A NON-LINEAR REGIME SWITCHING MODELS IN FINANCIAL SERIES WITH TWO REGIMES

BY

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This Project was carried out by Umah, Obianuju Glory with registration number PG/M.Sc/08/48870 of the Department of Statistics, University of Nigeria, Nsukka. This work has not been used for any Diploma, Degree or Masters programme previously in this department or any other institution.

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This research work is dedicated to God the Father, God the son and God the Holy Spirit.

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ABSTRACT

In this study, two economic series which have changes in regimes were considered. Models considered for the two series are Simple Switching Mixture (SSM) model and Markov Switching Autoregressive (MS-AR) model. Predictions of future transition regime probabilities were performed using the Hamilton filter of m-period transition matrix for MS-AR model, while, the two state ergodic m-step ahead transitions probabilities for SSM model. Subsequently, forecast evaluation measures for the two models were carried out with Mean Absolute Percentage Error (MAPE) and Root Mean Square Error (RMSE). Consumer Price Index (CPI), had a better forecast with SSM model while, Nominal Effective Exchange Rate (NEER) had a better forecast with the MS-AR model.

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CHAPTER ONE

INTRODUCTION

1.1 Overview

Most economic series, as have been observed, undergo series of random shocks that results in switching among different regimes. Regime shifts cannot be modelled using simple linear time series model but the application of non-linear model is more appropriate. Most variables have changes in their behaviour which might be as a result of changes in policies, both monetary and fiscal (or governmental control).

A switching mechanism is controlled by an unobservable state variable that follows a first-order Markov chain. In particular, Kuan (2002), views that the Markovian property requires that the current value of the state depends on its immediate past value. As such, a structure may prevail for a random period of time, and it will be replaced by another structure when a switching takes place.

Markov switching autoregressive model basically, captures a random variable changing over time. This describes the consequences of a dramatic change in the behaviour of a single variable. i.e (Y_t) looking at the past (Y_{t-1}), Modh and Zaidi (2008). Many authors from various fields have looked into regime switching models and have observed that most of our economic variables exhibit switching in their behaviour. Goldfeld and Quandt (1973) gave an extensive review of the switching regression model to simultaneous equation systems, their extension was on the framework of exogenous switching. Also, Maddala and Nelson (1975) presented many practical problems one encounters using ï disequilibrium models, selectivity models etc. They pointed out that these involve switching regression models with endogenous switching. Mean while Lee, Maddala and Trost (1982), extended the D-method suggested by Goldfeld and Quandt (1973) to switching simultaneous equations systems with endogenous switching.

Hamilton (1989) majorly, focused on the real growth of GNP which he found that is autocorrelated. He classified the regimes into expansion and recession regimes using US post war data. Hamilton (1994: pp 697-699), went further to model time series with changes in regime. His study of the behaviour of U.S. real GNP had been a veritable ground for other writers to build upon. It is also, based on his procedure that this work derived its root.

Switching Models are designed to capture sudden shifts in those series that generate the data. This captures more appropriately economic melt down, resulting in fiscal cliffs. Lingyun (2012), views financial market changes in patterns over time, and its variables exhibiting dramatic changes to be as a result of unexpected events, such as natural hazards and financial crisis. Price statistics constitute an important tool for analysis in every economy, it guides decision-making in policy formulation and monitoring. Adetiloye (2010), stated that Consumer Price Index (CPI), is simply a relative movement in the general prices of goods and commodities consumed by the public relative to a base period. The CPI changes purely in response to price changes, while the quantities of the goods and services in the basket remain fixed.

Again, Adetiloye (2010) citing Benita and Lauterbach (2004), showed that exchange rate volatility has real economic costs that affect price stability, firm profitability and a countryꞌs financial system stability. Consequently, Modh and Zaidi (2008), in their study showed that financial series especially prices always undergo episodes in which the behaviour of the series seems to change quite dramatically. For a developing country like Nigeria, Consumer Price Index plays a significant role in the economy in order to attain optimal productivity. It is used to dictate the state of an economy indexed by time, whether there is inflation or price stability. These trends in an economy confer a nationꞌs economic status, showing whether the nation is growing, stable or falling.

