Price of First Bank Nigeria, Plc (FBN3)

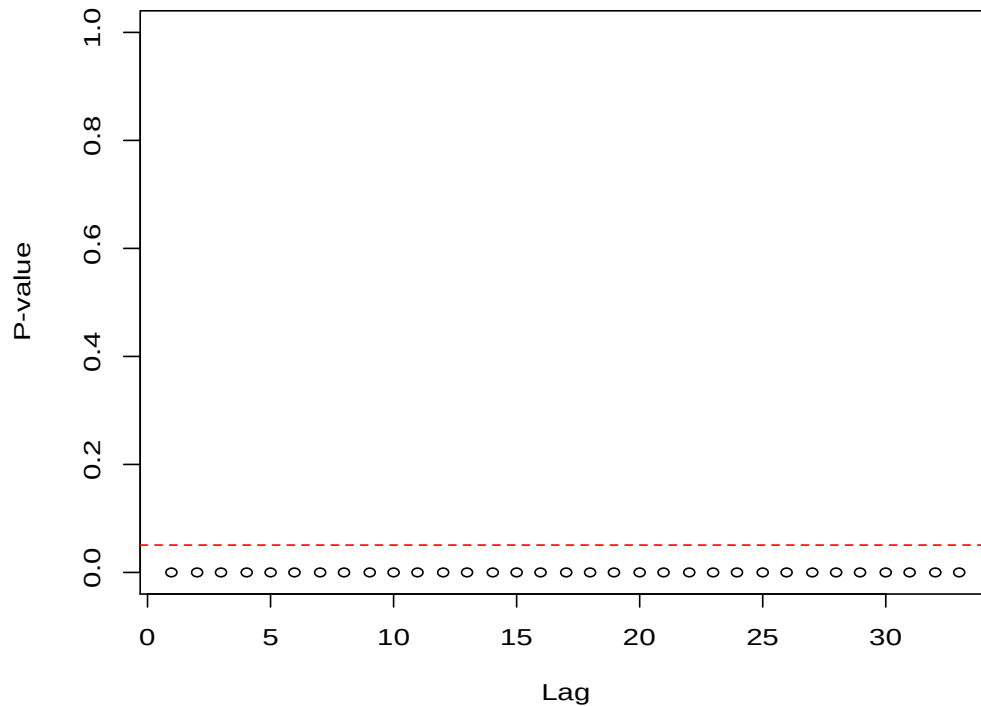


Figure 8: McLeod – Li Test for Daily Stock Price of First Bank Nigeria, Plc (FBN4)

The tests at different lags shown in the Figure 5 to 8 above indicate that there is a significant ARCH effect at 5 % level of significant. Hence, the use of GARCH model for the stock series. In this study the GARCH model is combined with ARMA model because the assumption of conditional mean being zero is not always true in real life situation, especially in financial time series (Cryer and Chan, 2008). The model in general form is given as $ARMA(u,v)+GARCH(p,q)$.

4.3. The Transformation of the Time Series

The series presented in the four graphs of Figure 1 to Figure 4 has to be transformed for stationarity. The transformation method will be determined as stated in table 1 of chapter three.

Table 2: Type of Transformation of the Series

S/N	Series	Slope of regressing $\log(\text{variance})$ against $\log(\text{mean})$	Type of Transformation
1	FBN1	2	Logarithm
2	FBN2	2	Logarithm
3	FBN3	2	Logarithm
4	FBN4	2	Logarithm

The slope of the four series after the regression of $\log(\text{variance})$ against $\log(\text{mean})$ are approximately 2 satisfying the use of logarithm transformation. The variances and means are computed from each of the weeks of the four series.

The behavior of the logarithm transformation of the four series is shown in the plots of the log of the four series from Figure 9 to 12.

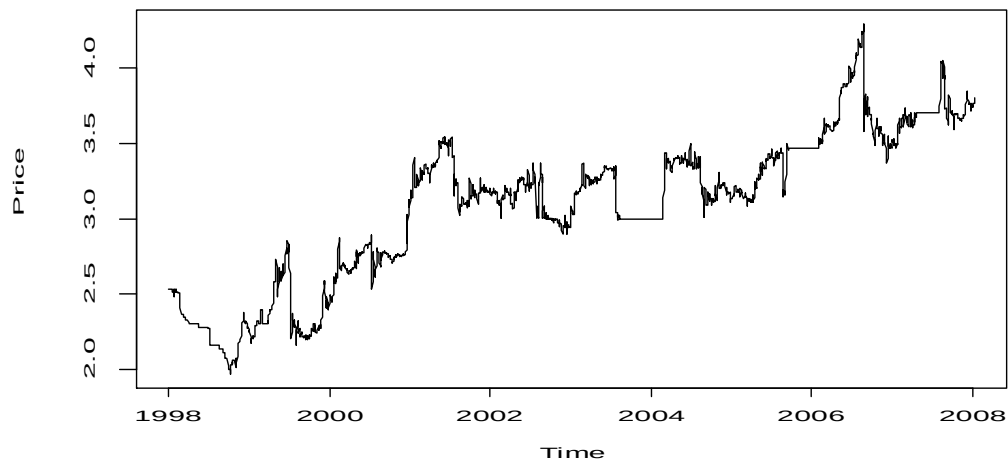


Figure 9: Time plot of Log Daily Stock Price of First Bank Nigeria, Plc (FBN1)

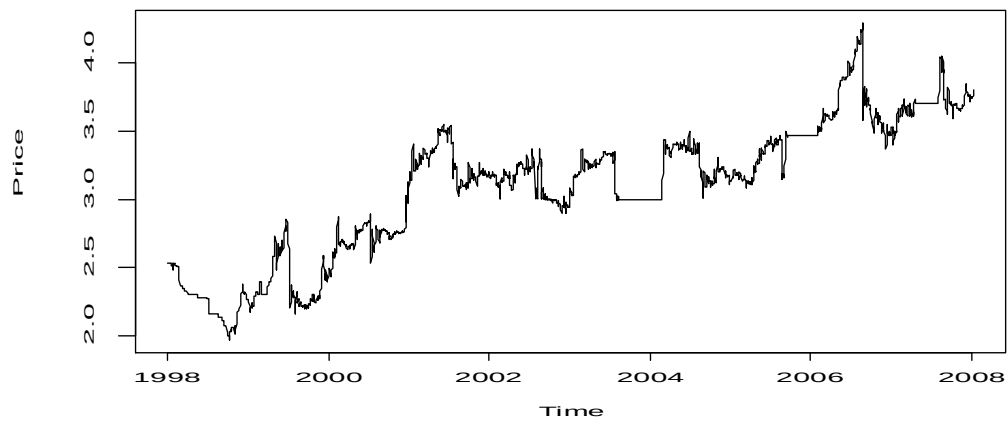


Figure10: Time plot of Log Daily Stock Price of First Bank Nigeria, Plc (FBN2)

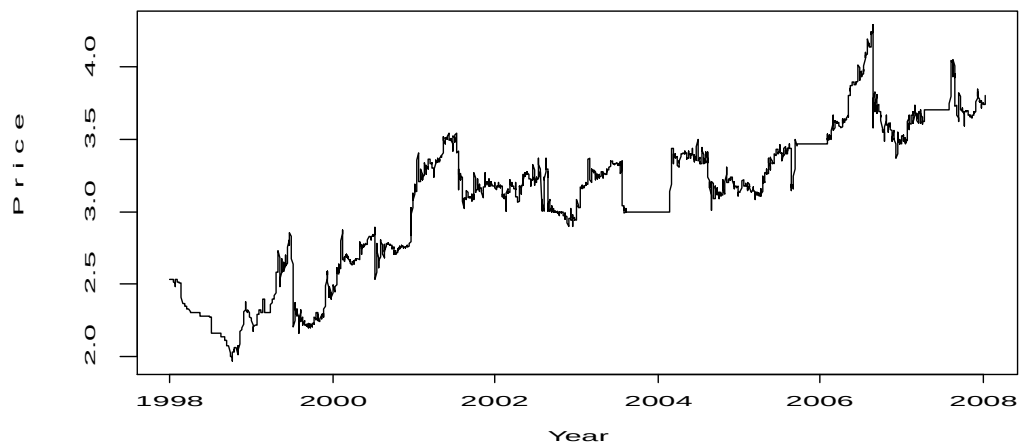


Figure11: Time plot of Log Daily Stock Price of First Bank Nigeria, Plc (FBN3)

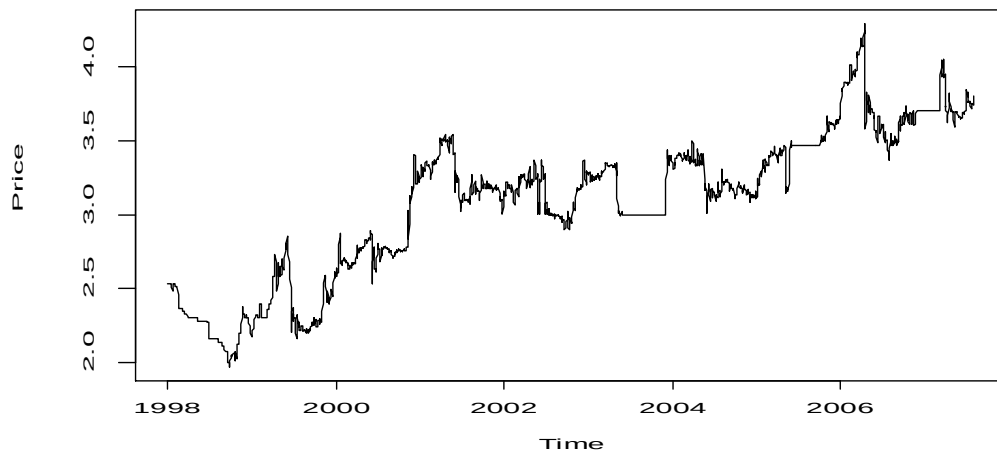


Figure12: Time plot of Log Daily Stock Price of First Bank Nigeria, Plc (FBN4)

The logarithm transformation exhibited trend in all the four series which implies that the series are still non-stationary. Thereby prompting differencing which give the returns series. Let's see the behaviour of the four returns series from Figure 13 to 16.

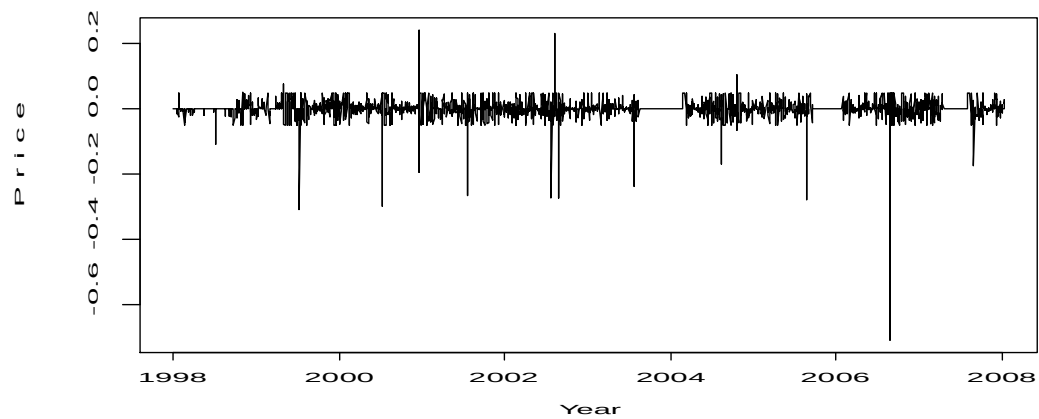


Figure 13: Time plot of Returns of Daily Stock Price of First Bank Nigeria, Plc (FBN1)

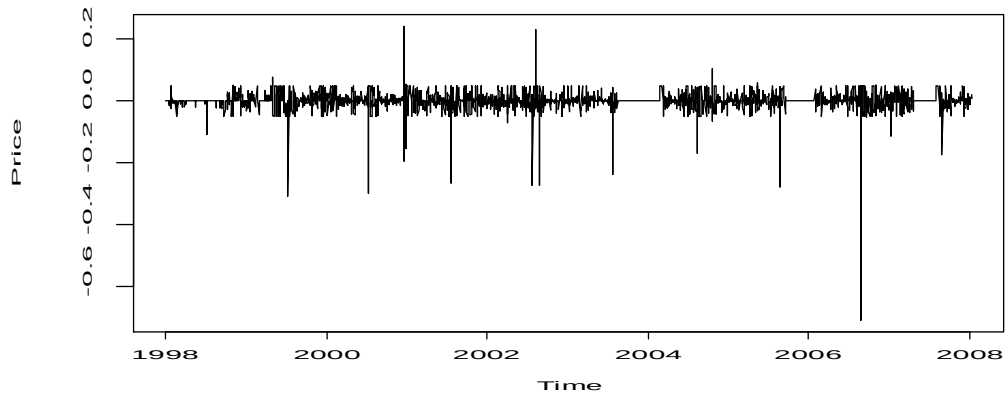


Figure 14: Time plot of Returns of Daily Stock Price of First Bank Nigeria, Plc (FBN2)

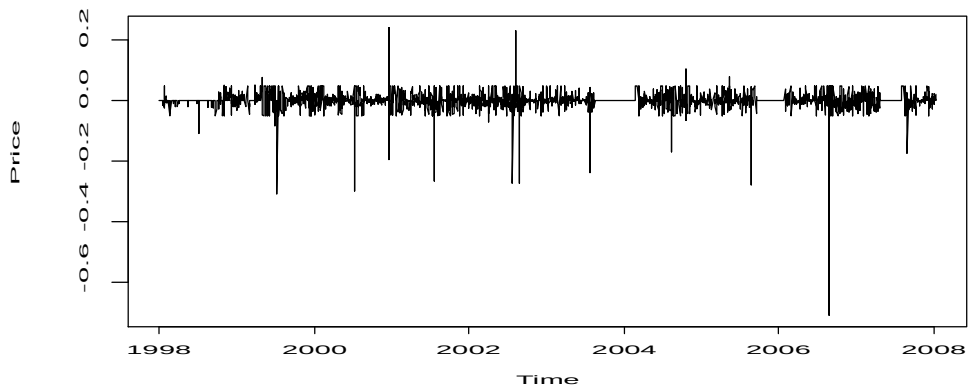


Figure 15: Time plot of Returns of Daily Stock Price of First Bank Nigeria, Plc (FBN3)

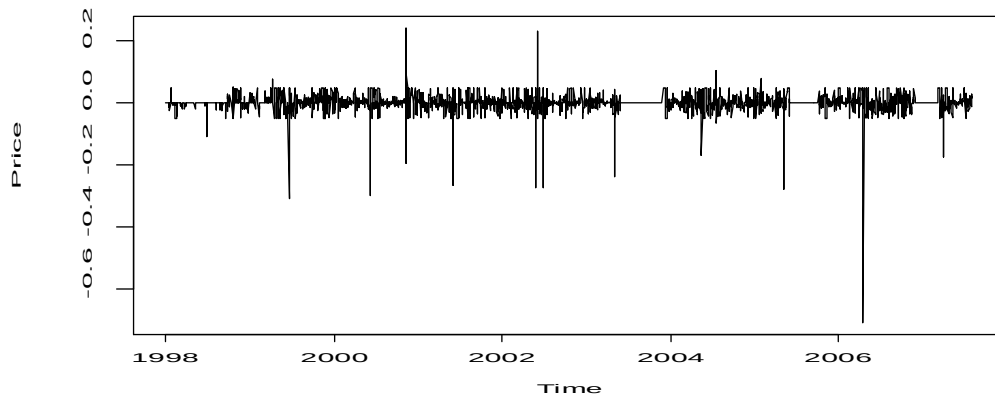


Figure 16: Time plot of Returns of Daily Stock Price of First Bank Nigeria, Plc (FBN4)

The plot of the returns of the four series in Figure 13 to 16 shows the removal of trend making it stationary. According to Cryer and Chan, (2008) differencing is a useful tool to achieve stationarity. In order to further verify the stationarity condition, the Augmented Dickey Fuller (ADF) unit-root test is considered using the log of the original series. The null hypothesis is that the log of the series is non-stationary and will be stationary after first differencing against the alternative hypothesis that the log of the series is stationary. The test statistics and the p-values from the four series are shown below.

Table 3: ADF Test for the Four Series

S/N	Series	Dickey Fuller	p-value
1	FBN1	-3.1465	0.09713
2	FBN2	-3.1636	0.09417
3	FBN3	-3.1406	0.09814
4	FBN4	-3.1237	0.1026

The p-values of the test in all the four series are in favour of the null hypothesis being accepted at 5% level of significant, meaning that there will be stationarity after log difference (returns).

4.4. Order of ARMA(u,v)+GARCH(p,q) Model.

The order of the ARMA+GARCH model from the returns series will be determined by using the Extended Autocorrelation Function (EACF) method (Tsay and Tiao, 1984). The tentative model for each of the four series is in the last column of the table 4 below.

Table 4: Order of the Model

S/N	Series	Order of the model (ARMA+GARCH)
1	FBN1	ARMA(1,1)+GARCH(1,1)
2	FBN2	ARMA(2,1)+GARCH(2,1)
3	FBN3	ARMA(2,1)+GARCH(2,1)
4	FBN4	ARMA(1,1)+GARCH(1,1)

The models will be considered in two versions. These are ARMA+GARCH conditional distribution version of normal distribution and student t-distribution. We will make use of eight models made up of four each from normal distribution version and student t-distribution version where the best model will be selected tentatively. The models can be written generally as $ARMA(u,v)+GARCH(p,q)_{Norm}$ for normal distribution and $ARMA(u,v)+GARCH(p,q)_t$ for student t-distribution.

4.5. Model Selection and Estimation of Parameters

The next step is to estimate the parameters of each of the eight tentative models, test for significance of the parameters and also conduct residual test before the best model is selected among the eight models. The estimates and test are shown in the tables below:

Table 5: The parameters, p-values and residual tests of the ARMA+GARCH model with Normal Distribution Version.

Parameter	m1	p-value	m2	p-value	m3	p-value	m4	p-value
ar1	-0.1642	0.295	-1	0	0.8124	0	-0.1959	0.414
ar2	--	--	-0.0875	0.0156	0.04565	0.128	--	--
ma1	0.0568	0.755	0.93	0	-0.8926	0	0.131	0.595
W	0.000124	0	0.00014	0.00281	0.00013	0.0002	0.00013	0
alpha1	1	0	1	0	1	0	1	0
alpha2	--	--	0	1	0	1	--	--
beta1	0.3537	0	0.328	0.0677	0.3486	0.01352		
Jarque-Bera Test	392814.4	0	369418.1	0	383562.2	0	366355.6	0
Shapiro-Wilk Test	0.7042653	0	0.71134	0	0.69985	0	0.6984029	0
Ljung-Box Test R Q(15)	24.07747	0.0638	32.25324	0.00594589	26.97602	0.02893	28.30842	0.01971626
Ljung-Box Test R ² Q(15)	2.180998	0.9999	1.613111	0.999993	2.009059	0.999969	1.811148	0.9999847
AIC	-4.556857	-----	-4.547152	-----	-4.577212	-----	-4.506181	-----

ARMA(1,1)+GARCH(1,1)_{Norm}= model 1 (m1)

ARMA(2,1)+GARCH(2,1)_{Norm}= model 2 (m2)

ARMA(2,1)+GARCH(2,1)_{Norm}= model 3 (m3)

ARMA(1,1)+GARCH(1,1)_{Norm}= model 4 (m4)

The best of the models out of the four models in the normal distribution version is ARMA(2,1)+GARCH(2,1) of FBN3 (Estimation of unobserved values by preceding value estimates) with a minimum AIC of -4.577212.

Table 6: The parameters, p-values and residual tests of the ARMA+GARCH model with Student t- Distribution Version

Parameter	m5	p-value	m6	p-value	m7	p-value	m8	p-value
ar1	0.1350	0.1818	-0.8958	0	0.5528	0.00176	0.1236	0.480
ar2	--	--	0.04074	0	-0.04035	0.01585	--	--
ma1	-0.1231	0.834	0.9391	0	-0.4940	0.00501	-0.05858	0.738
W	9.647×10^{-10}	0.981	9.776×10^{-10}	0.98	9.524×10^{-10}	0.98031	9.985×10^{-10}	0.981
alpha1	1	0	1	0	1	0	1	0
alpha2	--	--	1	0	1	0	--	--
beta1	0.4449	0	0.2819	0	0.298	0	0.443	0
Shape	2.486	0	2.311	0	2.275	0	2.489	0
Jarque-Bera Test	48699727	0	41201599	0	43195034	0	42456129	1
Shapiro-Wilk Test	0.03558634	0	0.03838284	0	0.03741205	0	0.03650709	1
Ljung-Box Test R Q(15)	0.3585102	1	5.496164	0.9870268	3.705121	0.9985541	0.3711243	1
Ljung-Box Test R ² Q(15)	0.08390692	1	0.2107826	1	0.1326831	1	0.08787515	1
AIC	-6.642874	---	-6.772759	---	-6.837350	---	-6.573305	---

ARMA(1,1)+GARCH(1,1)_t= model 5 (m5)

ARMA(2,1)+GARCH(2,1)_t= model 6 (m6)

ARMA(2,1)+GARCH(2,1)_t= model 7 (m7)

ARMA(1,1)+GARCH(1,1)_t= model 8 (m8)

In the student t-distribution the best model is still that of $ARMA(2,1)+GARCH(2,1)$ of the same FBN3 by having the minimum AIC of -6.837350. From the table 5 and table 6 above, the two models, m6 and m7, out of the eight models have all parameters significant except the constant term. These models are from FBN2 and FBN3 series which gave $ARMA(2,1)+GARCH(2,1)_t$ with FBN3 having a better AIC. Also, the student t-distribution version of the model from FBN3 is better than that of the normal distribution. Therefore, the tentative model is model 7 (m7) that of $ARMA(2,1)+GARCH(2,1)$ of student t-distribution version written as $ARMA(2,1)+GARCH(2,1)_t$.

Even though there is no enough evidence to accept the normality assumption from the Jarque-Bera Test and Shapiro-Wilk Test of m7. But the Ljung-Box Test statistic of the residuals is 3.705121 and p-value is 0.9985541 being an indication that there is no intercorrelation . Also, the Ljung-Box Test statistic of the square residuals is 0.1326831 and p-value is 1, meaning that there is no conditional heteroscedasticity in the standardized residuals. The selected model, m7, seems adequate. The model will be used to forecast while the residuals without ARCH effect (Bystrom, 2001), will be subjected to a test to determine the distribution since it is not normally distributed.

Further extension of the model, $ARMA(1,1)+GARCH(1,1)$, from FBN1 series and FBN4 series to $ARMA(2,1)+GARCH(2,1)$ for both the normal distribution and the student t ï distribution versions gave the following AIC.

Table 7: AIC of Model 9 to 12.

S/N	Model	AIC
1	m9	-4.554653
2	m10	-4.506617
3	m11	-6.773584
4	m12	-6.705159

ARMA(2,1)+GARCH(2,1)_t= model 9 from FBN1(m9)

ARMA(2,1)+GARCH(2,1)_t= model 10 from FBN4 (m10)

ARMA(2,1)+GARCH(2,1)_t= model 11 from FBN1 (m11)

ARMA(1,1)+GARCH(1,1)_t= model 12 from FBN4 (m12)

The extension of the order has not changed the position based on Table 5 and 6, that is, m7 is still the most appropriate model with minimum AIC of -6.837350.

.

4.6. Forecasting

The fitted model, ARMA(2,1)+GARCH(2,1)_t, model 7 (m7), parameters are in table 6 above. Using the fitted model, the forecast of the returns and volatility are shown in table 8 below.