Regime Switching Models have been applied across many areas such as education, marketing, health services, finance, accounting, production, meteorology etc. The approach applied in this study, generalizes to processes in which the probability that the regime depends not only on the value of the immediate past regime but also on a vector of other observed variables.

1.2 Statement of Problem

Many nations recognize the need for a control system in order to minimize inflation and maximize efficiency. Price stability in a country is as crucial as economic growth itself, a deviation from this, results to inflation which discourages long term planning, reduces investments and results to misleading alterations in the economy.

Nigerian financial series have shown a sign of instability over the years and this may be as a result of some factors which include the introduction of some polices such as the Structural Adjustment Programme (SAP). Yaqud (2010) observed that, in the last few decades, the foreign exchange rate management in Nigeria has also undergone tremendous changes especially after the adoption of the Structural Adjustment Programme (SAP), sponsored by the International Monetary Fund (IMF) and the World Bank. Some of these policies were introduced to reduce dependence on monocultural economy. Ike (1988), had a view that, the steady decline in the Nigerian economy which ñSAPò was designed to curtail and reverse has continued to manifest. In the real sense, SAP has raised new problems, fuelled inflation, worsened unemployment and exacerbated decline in living standards.

Consequently, Hamilton (1994: p.677), observed that if a process has changed in the past, clearly it could also change again in the future. Moreover; the change in regime is itself a random variable. This important feature should be taken into account in forecasting knowing that prediction of future events must be incorporated into decision making process.

Some authors like Adetiloye (2010), adopted the method of correlation and granger causality to find the significance of the relationship between consumer price index and exchange rate in Nigeria. With the variables, they established that a string of causal relationships exist. Again, Adeoye and Saibu (2014) used the method of Least Square to analyze the effect of monetary policy shocks on Exchange rate volatility. They also, used the cointegration test to determine the long-run relationship between the two. They noted that inflation rate causes volatility in nominal exchange rate in Nigeria. These authors viewed the relationship of the two variables this study will proceed to forecast future regime probabilities of the two series being considered having established their relationship.

In this regard, the application of regime switching model to the variables of interest is to assist policy makers in decision making process.

1.3 Objectives of the Study

I) To estimate the average time and volatility of being in a particular regime.

II) To obtain the long run probability distributions of the two series.

III) To forecast the future trend of the regime transition probabilities using the models presented.

IV) To determine the more appropriate model for the two financial series.

1.4 Significance of the Study

The ability to have knowledge of the behaviour of financial series is very essential to ensure price stability in an economy.

The use of regime switching models is being advocated for better planning purpose. Predictions of future behaviours of these are very essential and strategic for national planning leading to better living standards of the populace.

1.5 Scope of the Study

Two financial series were analyzed; namely the monthly Consumer Price Index (CPI) and Nominal Effective Exchange Rates (NEER) in Nigeria. The period ranges from Jan 1970 to Dec 2012.The log transform of these variables were subjected to Simple Switching Mixture (SSM) model and Markov Switching Autoregressive (MS-AR) model.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

In recent time, extensive interest has been on both time-invariant and time-varying parameter models of macro-economic and financial time series. Financial market behaviour is one of the areas where Regime Switching model has been widely applied. The study of regime changes by Hamilton (1989) gave rise to other studies by different authors.

Cox and Miller (1965), in their view of the two state Markov chain transition probabilities, assumed that transition probabilities are independent of time. They stated that if a system is in state "0" at time n, then there is a probability of $1-\alpha$ of being in state "0" at time $n+1$ and a probability α of being in state "1" at time $n+1$. Similarly, if the system is in state "1" at time n, then the probabilities of being in state "1", state "0" at time $n+1$ are $1-\beta$, β respectively. This is given as

$$P = \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}$$

Hamilton (1994) also, viewed two state Markov chain transition probabilities $\left(p_{ij}\right)$ as a representation of the probability transition from one state to another, ie. (*i, j = 1,2*). Each row of this transition matrix represents the probability distribution of the regime indicated as s_t, conditional on the value of s_{t-1}. Where s_t and s_{t-1} are the regimes. He summed the column probabilities to one (1) having,

$$p = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}$$

Other authors such as Kuan (2002), Modh and Zaidi (2008) etc., presented their transition matrix as that of Cox and Miller (1965), having the rows of the transition matrix sum to ñ1ò. Hamilton (1994) had an alternative to theirs with the columns summing to ñ1ò instead of the rows. This study allows the rows of the transition matrix to sum to ñ1".