Table 8: Forecast of Returns and Volatility

Step ahead	Days/Dates	Returns	Volatility
1	Tuesday, 1 st Jan, 2008	0.0008820012	0.03005467
2	Wednesday, 2 nd Jan, 2008	-0.0003332052	0.03956889
3	Thursday, 3 rd Jan, 2008	-0.0002197839	0.05419313
4	Friday, 4 th Jan, 2008	-0.0001080485	0.07333283
5	Monday, 7 th Jan, 2008	-0.00005085922	0.09958383
6	Tuesday, 8 th Jan, 2008	-0.00002375439	0.13509061
7	Wednesday, 9 th Jan, 2008	0.00001107887	0.18331388
8	Thursday, 10 th Jan, 2008	-0.00005165729	0.24872877
9	Friday, 11 th Jan, 2008	-0.00002408498	0.33749580
10	Monday, 12 th Jan, 2008	-0.00001122941	0.45793861

The forecast shows a fall on returns on a decreasing pattern with only the first step being positive. This was followed by a pattern of increase in volatility.

4.7. Residuals of ARMA (2,1)+(2,1)_t of FBN3(m7).

The residuals of the selected model, m7, are shown in appendix A2 which will be used henceforth in this work.

4.8. Symmetric Test.

The Kurtosis (K) of the residuals is 128.2811, which means $K > 0$, hence K is heavy tailed as described by Cryer and Chan, (2008). Therefore, the generalized extreme value distribution of a heavy tailed behavior is that of Frechet type of distribution, having the shape parameter $\mathfrak{z} > 0$. This according to Tsay (2005) is good for risk management and Bystrom

(2005) recommended this distribution for financial time series which is usually fat tailed. This is approximated to Generalized Pareto Distribution (GPD).

4.9. The Generalized Pareto Distribution (GPD).

Using the mean plot, the threshold (u) of choice at $u \rightarrow 0$ is 0.05, where the mean excess becomes approximately linear for all the values of $u \rightarrow 0.05$. This gives 50 values above the threshold (exceedances). The tail index is 2.175344675 and the scaling parameter is 0.001394252.

4.10. Model Diagnosing of GPD

The model with $u = 0.05$ will be diagnosed through the following plots.

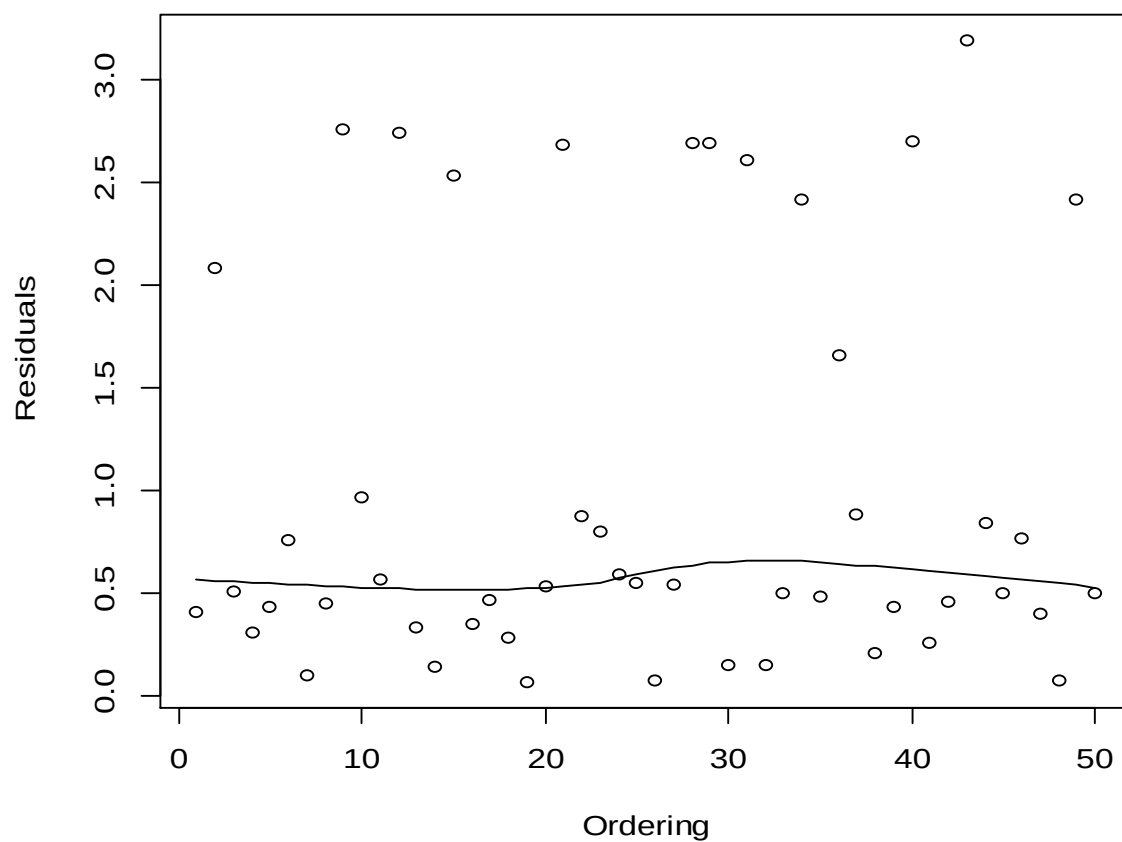


Figure 17: Scatter plot of Residuals

The residual in Figure 17 plots does not show any form of trend but a rectangular scatter points.

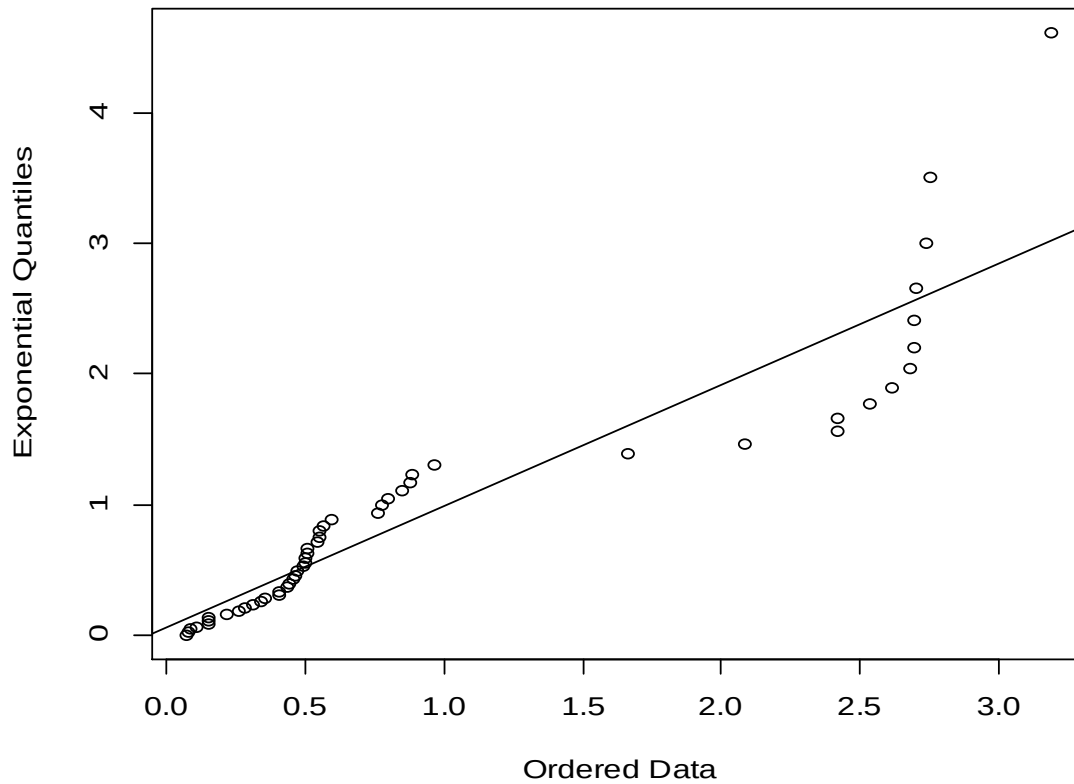


Figure 18: QQ plot of Residuals

The model is adequate with Figure 18 showing an iid standard exponential distribution.

4.11. VaR and ES

The parameters of the GPD is used in computing the Value at Risk (VaR) and Expected Short Fall. The quintiles of the two measures are stated below.

$$VaR_q = 0.04943879$$

$$ES_q = 0.04929124$$

Therefore,

$$VaR = 0.04943879 \times 1000000 = 49438.79$$

$$ES = 0.04929124 \times 1000000 = 49291.24$$

These values implies that the value at risk (VaR) is ~~N~~49438.79 while that of expected short fall is ~~N~~ 49291.24. The two risk measures were computed at 95% confidence interval and a position of ~~N~~1,000,000.00.

CHAPTER FIVE

SUMMARY OF FINDINGS, CONCLUSION AND RECOMMENDATIONS

5.1. Summary of Findings

The time plot of the daily stock price of First Bank Nigeria, Plc shows an evidence of trend. This is an indication that a non-stationary model is required. In determining the appropriate method of transformation, the log transformations were suitable for the four series. The log transformation did not remove the trend but stationarity was achieved in log difference (returns). This is in agreement with most research works and articles that asset returns is described as the log difference. Also, as expected in financial time series, the ARCH effect is present in the data which McLeod $\hat{\imath}$ Li test indicated strong presence of ARCH effect from lag 1 to lag 35 with p-values less than 0.05. Thereby, satisfying the use of Generalised Autoregressive Conditional Heteroscedastic (GARCH) model. But the mixed model of ARMA+ GARCH was adopted based on the fact that financial time series has a non constant conditional mean and conditional variance structures. The approaches in handling the unobserved values contributed immensely in selecting the best model. This can be seen in the performance of models where we used deleting approach as against other approaches. The AIC of the ARMA+GARCH model from deleting unobserved values series has the maximum value for both normal distribution and student t $\hat{\imath}$ distribution versions. The ARMA(2,1)+GARCH(2,1) with student t-distribution has AIC of -6.837350 which is the minimum, hence FBN3 was selected as the series with the most suitable model.

The residuals without ARCH effects were estimated from ARMA(2,1)+GARCH(2,1). The kurtosis of the residuals is 128.2811, that is, heavy tailed resulting in the generalized extreme value distribution of Frechet type. This distribution was approximated to Generalised Pareto Distribution (GPD) and the model parameters used in computing risk measures. The study adopted the use of second stage approach; finding the residuals and

using its series in GPD. The use of GPD on the residuals was satisfactory since it gave a good fit. Meaning that all the factors that determined the price of stock are appropriately represented in the model.

In the computation of Value at Risk (VaR) and Expected Shortfall (ES), the position of ₦1000000 was considered because most single individual investments were about a million naira. This is because most of the investors were from the middle income earners, who are mostly civil servants and who can assess such loans from their salary accounts. Considering the position ₦1000000, the value of VaR is ₦49438.79 while ES is N 49291.24

5.2. Conclusion

The daily stock price of First Bank Nigeria, Plc exhibited the same features of other financial series. It is non-stationary in the observed prices but with the returns on the daily stock price, the trend was removed. The determination of method of transformation gave the confidence of using returns as log difference. Also, the approach to unobserved values contributed immensely to the selection of the appropriate model used in estimating the residuals. The series, daily stock price of FBN, has both conditional mean and conditional variance structure not constant. This implies that the ARMA model alone or the GARCH model alone would not have fit the data appropriately.

Also, as expected in financial time series, the ARCH effect is present in the data which the test revealed. The volatility nature of a series is exhibited, hence the use of GARCH model combined with ARMA model. Eventually, the model exhibited significant parameters except the constant term and the residuals series did not show normality and was not intercorrelated that led to the fitness of the model. The residuals series was subjected to the test to determine its distribution. The test revealed that the residual follows the

Generalised Extreme Value Distribution (GEVD). The approximation of the GEVD to Generalised Pareto Distribution (GPD) was adopted.

The study adopted the use of the second stage approach; finding the residuals without ARCH effect and using the series in GPD. In the computation of VaR and ES the position of N1000000 determines the risk of investing N1000000 in the stock of First Bank Nigeria, Plc at long and short position.

There is enough evidence exhibited by the measures and forecast that pointed to the fact that in 2008 and beyond the market experienced hard times.