Hamilton (1994, pp. 685-688), considered a special case of these Markov switching processes known as independent identically distributed mixture distributions. Here there is no inclusion of autoregressive coefficient. The regime that a given process is in at date ó**t'** be indexed by an unobserved random variable s_t where there are ó**N'** possible regimes (s_t = 1, 2, . . . , or N).When the process is in regime 1, the observed variable y, is presumed to have been drawn from a $N\left(\mu_1, \sigma_1^2\right)$ distribution. If the process is in regime 2, then y, is drawn from a $N\left(\mu_2, \sigma_2^2\right)$ distribution. These he noted are used to represent a broad class of different densities. He applied it to density of mixture of two Gaussian distributions with *y,\s,* = 1 ~ N(0, 1), *y,\s,* = 2 ~ N(4, 1), and P(s, = 1) = 0.8, and another density of mixture of two Gaussian distributions with *y,\s,* =1 ~ N(0, 1), *y,\s,* = 2~ N(2, 8), and P(s, = 1) = 0.6. The first model had bimodal while the second had a unimodal pattern. He emphasised that a mixture of two Gaussian variables need not have the bimodal appearance but Gaussian mixtures can also produce a unimodal density, allowing skew or kurtosis different from that of a single Gaussian variable. Andrew and Allan (2011) also, viewed an attractive feature of regime switching models in capturing central statistical features of series. To illustrate this, they considered a simple two-regime switching model given as;

$$y_t = \mu_{s_t} + \sigma_{s_t} e_t \qquad where \qquad e_t \sim N\left(0,1\right).$$

The unconditional probability that $s_t = 0$ is π_0 and $s_t = 1$ with probability ó1ô to be π_1. This they also viewed as a special case of a regime switching model (1) with no autoregressive terms.

Markov Switching Autoregressive Models had been applied by many authors. Hamilton (1989), viewed the parameter of autoregression as the outcome of discrete-state Markov process, which changes between regimes. He showed this by presenting an algorithm for drawing probabilistic inference in the form of a nonlinear iterative filter. This he applied to post war U.S. real GNP, suggesting that the periodic shift from a positive growth rate to a negative growth rate is a recurrent feature of the U.S. business cycle. Meanwhile, Hamilton (1990), builded on an approach introduced in his Hamilton (1989) study for analyzing discrete qualities of time series. He regarded the parameters of a vector autoregressive coefficient as subject to occasional discrete shifts

$$y_t = c_{s_t} + \phi_{s_t} y_{t-1} + a_t \,.$$

Where, y_t is the variable of interest, c is the constant, ϕ is the autoregressive coefficient, a_t is the white noise process assumed to follow $N\left(0,\sigma^2\right)$ and s_t is the regime switching parameter. The probability law governing these shifts is also stated explicitly and presumed to exhibit dynamic behaviour of its own. He noted that as an analyst, oneĝs task is to determine when the shifts occurred, to estimate parameters characterizing the different regimes and the probability law for the transition between regimes. The expectations for the future that one would make, was also considered.

Again, Hamilton (1994), considered autoregressive processes in which the parameters of the autoregression can change as the result of a regime-shift variable. The regime itself will be described as the outcome of an unobserved Markov chain. Consequently, Hamilton (2005), viewed the autoregressive coefficient as not switching. His approach to modeling changes in regime typically relies on the assumption that the latent state variable ñs_tò controlling regime change is exogenous. This exogenity of the state variable as explained by an AR (1) process given as

$$y_t = c_{s_t} + \phi y_{t-1} + a_t$$

The parameters are same as in his Hamilton (1990) work. Here, the state variable $\left(s_t\right)$ was not observed, but seen as one of the many factors influencing the series. Following this, Kim, Piger and Startz (2008) relaxed the assumption of Hamilton (2005) of regime change being exogenous. They developed a model of endogenous Markov regime-switching, which inference was via maximum likelihood. Their estimation was possible with relatively minor modifications to existing recursive filters. The model tests the exogenous switching model, yielding straightforward tests for endogeneity. Monte Carlo experiments and maximum likelihood estimates of the endogenous switching model parameters were quite accurate in their outcome.

Mohd and Zaidi (2008), in their work, applied a univariate 2-regime Markov switching autoregressive model (MS-AR), to capture regime shifts behaviour in both the mean and the variance in four indices of Bursa [Malaysia]. These were the Composite Index, the Industrial Index, the Financial Index and the Property Index between 1974 and 2003 (except for Composite Index between 1977 and 2003). The MS-AR model was found to successfully capture the timing of regime shifts in the four series. They assumed that these regime shifts occurred because of global economic and financial crises such as the 1974 oil price shock, the 1987 stock market crash and the financial crisis in 1997. Furthermore, the significant result of the likelihood ratio test (LR test) justified the use of nonlinear MS-AR. Ronald and Ronald (2006), studied a regime switching model in which they assumed that the exchange rate switches between two regimes over time. The first regime is a UIP regime in which changes in exchange rates are described by the observed interest rate differential between the two currencies involved. The second regime is a random walk with drift. The latent regime indicator follows a Markov process, and the transition probabilities allow for switching between the regimes. Their empirical results provide evidence that exchange rate movements were consistent with UIP over some periods, but not all. Masoud, Hamidreza and Maryann (2012), in their study, considered Markov switching autoregressive (*MS*-AR) model and six different time series modelling approaches. These models were compared according to their performance for capturing the Iranian exchange rate series. The series had dramatic jump in early 2002 which coincides with the change in policy of the exchange rate regime. Their criteria were based on the AIC and BIC values. The results indicate that the MS-AR model can be considered as useful model, with the best fit, to evaluate the behaviours of Iranǿs exchange rate.

Gianni and Roberta (2013), in their analysis tackled monetary policy, primarily they analyzed the features of a relationship between money growth and inflation in a Bayesian Markov Switching framework. They applied Bayesian Vector Autoregressive (BVAR) model which allows regime switches to a set of four countries: the U.S, UK, Japan and Euro zone. They found out first; that observing monetary developments to a certain extent contribute to improve the signal of entering the high inflation regime. Second; the inclusion of the monetary indicator seems to have a relevant influence on the signal of inflational risk. This was mainly seen in the first part of their sample (i.e. early 70s and 80s), which will help to reduce inflation in the economy. Also, Ozge (2013), in his first thesis implemented a Markov Switching GARCH methodology which allows for regime shifts in inflation series while generating the proxy for inflation uncertainty. He allowed both inflation and inflation uncertainty to exert a regime dependent impact on output growth. He employing a Markov Regime Switching Model concluding that, inflation uncertainty has a greater negative impact on output growth during the low growth regime. The authors cited above investigations, provide evidence that the magnitude of inflation uncertainty on output growth changes significantly across low and high-growth regimes.

In the usability of the Switching Models to forecast, Michael and Krolzig (1997), had a view that though there had been a great deal of interest in the modelling of non-linearity in economic time series; they stated that there is no clear consensus regarding the forecasting abilities of non-linear time series models. They evaluated the performance of two leading non-linear models in forecasting post-war US GNP, using the self-exciting threshold autoregressive model and the Markov-switching autoregressive model. The methods of analysis employed, yielded an empirical forecast accuracy comparison of the two models, and a Monte Carlo study. The latter allows control for factors that may otherwise undermine the performance of the non-linear models.

The above literatures reviewed, apply to regime switching models in different fields with different variables. Researchers have seen the importance and enhanced their works on nonlinear modelling using different switching model. This basically depends on the dataset to which the chosen model is applied.

CHAPTER THREE

METHODOLOGY

3.1 Introduction

In this section, we are interested in the statistical method governing this study. Firstly, we subjected the financial series of interest to different tests to determine if they are stationary or not. There was no evidence of stationarity. The data were therefore, analyzed using two non-linear models, the Simple Switching Mixture (SSM) and Markov Switching Autoregressive (MS-AR) models. The forecast usability of the models was compared using the Mean Absolute Percentage Error (MAPE) and Root Mean Squared Error (RMSE).