5.3. Recommendations

The volatility nature of the stock series should be given adequate attention by using volatility models for financial time series. Moreover, the extension of GARCH models to other useful models should not be under estimated. The extended models will be able to capture other characteristics exhibited by financial time series. In this research work, the ARMA (2,1)+GARCH (2,1) is most suitable for the First Bank Nigeria, Plc stock price series from 1998 to 2007.

The least of all the models is from the deleting approach with maximum AIC for both normal distribution and student t - distribution versions. The best model is from imputation by repetition with minimum AIC for both normal distribution and student t - distribution versions. Based on this, imputing an unobserved value with the preceding value is the best option.

The risk of investing in the stock of the First Bank Nigeria, Plc is eminent from the values of risk measures and forecast. The Value at Risk (VaR) is ₦49438.79 and Expected Shortfall (ES) is ₦49291.24. The forecast of the returns diminishes as days pass by while volatility forecast is in the increasing pattern. Therefore, First Bank Nigeria, Plc has to improve its performance to restore investors confidence.

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Appendix A1

The daily stock price of First Bank Nigeria, Plc from 1998 to 2007 in naira

Day	Price	Day	Price	Day	Price	Day	Price	Day	Price
1		35	11.12	69	10	103	9.8	137	8.7
2	12.6	36	11.12	70	10	104	9.8	138	8.7
3	12.6	37	11.12	71	10	105	9.8	139	8.7
4	12.6	38	11.12	72		106	9.8	140	8.7
5	12.6	39	10.65	73		107	9.8	141	8.7
6	12.6	40	10.65	74	10	108	9.8	142	8.7
7	12.6	41	10.65	75	10	109	9.8	143	8.7
8	12.6	42	10.65	76	10	110	9.8	144	8.7
9	12.6	43	10.65	77	10	111	9.8	145	8.7
10	12.6	44	10.65	78	10	112	9.8	146	8.7
11	12.6	45	10.65	79	10	113	9.8	147	8.7
12	12.4	46	10.65	80	10	114	9.8	148	8.7
13	12.4	47	10.65	81	10	115	9.8	149	8.7
14	12.3	48	10.55	82	10	116	9.8	150	8.7
15	12	49	10.5	83	10	117	9.8	151	8.7
16	12	50	10.5	84	10	118	9.8	152	8.7
17	12	51	10.5	85	10	119	9.8	153	8.7
18	12	52	10.5	86	10	120	9.8	154	8.7
19	12.6	53	10.3	87		121	9.8	155	8.7
20		54	10.3	88	10	122	9.8	156	8.7
21		55	10.3	89	10	123	9.8	157	8.7
22	12.6	56	10.3	90	10	124	9.8	158	8.7
23	12.6	57	10.3	91	10	125	9.8	159	8.7
24	12.6	58	10.3	92	10	126	9.8	160	8.7
25	12.5	59	10.3	93	10	127	9.8	161	8.7
26	12.4	60	10.3	94	10	128	9.8	162	8.7
27	12.4	61	10.3	95	10	129	9.7	163	8.7
28	12.4	62	10.3	96	9.8	130	9.7	164	8.5
29	12.4	63	10.1	97	9.8	131	9.7	165	8.5
30	12.4	64	10.1	98	9.8	132	9.7	166	8.5
31	12.3	65	10	99	9.8	133	9.7	167	8.5
32	12.3	66	10	100	9.8	134	8.7	168	8.5
33	12.3	67	10	101	9.8	135	8.7	169	8.5
34	11.69	68	10	102	9.8	136	8.7	170	8.5

Day	Price	Day	Price	Day	Price	Day	Price	Day	Price
171	8.5	205	7.8	273		307	10	341	12.6
172	8.5	206	7.7	274		308	10	342	13.2
173	8.5	207	7.7	275	9.2	309	10	343	13.2
174	8.5	208	7.9	276	9.2	310	10	344	13.9
175	8.5	209	7.9	277	9.3	311	10	345	15
176	8.3	210	7.9	278	9.5	312	10	346	15.33
177	8.3	211	7.9	279	9.5	313	10	347	15.33
178	8.3	212	7.9	280	9.8	314	10	348	15.33
179	8.3	213	7.9	281	9.9	315	10	349	14.6
180	8.1	214	7.9	282	9.9	316	10	350	13.88
181	8.1	215	7.51	283	10.1	317	10	351	13.19
182	8.1	216	7.89	284	10.1	318	10	352	12.54
183	8	217	8	285	10.1	319	10	353	12
184	8	218	8	286	10.2	320	10.1	354	12.6
185	8	219	7.6	287	10.2	321	10.2	355	13.22
186	8	220	7.75	288	10.2	322		356	13.88
187		221	8.1	289	10	323	10.6	357	14.57
188	8	222	8.2	290	10	324	10.6	358	13.85
189	8	223	8.2	291	10	325	10.7	359	13.25
190	8	224	8.4	292	10	326	10.7	360	13.25
191	7.6	225	8.4	293	10	327	10.7	361	13.51
192	7.4	226	8.8	294	10	328		362	14.18
193	7.4	227	8.9	295	10.5	329	10.9	363	14.1
194	7.4	228	8.9	296	10.8	330	11	364	13.41
195	7.4	229	9	297	11	331	11	365	13.35
196		230	9	298	11	332	11	366	13.4
197	7.4	231	9	299	11	333	11	367	13.8
198	7.4	232	9	300	11	334	11.2	368	14.2
199	7.2	233	9.25	301	10.5	335	11.3	369	14.9
200	7.4	234	9.25	302	10.4	336	11.3	370	14.9
201	7.5	235	9.4	303	10.2	337	11.5	371	14.16
202	7.65	236	9.6	304	10	338	11.5	372	13.8
203	7.65	237	10	305	10	339	11.75	373	14.07
204	7.71	238	10.1	306	10	340	12	374	14.6

Day	Price	Day	Price	Day	Price	Day	Price	Day	Price
375	14.6	409	10	443	9.18	477	9.4	511	11.65
376	14.6	410	9.7	444	9.06	478	9.4	512	12
377	15	411	9.3	445	9.3	479	9.42	513	11.5
378	15.75	412	9	446	9.28	480	9.5	514	11.5
379	16.53	413	8.7	447	9.15	481	9.61	515	11.25
380	16.52	414	9.13	448	9.03	482	9.6	516	11.01
381	15.9	415	9.58	449	9.15	483	9.85	517	11.5
382	16.52	416	10.05	450	9.1	484	9.85	518	11.11
383	17.34	417	10.2	451	9.23	485	9.8	519	11.5
384	17.33	418	10	452	9.13	486	9.7	520	11.52
385	17	419	9.5	453	9.11	487	9.6	521	11.69
386	16.16	420	9.45	454	9.19	488	9.89	522	11.69
387		421	9.5	455	9.25	489	9.99	523	11.9
388	15.6	422	9.75	456	9.3	490	9.75	524	12.17
389	15.4	423	9.5	457		491	9.93	525	11.59
390	14.63	424	9.2	458	9.3	492	9.95	526	
391	14.5	425	9.2	459	9.24	493	9.97	527	
392	14	426	9.2	460	9.4	494	10.25	528	11.67
393	14	427	9.3	461	9.5	495	10.5	529	11.89
394	13.65	428	9.4	462	9.6	496	11	530	12
395	12.97	429	9.45	463	9.6	497	11.01	531	12.11
396	12.33	430	9.46	464	9.75	498	11.03	532	12.69
397	9.06	431	9.46	465	9.6	499	11.53	533	13.15
398	9.51	432	9.3	466	9.57	500	12.1	534	13.56
399	9.51	433	9.18	467	9.57	501	12.7	535	13.65
400	9.98	434	9.18	468	9.57	502	13.33	536	13.16
401	10.47	435	9.16	469	9.57	503	13	537	13.06
402	10.74	436	9.15	470	10.04	504	13.29	538	13.5
403	10.3	437	9.25	471	10	505	13	539	13.5
404	9.79	438	9.25	472	9.54	506	12.36	540	13.21
405	10	439	9.27	473	9.85	507	11.75	541	13.35
406	10.3	440	9.27	474	9.9	508	11.17	542	14
407	10	441	9.2	475	9.7	509	11.72	543	13.55
408	10	442	9.02	476	9.5	510	11.14	544	13.58

Day	Price	Day	Price	Day	Price	Day	Price	Day	Price
545	14.15	579	14.06	613	15.89	647	17	681	15.19
546	14.85	580	13.9	614	15.9	648	17	682	14.98
547	14.9	581	13.9	615	15.51	649	17.18	683	14.7
548	15.64	582	14.2	616	15.7	650	17.18	684	14.58
549	16.42	583	14.25	617	15.75	651	16.96	685	15.3
550	17.24	584	14.25	618	16	652	17	686	16
551	17.75	585	14.27	619	16	653	17.15	687	15.25
552	16.87	586	14.21	620	15.93	654	17.99	688	15.35
553	16.03	587	13.99	621	15.99	655	17.65	689	15.31
554	15.23	588	14.1	622	15.75	656	17.54	690	15.79
555	14.47	589	14.3	623	15.99	657	17.05	691	15.92
556	14.47	590	14.35	624	16	658	12.64	692	16
557	14.7	591	14.28	625	15.99	659	13.27	693	15.99
558	14.3	592	14.5	626	16.1	660	13.93	694	16
559	14.47	593	14.5	627	16.07	661	14.62	695	16.18
560	14.62	594	14.39	628		662	15.35	696	16
561	14.75	595	14.48	629	16.02	663	16.11	697	16.2
562	14.86	596	14.5	630	16	664	15.4	698	16.05
563	14.7	597	14.5	631	16.1	665	14.63	699	16
564	15	598	14.5	632	16.2	666	13.9	700	16.02
565	15	599	14.55	633	16.3	667	13.91	701	16
566	14.76	600	14.5	634	16.5	668	13.9	702	16
567	14.96	601	14.49	635	17	669	13.63	703	15.9
568	14.75	602		636	16.9	670	14.31	704	15.98
569	14.7	603		637	16.95	671	14.74	705	15.99
570	14.55	604	14.48	638	16.95	672	15	706	15.92
571	14.7	605	14.9	639	16.95	673	15.32	707	15.95
572	14.6	606	15.34	640		674	16	708	15.9
573	14.5	607	16.1	641	16.8	675	16.5	709	15.9
574	14.25	608		642	16.55	676	16.2	710	15.9
575		609	16.35	643	16.8	677	16	711	15.77
576		610	15.7	644	16.8	678	15.9	712	15.5
577	14.4	611	15	645	17	679	15.98	713	15.7
578	14.15	612	15.4	646	17	680	15.98	714	15.69

Day	Price	Day	Price	Day	Price	Day	Price	Day	Price
715	15.7	749	15.73	783		817	26.8	851	28.5
716	15.5	750	15.73	784	24	818	26	852	28.06
717	15.52	751	15.62	785	24.25	819	26.35	853	28.2
718	15	752	15.66	786	23.09	820	26.8	854	28.3
719	15.36	753	15.65	787	23.09	821	27.7	855	28.53
720	15.39	754	15.75	788	24.1	822	27.3	856	29
721	15.19	755	15.7	789	25.3	823	27.3	857	
722	15.19	756	15.8	790	26.56	824	26.88	858	
723	15.1	757	15.9	791	27.25	825	27.52	859	28.29
724	15.02	758	15.93	792	28.61	826	27.6	860	28.29
725	15.08	759	15.7	793	30.04	827	28.5	861	28.51
726	15.15	760	15.76	794	29.99	828		862	28.75
727	15.2	761	15.76	795	28.75	829	28.8	863	28.6
728	15.5	762	15.77	796	27.32	830	28.9	864	28.55
729	15.45	763	15.8	797	25.96	831	28.88	865	29
730	15.4	764	15.98	798	24.67	832	28.09	866	29.62
731	15.5	765	16	799	24.7	833	28.03	867	29.16
732	15.7	766	16.18	800	25.93	834	28.03	868	29.16
733	15.5	767	16.18	801	27	835	28.02	869	
734	15.8	768	16.18	802	27.01	836	28.04	870	29.2
735	15.65	769	16.29	803	27	837	28.07	871	29.55
736	15.65	770	16.29	804	26.99	838	28.01	872	30.06
737	15.62	771	20.7	805	26	839	28	873	31
738	15.72	772	17.05	806	26.01	840	27.99	874	32.55
739	15.72	773	18.79	807	25.06	841	27.1	875	33.61
740	15.98	774	19.72	808	26	842	27.12	876	32.88
741	15.85	775	20.7	809	24.99	843	27	877	32.8
742	15.87	776	21.73	810	25.46	844	25.73	878	32.15
743	15.85	777	22.81	811	25.8	845	25.5	879	32.1
744	15.87	778		812	25.89	846	26.2	880	32.1
745	15.61	779		813	25.89	847	26.77	881	33.03
746	15.63	780	21.67	814	26.1	848	27.99	882	33.5
747	15.73	781	22.73	815	27	849	28.23	883	33.2
748	15.75	782	23.8	816	28.15	850	28.7	884	33.45

Day	Price	Day	Price	Day	Price	Day	Price	Day	Price
885	33.02	919	31.26	953	22.15	987	24.1	1021	24
886	33.5	920	30.11	954	22.12	988	23.11	1022	25.2
887	33.5	921	30.5	955	22.12	989	24.25	1023	24
888	33.61	922	23.36	956	22.15	990	24	1024	24
889		923	24.52	957	22.2	991	24	1025	24.15
890	34.56	924	25.74	958	22.17	992	23.5	1026	24.17
891	34.41	925	27.02	959	22.17	993	23.5	1027	24.65
892	34.48	926	25.97	960	22.19	994	23.7	1028	24.2
893		927	25	961	22.13	995	23.62	1029	24.49
894	34.6	928	25.75	962	22.19	996	23.51	1030	24
895	32.9	929	25.05	963	22.23	997	23.56	1031	24
896	33.2	930	25	964	22.16	998	23.5	1032	24
897	32.85	931	26.1	965	22.16	999	23	1033	
898	33.2	932	25.5	966	21.78	1000	22.17	1034	24
899	32.85	933	25.45	967	22.69	1001	23.27	1035	24
900	33	934	25.1	968	21.56	1002	23.3	1036	23.99
901	32.9	935	24.55	969	22.63	1003	23.62	1037	24.6
902	32.12	936	23.9	970	21.87	1004	23.73	1038	24
903	30.7	937	22.71	971	22.8	1005	23.83	1039	
904	30.5	938	21.6	972	23.93	1006	23.84	1040	
905	30.8	939	20.53	973	25.12	1007	25.03	1041	23.6
906	32.32	940	21.55	974	26.36	1008	26.28	1042	23.32
907	33.93	941	22.04	975	25.39	1009	24.98	1043	23.55
908	33.45	942	21.01	976	24.13	1010	24	1044	
909	33.46	943	22.05	977	22.95	1011	23.99	1045	23.85
910	33.5	944	22.06	978		1012	24.5	1046	23.6
911	32.51	945	21.91	979	22.9	1013	24.5	1047	23.89
912	32.35	946	21.9	980	23.43	1014	24.5	1048	24
913	33.2	947	21.93	981	23.43	1015	24.69	1049	24.1
914	34	948	22.91	982	23.93	1016	24.4	1050	24
915	34.5	949	22.99	983	23.99	1017	24.4	1051	23.99
916	32.83	950	23.3	984	24.5	1018	23.97	1052	24.5
917	32.84	951	22.99	985	25.25	1019	23.99	1053	23.8
918	32.9	952	22.11	986	25.24	1020	23.99	1054	23.99

Day	Price	Day	Price	Day	Price	Day	Price	Day	Price
1055	23.99	1089	24.5	1123	24	1157	25.01	1191	21.71
1056	24	1090	24.21	1124	23.3	1158	25	1192	21.86
1057	23.6	1091	23.99	1125	22.71	1159	25.5	1193	21.25
1058	24.05	1092	24.01	1126	23.6	1160	24.99	1194	20.2
1059	24	1093	24.5	1127	24.77	1161	25.4	1195	20.3
1060	23.9	1094	24.55	1128	26	1162	26.48	1196	25.55
1061	23.9	1095	24.5	1129	24.75	1163	27.8	1197	24.99
1062	23.9	1096	24	1130		1164	26.7	1198	25.2
1063	23.98	1097	23.99	1131	24.15	1165	25.38	1199	25.21
1064	23.3	1098	24	1132	25.01	1166	24.12	1200	26.47
1065	22.3	1099	24	1133	26.25	1167	25.2	1201	27.79
1066	22.65	1100	24	1134	25.7	1168	24.69	1202	29
1067	22.3	1101	24	1135	26.1	1169	24.9	1203	28
1068	22.65	1102	23.55	1136	26.3	1170	24.55	1204	27.8
1069	22.02	1103	23.54	1137	26.22	1171	25.1	1205	27
1070	22.3	1104	23.35	1138	25.82	1172	25.15	1206	27
1071	22.2	1105	24.5	1139	25.82	1173	25.55	1207	26
1072	22.15	1106	23.44	1140	26.1	1174	24.99	1208	25.48
1073	22.25	1107		1141	26.03	1175	25.2	1209	25.95
1074	21.14	1108		1142	26.03	1176	25.21	1210	26.51
1075	20.15	1109	22.3	1143	26.3	1177	26.47	1211	20.17
1076	21.15	1110	22	1144	26.4	1178	27.79	1212	21.17
1077	22	1111	22	1145	26.35	1179	29	1213	21.71
1078	22.8	1112	22	1146	26.01	1180	28	1214	21.71
1079	23.5	1113	21.9	1147		1181	27.8	1215	21.86
1080	22.75	1114	21.48	1148	25.52	1182	27	1216	21.25
1081	22.95	1115	21.61	1149	25.5	1183	27	1217	20.2
1082		1116	22.5	1150		1184	26	1218	20.3
1083	22.8	1117	22.5	1151	25.15	1185	25.48	1219	20.6
1084	23	1118	23.62	1152	25.2	1186	25.95	1220	20.74
1085	22.99	1119	24	1153	25	1187	26.51	1221	20.45
1086	24	1120	23.8	1154	24.98	1188	20.17	1222	20
1087	24.99	1121	24	1155	24.23	1189	21.17	1223	20.95
1088	24.95	1122	23.86	1156	23.9	1190	21.71	1224	20.15

Day	Price	Day	Price	Day	Price	Day	Price	Day	Price
1225	20.49	1259	19.65	1293	19.3	1327	24	1361	24.97
1226	20.19	1260	19.5	1294	19.14	1328	24.15	1362	25
1227	20.06	1261	19.57	1295	19	1329	24.51	1363	26.25
1228	20.27	1262	19.69	1296	19	1330	24.3	1364	25.99
1229	20.08	1263	19.85	1297	19.07	1331	24.18	1365	25.95
1230	20	1264	19.85	1298	18.9	1332	24.2	1366	25.2
1231	19.99	1265	19.05	1299	19.8	1333	24.3	1367	25.3
1232	19.99	1266	19.5	1300		1334		1368	26
1233	19.99	1267	19.45	1301		1335		1369	25.96
1234	20.09	1268	19.45	1302	20.78	1336	25	1370	25.5
1235	20.13	1269	19.45	1303	21.81	1337	26.23	1371	25.54
1236	19.99	1270	19.4	1304	21.05	1338	27.54	1372	25.3
1237	20	1271	19.1	1305		1339	28.85	1373	25.55
1238	19.96	1272	18.59	1306	20.8	1340	28.95	1374	25.95
1239		1273	18.1	1307	20.8	1341	27.51	1375	25.5
1240	20	1274	18.32	1308	20.9	1342	26.98	1376	25.52
1241	19.99	1275	18.4	1309	20.7	1343	26.3	1377	25.69
1242	20.1	1276	18.76	1310	20.66	1344	25.5	1378	26.01
1243	19.5	1277	19.15	1311	21	1345	24.99	1379	26.1
1244	20.08	1278	19.2	1312	21.45	1346	24.6	1380	26.11
1245	19.86	1279	19.2	1313	22.52	1347	25.83	1381	26.48
1246	20	1280	19.44	1314	23.64	1348	26.99	1382	
1247	20.05	1281	19.99	1315	23.8	1349	26.6	1383	
1248	20	1282	20.49	1316	23.55	1350	26.8	1384	26.48
1249	19.99	1283	19.98	1317	23.8	1351	26.61	1385	26.3
1250	20	1284	18.99	1318	23.8	1352	26.55	1386	26
1251	19.98	1285		1319	24.05	1353	26.25	1387	25.39
1252	19.93	1286		1320	23.6	1354	26.6	1388	26.2
1253	19.97	1287	18.41	1321	23.65	1355	26.5	1389	26.1
1254	20	1288	18.1	1322	23.69	1356	26.55	1390	26
1255	19.99	1289	19	1323	23.82	1357	26.8	1391	
1256	19.9	1290	19.65	1324	23.8	1358	26.75	1392	26
1257	19.8	1291	19.2	1325	23.9	1359	26.49	1393	25.99
1258	19.8	1292	19.3	1326	23.8	1360	25.59	1394	25.98

Day	Price	Day	Price	Day	Price	Day	Price	Day	Price
1395	25.98	1429	28.01	1463	20	1497	20	1531	20
1396	26.05	1430	28.05	1464	20	1498	20	1532	20
1397	26.9	1431	28	1465	20	1499	20	1533	20
1398	26.48	1432	27.61	1466	20	1500		1534	20
1399		1433	27.97	1467	20	1501	20	1535	20
1400	26.82	1434	27.9	1468	20	1502	20	1536	20
1401	26.82	1435	27.7	1469	20	1503	20	1537	20
1402	26.8	1436	27.8	1470	20	1504	20	1538	20
1403	26.81	1437	27.94	1471	20	1505	20	1539	20
1404	26.8	1438	27.7	1472	20	1506	20	1540	20
1405	26.95	1439	28.38	1473	20	1507	20	1541	20
1406	27.29	1440	28.15	1474	20	1508	20	1542	20
1407	27.39	1441	27.99	1475	20	1509	20	1543	20
1408	27.4	1442	26.97	1476	20	1510	20	1544	20
1409	28.5	1443	26.1	1477	20	1511	20	1545	20
1410	28.4	1444	26.8	1478	20	1512	20	1546	20
1411		1445	27.97	1479	20	1513	20	1547	20
1412	28.26	1446	28.4	1480	20	1514	20	1548	20
1413	28.49	1447	22.41	1481	20	1515	20	1549	20
1414	28.3	1448	21.29	1482	20	1516	20	1550	20
1415	28.03	1449	20.85	1483	20	1517	20	1551	20
1416	27.94	1450	20.85	1484	20	1518	20	1552	20
1417	27.95	1451	20.7	1485	20	1519	20	1553	20
1418	27.98	1452	20.24	1486	20	1520	20	1554	20
1419	27.75	1453	20.7	1487	20	1521	20	1555	20
1420	27.88	1454	19.95	1488	20	1522	20	1556	20
1421	27.85	1455	19.96	1489	20	1523	20	1557	20
1422	27.98	1456	19.97	1490	20	1524	20	1558	20
1423	28	1457	19.97	1491	20	1525	20	1559	20
1424	27.97	1458	20	1492	20	1526	20	1560	20
1425	28	1459	19.97	1493	20	1527	20	1561	
1426	28	1460	20.5	1494	20	1528	20	1562	
1427	28	1461	20	1495	20	1529	20	1563	20
1428	28	1462	20	1496	20	1530	20	1564	20

Day	Price	Day	Price	Day	Price	Day	Price	Day	Price
1565	20	1599	21	1633	29.31	1667	29.04	1701	29.55
1566		1600	22.05	1634	29.29	1668	29.6	1702	31.02
1567	20	1601	23.15	1635	29.7	1669	30	1703	31.2
1568	20	1602	24.3	1636	29.97	1670	29.96	1704	30.68
1569	20	1603	25.51	1637		1671	29.9	1705	29.8
1570	20	1604	26.78	1638		1672	30.98	1706	29
1571	20	1605	28.11	1639	30	1673	30.5	1707	28.99
1572	20	1606	29.51	1640	30	1674	30	1708	27.88
1573	20	1607	30.98	1641	30.09	1675	29.03	1709	28
1574	20	1608	31	1642	30.2	1676	28.7	1710	29
1575	20	1609	29.45	1643	30.15	1677	29.4	1711	28.48
1576	20	1610	28.02	1644	30.01	1678	29.7	1712	28.4
1577	20	1611	28	1645	29.99	1679	28.23	1713	28.17
1578	20	1612	29.4	1646	30.4	1680	28.4	1714	29.5
1579	20	1613	29.99	1647	30.3	1681	29.4	1715	30.29
1580	20	1614	29.57	1648	30	1682	28.95	1716	29.98
1581	20	1615	29.11	1649	30.12	1683	28.61	1717	28.98
1582	20	1616	29.49	1650	30.14	1684	28.8	1718	28.06
1583	20	1617	29.88	1651	30.05	1685	29.9	1719	28.8
1584	20	1618	29.05	1652	29.99	1686	31.39	1720	29
1585	20	1619	29.1	1653	29.25	1687	32.95	1721	30.3
1586	20	1620	29.39	1654	28.91	1688	32.5	1722	25.55
1587	20	1621	28.75	1655	28.98	1689	31.6	1723	24.85
1588		1622	28.5	1656	29	1690	30.02	1724	23.61
1589	20	1623	28.25	1657	28.99	1691	28.57	1725	24.7
1590	20	1624	28.25	1658	29	1692	28.57	1726	24.5
1591	20	1625	27.64	1659	29.31	1693	29.2	1727	24.25
1592	20	1626	27.21	1660	30.44	1694	28	1728	23.7
1593	20	1627	27.5	1661	30	1695	28.3	1729	22.52
1594	20	1628	27.85	1662	29.99	1696	28.22	1730	21.4
1595	20	1629	28.5	1663	29.99	1697	28	1731	20.34
1596	20	1630	28.1	1664	29.5	1698	28	1732	21.35
1597	20	1631	28.51	1665	29.5	1699	28	1733	22.41
1598	20	1632	29.39	1666	29.9	1700	28.15	1734	23.53

Day	Price	Day	Price	Day	Price	Day	Price	Day	Price
1735	24.7	1769	23.38	1803	24.01	1837	23.99	1871	22.97
1736	24.4	1770	23.63	1804	24	1838	24	1872	22.9
1737	23.76	1771	23.63	1805	23.96	1839	23.95	1873	23
1738	23.58	1772	22.61	1806	23.48	1840	24	1874	22.95
1739	22.95	1773	22.65	1807	24.65	1841		1875	22.91
1740	22	1774	22.83	1808	23.99	1842		1876	22.55
1741	22	1775	23.1	1809	24.01	1843	24.47	1877	22.95
1742	21.99	1776	23.55	1810	23.98	1844	24.09	1878	22.94
1743	23	1777	24.72	1811	24	1845	23.8	1879	23
1744	23.26	1778	25.95	1812	24	1846	23.22	1880	22.98
1745	23.8	1779	27.24	1813	24	1847	23.6	1881	22.94
1746	24	1780	26.1	1814	23.7	1848	23.42	1882	22.8
1747	23.7	1781	25.01	1815	23.9	1849	23.6	1883	22.51
1748	22.55	1782	25	1816	23.97	1850	24.49	1884	22.75
1749	23.67	1783	24.5	1817	23.9	1851	24.3	1885	22.5
1750	23.48	1784	24.5	1818	22.75	1852	23.88	1886	22.35
1751	23.4	1785	25	1819	22.45	1853	23.94	1887	
1752	22.95	1786		1820	23	1854	23.85	1888	
1753	22.97	1787	24.83	1821	23	1855	23.86	1889	22.45
1754	22.98	1788	25.25	1822	22.96	1856	23.73	1890	22.46
1755	23	1789	25.1	1823	22.97	1857	23	1891	23.58
1756	22.98	1790	25	1824	23	1858	22.98	1892	22.41
1757	22.04	1791	25.26	1825	22.9	1859	22.99	1893	22.6
1758	23	1792		1826	23.1	1860	23	1894	23.06
1759	23	1793	25	1827	23.6	1861	23	1895	23.7
1760	22.89	1794	25.29	1828		1862	23.1	1896	24.8
1761	22.35	1795	25.5	1829	23.98	1863	23.2	1897	25.68
1762		1796	25.5	1830	24	1864	23.78	1898	
1763	22.65	1797	25.49	1831	24.7	1865	23.7	1899	24.4
1764	22.5	1798	25.49	1832	24.9	1866	23.45	1900	24.7
1765	22.53	1799	25.5	1833	25	1867	23.3	1901	24.49
1766	22.55	1800	25	1834	24.8	1868	23.35	1902	24.5
1767	22.55	1801	24.94	1835	24.45	1869	22.89	1903	25.03
1768	25	1802	24	1836	24.3	1870	21.88	1904	25.4

Day	Price	Day	Price	Day	Price	Day	Price	Day	Price
1905	26.24	1939	29.99	1973	29.98	2007	32	2041	32
1906		1940	30.26	1974	31.15	2008	32	2042	32
1907	27	1941	30.85	1975	31	2009	32	2043	32
1908	27	1942	30.83	1976	30.5	2010	32	2044	32
1909	26.98	1943	30.43	1977	30.99	2011	32	2045	
1910	26	1944	30.05	1978	31	2012	32	2046	
1911	26.09	1945	29.96	1979	31	2013	32	2047	32
1912	26.17	1946	30.05	1980	30.99	2014	32	2048	32
1913		1947	30.2	1981	31.5	2015	32	2049	32
1914	27.39	1948	30.56	1982	31.99	2016	32	2050	32
1915	28.75	1949	30.99	1983	31.7	2017	32	2051	32
1916	28.98	1950	31.7	1984	31	2018	32	2052	32
1917	28.07	1951	31.79	1985	31	2019	32	2053	32
1918	27.5	1952	31.07	1986	30.61	2020	32	2054	32
1919	26.77	1953	30.22	1987	23.2	2021	32	2055	32
1920	27.5	1954	30.15	1988	24.29	2022	32	2056	32
1921	28.24	1955	29.99	1989	23.75	2023		2057	32
1922	29.22	1956	30	1990	24.11	2024	32	2058	32
1923	28.99	1957	30.2	1991	24.05	2025	32	2059	32
1924	28.5	1958	29.99	1992	23.7	2026	32	2060	32
1925	28.31	1959	29.99	1993	23.49	2027	32	2061	32
1926	28.3	1960	28.5	1994	23.8	2028	32	2062	32
1927	28.02	1961	29.92	1995	24.7	2029	32	2063	32
1928	28.08	1962	29.99	1996	25.6	2030	32	2064	32
1929	28.05	1963	29.5	1997	26.88	2031	32	2065	32
1930	27.95	1964	29.8	1998	28.22	2032	32	2066	32
1931	27.95	1965	30	1999	29.63	2033	32	2067	32
1932	28.6	1966	31.08	2000	31.11	2034	32	2068	32
1933		1967	31.5	2001	32.66	2035	32	2069	32
1934	29.4	1968	31.46	2002	33.02	2036	32	2070	32
1935	29.99	1969	30.2	2003	31.6	2037	32	2071	32
1936	29.65	1970	30.69	2004	32	2038	32	2072	32
1937	29.75	1971	30	2005	32	2039	32	2073	32
1938	29.96	1972	29.95	2006	32	2040	32	2074	32

Day	Price	Day	Price	Day	Price	Day	Price	Day	Price
2075	32	2109	33.12	2143	36	2177	47.03	2211	52.73
2076	32	2110	33.18	2144		2178	48	2212	52.8
2077	32	2111	33.98	2145		2179	48	2213	52.56
2078	32	2112	33.52	2146		2180	48	2214	51
2079	32	2113	33	2147		2181	48	2215	51.6
2080	32	2114	33.82	2148	35.85	2182	49.15	2216	51.97
2081	32	2115	33.4	2149	35.99	2183	48.96	2217	53.5
2082	32	2116	33.69	2150	36.52	2184	49.02	2218	52.99
2083		2117	34.3	2151	38	2185	49.15	2219	52.94
2084		2118	35.1	2152	37	2186	49	2220	53.19
2085	32	2119	36.85	2153	37.89	2187	48.99	2221	54.95
2086	32	2120	38.69	2154	37.7	2188	48.99	2222	57.69
2087	32	2121	39	2155	37.98	2189	48.7	2223	60.02
2088		2122	39	2156	38	2190	48.76	2224	58.1
2089	32	2123	37.06	2157	37.99	2191	48.1	2225	58.9
2090	32	2124	35.22	2158		2192	48.22	2226	59.2
2091	32	2125	35.85	2159	37.1	2193		2227	58.98
2092	32	2126	36.01	2160	37.5	2194	48.99	2228	59.5
2093	32	2127	36.99	2161	38	2195	48.99	2229	59.8
2094		2128	37.5	2162		2196	49	2230	60.13
2095		2129	37.5	2163		2197	48.85	2231	60.2
2096	32	2130	37.11	2164	37.92	2198	49.4	2232	63
2097	32	2131	37.38	2165	37.98	2199	49.34	2233	66.15
2098	32	2132	36.93	2166	38.3	2200	50.2	2234	63
2099	32	2133	37.37	2167	38.98	2201	52.71	2235	63.2
2100	32	2134	37.05	2168	38.6	2202	55	2236	64.15
2101	33.25	2135	37	2169	38.66	2203	55.05	2237	63
2102	32.98	2136	36.98	2170	40.59	2204	53.99	2238	62.9
2103	33.61	2137	37.15	2171	42.61	2205	51.35	2239	62.5
2104	34.19	2138	37	2172	44.74	2206	49.81	2240	62.3
2105	32.99	2139	36.99	2173		2207	50	2241	63.25
2106	32.9	2140	36.97	2174	46.97	2208	52.5	2242	66.35
2107	32.99	2141	36.49	2175	46	2209	52.43	2243	68.9
2108	33.68	2142	36.25	2176	46.09	2210	52.7	2244	69.24

Day	Price	Day	Price	Day	Price	Day	Price	Day	Price
2245	68.1	2279	35.07	2313	35.1	2347	33.5	2381	41.01
2246	69.3	2280	35.33	2314	34.3	2348		2382	41.9
2247	72.76	2281	35.44	2315	33.02	2349		2383	39.87
2248	35.88	2282	34.8	2316	33.3	2350		2384	40.99
2249	37.67	2283		2317	33.01	2351	32.3	2385	40.17
2250	39.55	2284	33.85	2318	33	2352	33.1	2386	38.5
2251	41.52	2285	32.61	2319	31.99	2353	32.01	2387	37.99
2252	43.59	2286	32.59	2320	31	2354	32	2388	39.49
2253	45.76	2287	34.21	2321	31.12	2355	32	2389	38.51
2254	43.48	2288	35.92	2322	30.5	2356	32.99	2390	38.11
2255	42	2289	37.71	2323	29.6	2357	33.11	2391	37.55
2256	39.98	2290	39	2324	29	2358	32.92	2392	37
2257	40	2291	38.99	2325	29	2359	33.99	2393	36.99
2258	42	2292	39	2326	29.99	2360	35.68	2394	37.59
2259	44.1	2293	37.93	2327	31.48	2361	37.46	2395	38.5
2260	45	2294	36.56	2328	33.05	2362	38.2	2396	38.9
2261	42.77	2295	36.55	2329	34.63	2363	38.9	2397	39.99
2262	40.7	2296	34.8	2330	32.91	2364	39	2398	39.6
2263	39.5	2297	36.11	2331	32.5	2365	37.05	2399	38.2
2264	41.47	2298		2332	32.5	2366	35.25	2400	37
2265	42	2299		2333	32.01	2367	36.89	2401	37.99
2266	41.01	2300	36.9	2334	32.55	2368		2402	39.5
2267		2301	36.99	2335	32.56	2369	36.8	2403	40.5
2268	40.1	2302	36.02	2336	32	2370	36.89	2404	39.05
2269	39.99	2303	36.26	2337	32.95	2371	36.9	2405	38.6
2270	40.05	2304	36.9	2338	32	2372	37.1	2406	37.15
2271	39.99	2305	36.35	2339	32.6	2373	38.95	2407	37
2272	39.97	2306	35.42	2340	33	2374	39.01	2408	36.8
2273	38.5	2307	35.1	2341	33.19	2375	39.9	2409	37.01
2274	39	2308	34.99	2342	33.9	2376	38.9	2410	37.4
2275	38.86	2309	33.81	2343		2377	37.19	2411	37.3
2276	37.5	2310	32.61	2344		2378	36.98	2412	38
2277	35.63	2311	33	2345	33.99	2379	37.2	2413	
2278	35.4	2312	33.51	2346	32.59	2380	39.06	2414	38

Day	Price	Day	Price	Day	Price	Day	Price	Day	Price
2415	39.78	2449	40.4	2483	40.4	2517	39.98	2551	
2416	40.4	2450	40.4	2484	40.4	2518	39.98	2552	
2417		2451	40.4	2485	40.4	2519	38.98	2553	40
2418		2452	40.4	2486	40.4	2520	38.3	2554	39.82
2419	40.4	2453	40.4	2487	40.4	2521	37.4	2555	39.1
2420	40.4	2454		2488	40.4	2522	39.27	2556	38.99
2421		2455	40.4	2489	40.4	2523	41.23	2557	39
2422		2456	40.4	2490	40.4	2524	43.29	2558	38.9
2423	40.4	2457	40.4	2491	40.4	2525	45.45	2559	39.01
2424	40.4	2458	40.4	2492	40.4	2526	44.52	2560	39
2425	40.4	2459	40.4	2493	40.4	2527	44	2561	38.72
2426	40.4	2460	40.4	2494	40.4	2528	42.97	2562	38.6
2427		2461	40.4	2495	42.42	2529	41.49	2563	38.61
2428	40.4	2462	40.4	2496	44.54	2530	40.5	2564	38.4
2429	40.4	2463	40.4	2497	46.76	2531	40.5	2565	38.45
2430	40.4	2464	40.4	2498	49.09	2532	40.2	2566	38.49
2431	40.4	2465	40.4	2499	51.54	2533	41	2567	38.49
2432	40.4	2466	40.4	2500	54.11	2534	40	2568	39
2433	40.4	2467	40.4	2501	56.81	2535	39.99	2569	38.98
2434		2468	40.4	2502	56	2536	39.99	2570	40.1
2435	40.4	2469	40.4	2503	53.2	2537	40.24	2571	40
2436	40.4	2470	40.4	2504	50.8	2538	39.16	2572	39.9
2437	40.4	2471	40.4	2505	52.08	2539	38.01	2573	39.5
2438	40.4	2472	40.4	2506	54.68	2540	36.2	2574	39.51
2439	40.4	2473	40.4	2507	57.41	2541	38.01	2575	40
2440	40.4	2474	40.4	2508	54.91	2542	39.91	2576	40.01
2441	40.4	2475	40.4	2509	53	2543		2577	40.36
2442	40.4	2476	40.4	2510	50.52	2544	40.06	2578	40.41
2443	40.4	2477	40.4	2511	51.86	2545	40.3	2579	40.73
2444	40.4	2478	40.4	2512	49.5	2546	40.25	2580	42.75
2445	40.4	2479	40.4	2513	41.57	2547	40	2581	44
2446	40.4	2480	40.4	2514	41.82	2548	40	2582	45.99
2447	40.4	2481	40.4	2515	40.52	2549	40.25	2583	46.69
2448	40.4	2482	40.4	2516	40.52	2550	40.25	2584	44.36

Day	Price
2585	45.7
2586	44.5
2587	43.51
2588	42.9
2589	43.06
2590	43.15
2591	42.72
2592	43
2593	42.5
2594	43
2595	41.83
2596	41.11
2597	42
2598	41.95
2599	42.49
2600	
2601	
2602	42.49
2603	42
2604	
2605	
2606	43.15
2607	43.8
2608	44.7

Appendix A2

The residuals of ARMA(2,1)+GARCH(2,1)_t (m7)

[1] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
[6] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
[11] -1.600034e-02 9.401791e-04 -8.278395e-03 -2.430633e-02 1.315024e-03
[16] -3.467587e-04 -1.713092e-04 4.870553e-02 -2.908718e-03 5.318339e-04
[21] 2.627420e-04 1.298024e-04 6.412632e-05 -7.936489e-03 -7.548311e-03
[26] 3.894721e-04 -1.317110e-04 -6.506922e-05 -3.214616e-05 -8.113091e-03
[31] 4.679458e-04 -9.556718e-05 -5.091270e-02 -4.702296e-02 2.349807e-03
[36] -8.563085e-04 -4.230422e-04 -4.339439e-02 2.434321e-03 -5.400311e-04
[41] -2.667916e-04 -1.318031e-04 -6.511470e-05 -3.216862e-05 -1.589227e-05
[46] -7.851259e-06 -9.437911e-03 -4.198176e-03 1.713694e-04 -1.070393e-04
[51] -5.288063e-05 -1.925749e-02 1.117127e-03 -2.241481e-04 -1.107359e-04
[56] -5.470687e-05 -2.702684e-05 -1.335207e-05 -6.596326e-06 -3.258784e-06
[61] -1.609937e-06 -1.960927e-02 1.151800e-03 -1.017257e-02 4.748894e-04
[66] -1.669158e-04 -8.246142e-05 -4.073842e-05 -2.012601e-05 -9.942854e-06
[71] -4.912069e-06 -2.426710e-06 -1.198868e-06 -5.922768e-07 -2.926026e-07
[76] -1.445545e-07 -7.141427e-08 -3.528080e-08 -1.742978e-08 -8.610834e-09
[81] -4.254011e-09 -2.101610e-09 -1.038258e-09 -5.129310e-10 -2.534033e-10
[86] -1.251889e-10 -6.184708e-11 -3.055432e-11 -1.509475e-11 -7.457262e-12
[91] -3.684112e-12 -1.820062e-12 -8.991656e-13 -4.442149e-13 -2.020271e-02
[96] 1.187110e-03 -2.287713e-04 -1.130199e-04 -5.583523e-05 -2.758428e-05
[101] -1.362747e-05 -6.732379e-06 -3.325998e-06 -1.643143e-06 -8.117622e-07
[106] -4.010350e-07 -1.981233e-07 -9.787890e-08 -4.835512e-08 -2.388889e-08

[111] -1.180183e-08 -5.830459e-09 -2.880422e-09 -1.423015e-09 -7.030124e-10
[116] -3.473093e-10 -1.715813e-10 -8.476631e-11 -4.187711e-11 -2.068855e-11
[121] -1.022077e-11 -5.049367e-12 -2.494539e-12 -1.232378e-12 -6.088316e-13
[126] -3.007812e-13 -1.485949e-13 -1.025650e-02 6.026713e-04 -1.161425e-04
[131] -5.737788e-05 -2.834640e-05 -1.088169e-01 6.386331e-03 -1.235479e-03
[136] -6.103638e-04 -3.015381e-04 -1.489689e-04 -7.359512e-05 -3.635820e-05
[141] -1.796205e-05 -8.873792e-06 -4.383921e-06 -2.165789e-06 -1.069965e-06
[146] -5.285949e-07 -2.611418e-07 -1.290119e-07 -6.373576e-08 -3.148739e-08
[151] -1.555572e-08 -7.684993e-09 -3.796618e-09 -1.875643e-09 -9.266244e-10
[156] -4.577803e-10 -2.261573e-10 -1.117285e-10 -5.519724e-11 -2.726910e-11
[161] -1.347176e-11 -6.655453e-12 -2.325686e-02 1.366572e-03 -2.633559e-04
[166] -1.301057e-04 -6.427615e-05 -3.175435e-05 -1.568761e-05 -7.750150e-06
[171] -3.828807e-06 -1.891546e-06 -9.344808e-07 -4.616616e-07 -2.381088e-02
[176] 1.399000e-03 -2.696825e-04 -1.332313e-04 -2.445727e-02 1.400723e-03
[181] -2.922682e-04 -1.256691e-02 6.586139e-04 -1.759105e-04 -8.690509e-05
[186] -4.293373e-05 -2.121056e-05 -1.047865e-05 -5.176770e-06 -5.129585e-02
[191] -2.365552e-02 9.855658e-04 -5.892439e-04 -2.911042e-04 -1.438142e-04
[196] -7.104857e-05 -3.510013e-05 -2.741631e-02 2.900037e-02 1.149857e-02
[201] 1.916878e-02 -9.350807e-04 8.149677e-03 1.131304e-02 -1.341459e-02
[206] 9.739783e-04 2.560291e-02 -1.526270e-03 2.807250e-04 1.386866e-04
[211] 6.851536e-05 3.384865e-05 1.672225e-05 -5.061903e-02 5.233961e-02
[216] 1.037370e-02 -5.368332e-04 -5.099980e-02 2.270358e-02 4.251358e-02
[221] 9.644352e-03 -2.357451e-04 2.447622e-02 -1.228897e-03 4.688531e-02

[226] 8.746509e-03 -4.802198e-05 1.160555e-02 -4.429998e-04 2.320207e-04
[231] 1.146252e-04 2.745560e-02 -1.581986e-03 1.641022e-02 2.026829e-02
[236] 3.984615e-02 7.919104e-03 9.911417e-03 9.607987e-03 4.715333e-02
[241] -4.991687e-02 3.455870e-03 -2.055174e-04 -9.857707e-03 -1.927951e-02
[246] 1.028344e-03 -2.910620e-04 -1.437934e-04 -7.103827e-05 1.976753e-02
[251] -2.098357e-02 1.379277e-03 -1.176907e-04 -2.026085e-02 -9.098115e-03
[256] 3.597094e-04 -5.404143e-03 1.869931e-04 -1.161628e-04 -1.572350e-02
[261] 8.921888e-04 -1.914063e-04 -3.218288e-02 -9.090279e-03 2.557512e-04
[266] -1.136451e-02 -2.197903e-02 3.464127e-02 8.605043e-03 -4.375907e-04
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[331] 2.135661e-04 1.055081e-04 1.807063e-02 7.855932e-03 -3.055545e-04
[336] 1.775205e-02 -9.282716e-04 2.175558e-02 1.991290e-02 4.785746e-02

[341] 4.404189e-02 -1.988952e-03 5.256663e-02 7.356709e-02 1.808972e-02
[346] -1.933074e-05 8.685909e-04 -4.836105e-02 -4.749367e-02 -4.846611e-02
[351] -4.833314e-02 -4.201703e-02 5.032533e-02 4.414932e-02 4.594531e-02
[356] 4.622146e-02 -5.269765e-02 -4.034909e-02 2.503057e-03 1.888205e-02
[361] 4.698852e-02 -8.416165e-03 -4.925123e-02 -1.308464e-03 3.546111e-03
[366] 2.891831e-02 2.675104e-02 4.672693e-02 -2.362312e-03 -5.016542e-02
[371] -2.237653e-02 2.050174e-02 3.535493e-02 -2.192039e-03 4.091850e-04
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[386] 2.929982e-03 -3.586550e-02 -1.112616e-02 -5.108025e-02 -6.327017e-03
[391] -3.535293e-02 1.572547e-03 -2.595696e-02 -4.992861e-02 -4.804371e-02
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[406] -3.138201e-02 2.028939e-03 -1.904286e-04 -3.055328e-02 -4.036818e-02
[411] -3.068323e-02 -3.263343e-02 4.953805e-02 4.454909e-02 4.525459e-02
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[2181] 2.376561e-02 -5.220048e-03 1.742340e-03 2.675918e-03 -3.149190e-03
[2186] 3.660614e-05 7.569528e-06 -5.941662e-03 1.577921e-03 -1.376881e-02
[2191] 3.272670e-03 -3.105226e-04 1.578946e-02 -9.570085e-04 3.705963e-04
[2196] -2.995660e-03 1.141915e-02 -1.886693e-03 1.747144e-02 4.782037e-02
[2201] 3.987928e-02 -9.299821e-04 -1.868864e-02 -4.858208e-02 -2.752124e-02
[2206] 5.019864e-03 4.793681e-02 -4.469079e-03 5.635023e-03 4.597179e-04
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[2216] 2.855319e-02 -1.122320e-02 -2.291033e-05 4.835220e-03 3.229961e-02
[2221] 4.681213e-02 3.713536e-02 -3.408977e-02 1.640420e-02 4.313015e-03
[2226] -3.848958e-03 9.139533e-03 4.541972e-03 5.321136e-03 9.530904e-04
[2231] 4.551215e-02 4.619034e-02 -5.110692e-02 6.860762e-03 1.458828e-02
[2236] -1.900193e-02 -3.744081e-04 -6.416394e-03 -2.912543e-03 1.520915e-02
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[2246] 4.771165e-02 -7.096432e-01 9.087994e-02 3.815808e-02 4.250353e-02
[2251] 4.474493e-02 4.575483e-02 -5.339771e-02 -3.079837e-02 -4.742410e-02
[2256] 2.920890e-03 4.796770e-02 4.553712e-02 1.769752e-02 -5.128149e-02
[2261] -4.603232e-02 -2.729647e-02 4.972604e-02 9.153774e-03 -2.438755e-02
[2266] 1.650350e-03 -2.258685e-02 -1.501105e-03 1.370620e-03 -1.761740e-03
[2271] -4.813326e-04 -3.749269e-02 1.507426e-02 -4.794002e-03 -3.548426e-02
[2276] -4.913549e-02 -3.911281e-03 -9.782264e-03 7.469629e-03 2.337823e-03
[2281] -1.848916e-02 1.065144e-03 -2.788756e-02 -3.579696e-02 1.214949e-03
[2286] 4.794589e-02 4.562097e-02 4.616380e-02 3.152814e-02 -1.312016e-03
[2291] 1.107353e-03 -2.742433e-02 -3.494748e-02 1.674566e-03 -4.956978e-02

[2296] 3.957443e-02 -2.855858e-03 8.026107e-05 2.168137e-02 1.183978e-03
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[2311] 1.399571e-02 4.527349e-02 -2.569626e-02 -3.611098e-02 1.069734e-02
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[2336] 3.031846e-02 -3.114926e-02 2.054032e-02 1.089343e-02 5.130933e-03
[2341] 2.101974e-02 -1.084517e-03 3.183425e-04 2.808620e-03 -4.213897e-02
[2346] 3.007985e-02 -2.060739e-03 9.325172e-05 4.606916e-05 -3.645545e-02
[2351] 2.662075e-02 -3.533007e-02 1.730844e-03 -3.234074e-04 3.029620e-02
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[2366] 4.796630e-02 -5.893679e-03 2.736773e-04 2.479302e-03 1.456078e-04
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[2376] -4.305571e-02 -3.107427e-03 5.712606e-03 4.810496e-02 4.575095e-02
[2381] 1.911078e-02 -5.012283e-02 3.126064e-02 -2.208247e-02 -4.108307e-02
[2386] -1.097430e-02 3.896100e-02 -2.782626e-02 -8.734241e-03 -1.436056e-02
[2391] -1.408821e-02 3.290091e-04 1.580699e-02 2.282376e-02 9.038093e-03
[2396] 2.735187e-02 -1.114702e-02 -3.496789e-02 -2.969136e-02 2.792791e-02
[2401] 3.689051e-02 2.274536e-02 -3.746981e-02 -8.938710e-03 -3.776856e-02
[2406] -2.006876e-03 -5.720069e-03 5.697312e-03 9.932944e-03 -3.335238e-03

[2411] 1.884816e-02 -1.074417e-03 2.194817e-04 4.588654e-02 1.282916e-02
[2416] -3.639065e-04 4.442986e-04 2.194969e-04 1.084381e-04 5.357167e-05
[2421] 2.646602e-05 1.307501e-05 6.459449e-06 3.191162e-06 1.576530e-06
[2426] 7.788534e-07 3.847770e-07 1.900914e-07 9.391089e-08 4.639481e-08
[2431] 2.292043e-08 1.132338e-08 5.594093e-09 2.763650e-09 1.365326e-09
[2436] 6.745123e-10 3.332294e-10 1.646254e-10 8.132989e-11 4.017941e-11
[2441] 1.984984e-11 9.806418e-12 4.844666e-12 2.393411e-12 1.182417e-12
[2446] 5.841496e-13 2.885875e-13 1.425709e-13 7.043432e-14 3.479668e-14
[2451] 1.719061e-14 8.492677e-15 4.195638e-15 2.072772e-15 1.024012e-15
[2456] 5.058926e-16 2.499262e-16 1.234710e-16 6.099841e-17 3.013505e-17
[2461] 1.488762e-17 7.354934e-18 3.633559e-18 1.795088e-18 8.868273e-19
[2466] 4.381194e-19 2.164442e-19 1.069299e-19 5.282661e-20 2.609794e-20
[2471] 1.289317e-20 6.369612e-21 3.146780e-21 1.554604e-21 7.680213e-22
[2476] 3.794256e-22 1.874477e-22 9.260480e-23 4.574956e-23 2.260166e-23
[2481] 1.116590e-23 5.516291e-24 2.725214e-24 1.346338e-24 6.651314e-25
[2486] 3.285949e-25 1.623358e-25 8.019877e-26 3.962061e-26 1.957377e-26
[2491] 9.670033e-27 4.777287e-27 2.360124e-27 4.879016e-02 4.590081e-02
[2496] 4.632738e-02 4.659428e-02 4.680398e-02 4.682318e-02 4.689151e-02
[2501] -1.614845e-02 -4.936776e-02 -4.277627e-02 2.719996e-02 4.653585e-02
[2506] 4.578449e-02 -4.687043e-02 -3.198107e-02 -4.594812e-02 2.854123e-02
[2511] -4.887991e-02 -1.719394e-01 1.568702e-02 -3.418903e-02 8.081076e-04
[2516] -1.429143e-02 3.560293e-04 -2.569614e-02 -1.629094e-02 -2.312113e-02
[2521] 4.980237e-02 4.537887e-02 4.621908e-02 4.653861e-02 -2.263138e-02

[2526] -9.536097e-03 -2.273815e-02 -3.366302e-02 -2.236170e-02 8.884172e-04
[2531] -7.970616e-03 1.987733e-02 -2.606540e-02 1.317863e-03 -2.071416e-04
[2536] 6.119679e-03 -2.762745e-02 -2.816480e-02 -4.732550e-02 5.117788e-02
[2541] 4.512150e-02 1.047829e-03 4.122388e-04 6.328178e-03 -1.417046e-03
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[2556] 1.737956e-04 -2.736979e-03 2.901202e-03 -4.876521e-04 -7.190628e-03
[2561] -2.683654e-03 3.583219e-04 -5.545275e-03 1.586991e-03 8.844029e-04
[2566] -8.534485e-05 1.316297e-02 -1.286521e-03 2.850679e-02 -4.093605e-03
[2571] -2.002139e-03 -9.781820e-03 8.893302e-04 1.221849e-02 -5.170092e-04
[2576] 8.813551e-03 7.876544e-04 7.943832e-03 4.801852e-02 2.610392e-02
[2581] 4.315207e-02 1.313513e-02 -5.126817e-02 3.334003e-02 -2.865498e-02
[2586] -2.074465e-02 -1.300432e-02 4.195088e-03 1.532830e-03 -1.026192e-02
[2591] 7.083792e-03 -1.221191e-02 1.239207e-02 -2.840170e-02 -1.567226e-02
[2596] 2.216021e-02 -2.783775e-03 1.293782e-02 -7.267592e-04 1.570866e-04
[2601] 7.760549e-05 -1.156080e-02 7.005054e-04 -1.219889e-04 2.695253e-02
[2606] 1.333436e-02 1.975232e-02

Appendix B

```
library(TSA)

library(fGarch)

FBN1=read.csv('FBN1.csv',header=T)

FBN2=read.csv('FBN2.csv',header=T)

FBN3=read.csv('FBN3.csv',header=T)

FBN4=read.csv('FBN4.csv',header=T)

FBN1=ts(FBN1, start=c(1998,1), frequency=260)

FBN2=ts(FBN2, start=c(1998,1), frequency=260)

FBN3=ts(FBN3, start=c(1998,1), frequency=260)

FBN4=ts(FBN4, start=c(1998,1), frequency=260)

plot(FBN1)

plot(FBN2)

plot(FBN3)

plot(FBN4)

McLeod.Li.test(y=FBN1)

McLeod.Li.test(y=FBN2)

McLeod.Li.test(y=FBN3)

McLeod.Li.test(y=FBN4)

plot(log(FBN1))

plot(log(FBN2))

plot(log(FBN3))

plot(log(FBN4))

r.FBN1=diff(log(FBN1))

r.FBN2=diff(log(FBN2))
```

```
r.FBN3=diff(log(FBN3))
r.FBN4=diff(log(FBN4))
plot(r.FBN1)
plot(r.FBN2)
plot(r.FBN3)
plot(r.FBN4)
adf.test(log(FBN1))
adf.test(log(FBN2))
adf.test(log(FBN3))
adf.test(log(FBN4))
FBNi=read.csv('FBNi.csv',header=T)
FBNii=read.csv('FBNii.csv',header=T)
FBNiii=read.csv('FBNiii.csv',header=T)
FBNiv=read.csv('FBNiv.csv',header=T)
lm1 <- lm(log.variance.~log.mean., data=FBNi)
lm1
lm2 <- lm(log.variance. ~ log.mean., data=FBNii)
lm2
lm3 <- lm(log.variance. ~ log.mean., data=FBNiii)
lm3
lm4 <- lm(log.variance.~log.mean., data=FBNiv)
lm4
plot(r.FBN1)
plot(r.FBN2)
plot(r.FBN3)
plot(r.FBN4)
eacf((r.FBN1)^2)
```

```
eacf((r.FBN2)^2)
```

```
eacf((r.FBN3)^2)
```

```
eacf((r.FBN4)^2)
```

```
m1=garchFit(formula=~arma(1,1)+garch(1,1),data=r.FBN1,include.mean=F,trace=F)
```

```
m2=garchFit(formula=~arma(2,1)+garch(2,1),data=r.FBN2,include.mean=F,trace=F)
```

```
m3=garchFit(formula=~arma(2,1)+garch(2,1),data=r.FBN3,include.mean=F,trace=F)
```

```
m4=garchFit(formula=~arma(1,1)+garch(1,1),data=r.FBN4,include.mean=F,trace=F)
```

```
m5=garchFit(formula=~arma(1,1)+garch(1,1),data=r.FBN1,include.mean=F,trace=F,cond.dist="std")
```

```
m6=garchFit(formula=~arma(2,1)+garch(2,1),data=r.FBN2,include.mean=F,trace=F,cond.dist="std")
```

```
m7=garchFit(formula=~arma(2,1)+garch(2,1),data=r.FBN3,include.mean=F,trace=F,cond.dist="std")
```

```
m8=garchFit(formula=~arma(1,1)+garch(1,1),data=r.FBN4,include.mean=F,trace=F,cond.dist="std")
```

```
summary(m1)
```

```
summary(m2)
```

```
summary(m3)
```

```
summary(m4)
```

```
summary(m5)
```

```
summary(m6)
```

```
summary(m7)
```

```
summary(m8)
```

```
m9=garchFit(formula=~arma(2,1)+garch(2,1),data=r.FBN1,include.mean=F,trace=F)
```

```
m10=garchFit(formula=~arma(2,1)+garch(2,1),data=r.FBN4,include.mean=F,trace=F)
```

```
m11=garchFit(formula=~arma(2,1)+garch(2,1),data=r.FBN1,include.mean=F,trace=F,cond.dist="std")
```

```
m12=garchFit(formula=~arma(2,1)+garch(2,1),data=r.FBN4,include.mean=F,trace=F,cond.dist="std")
```

```
summary(m9)
```

```
summary(m10)
```

```
summary(m11)
```

```
summary(m12)
```

```
forecast= predict(m7, n.ahead = 10)
```

```
forecast
```

```
residuals(m7)
```

```
kurtosis(residuals(m7))
```

```
library(evir)
```

```
y=residuals(m7)
```

```
ny=-y
```

```
meplot(ny)
```

```
m13=gpd(ny,threshold=0.05)
```

```
m13
```

```
plot(m13)
```

```
selection:3
```

```
selection:4
```

```
riskmeasures(m13,0.95)
```