3.2 The Data

To empirically capture the changes in regimes using SSM and MS-AR, we used monthly log transform of Consumer Price Index (CPI) and Nominal Effective Exchange Rate (NEER) data of Nigeria. The original series were obtained from Central Bank of Nigeria Statistical Bulletin Volume 16, December (2005); National Bureau of Statistics May 2011 Consumer Price Index Statistical News and 2013 Central Bank of Nigeria Economic Report for the first half of the year under study. The Consumer Price Index and the Nominal Effective Exchange Rate have their series running from January 1970 to Dec 2012.

3.3 Model Identification

Identification of models involves testing coefficients for significance inclusion or elimination which involves different techniques. The stability of a process is realized when the plotting shows that the data fluctuates around its constant mean. Box, Jenkins and Reinsel (1994), had a notion that every stationary model assumes that the process remains in equilibrium about a constant mean level.

3.3.1 Data Stationarity

The variables time trend plots, Autocorrelation Function (ACF), the Partial Autocorrelation Function (PACF) and their first difference. These were applied to verify the stationarity behaviour of the variables.

3.3.2 CUSUM Test

The CUSUM test, Brown, Durbin and Evans (1975) was based on the cumulative sum of the recursive residuals. This option plots the cumulative sum together with the 5% critical lines. The test finds parameter instability if the cumulative sum goes outside the area between the two critical lines.

Let $y_t = x_t\beta + \mu_t \quad , \quad \mu_t \sim N\left(0,\sigma^2\right) \quad t = 1,2,\ldots,T$

Where β is a $k \times 1$ unknown parameter vector. y_t is a dependent variable while x_t is a $1 \times k$ vector of independent variables. μ_t is assumed to be normally distributed with mean zero and variance σ^2.

$$Y_t = \begin{pmatrix} y_1 \\ \vdots \\ y_t \end{pmatrix} \quad , \quad X_t = \begin{pmatrix} x_1 \\ \vdots \\ x_t \end{pmatrix}$$

The null hypothesis of no structural change for the model is specified as:

$$H_0 : \beta_1 = \beta_2 = \cdots = \beta_T = \beta \quad and \quad \sigma_1^2 = \sigma_2^2 = \cdots = \sigma_T^2 = \sigma^2$$

Where β_t denotes the vector of coefficients in period t and σ_t^2 the disturbance variance in that period. The null hypothesis would be violated if β remained constant but σ^2 varies.

$$w_t = \frac{y_t - x_t b_{t-1}}{\sqrt{1 + x_t\left(X_{t-1}^{'}X_{t-1}\right)^{-1} x_t^{'}}}$$

which has mean zero and variance σ^2

The CUSUM test is based on the statistic:

$$W_t = \sum_{i=k+1}^{t} w_i / \hat{\sigma} \tag{3.1}$$

$$where \quad \hat{\sigma}^2 = \frac{1}{T-k} . \sum_{i=k+1}^{t} w_i^2$$

which implies the unbiased estimate of σ^2, for t = k+1, … T where w is the recursive residual defined above, and σ is the standard deviation of the recursive residuals w_t. If the β vector remains constant from period to period, $E\left(W_t\right)$, but if β_t changes, W_t will tend to diverge from the zero mean value line. The significance of any departure from the zero line is assessed by reference to a pair of 5% significance lines, the distance between which increases with ***'t'***. The 5% significance lines are found by connecting the points.

$$k, \pm a\left(T-k\right)^{\frac{1}{2}} \text{ and } \left[T, \pm 3a\left(T-k\right)^{\frac{1}{2}}\right] \tag{3.2}$$

where a is a parameter depending on the significance level chosen for the test. It is known that we have a = 1.143 when α =0.01, a = 0.948 when α = 0.05 and a = 0.850 when α = 0.10. Movement of W_t outside the critical lines is suggestive of coefficient instability.

3.3.3 Unit Root Test

The formal method to test the stationarity of a series is the Unit Root Test. This includes specification of the test for a unit root in the level, first difference or second difference of the series. The approximate linear decay of the sample ACF is often taken as a symptom that the underlying time series is nonstationary, it is useful to quantify the evidence in the data-generating mechanism. Jonathan and Kung-sik (2008 p.128).

$$Y = \alpha Y_{t-1} + X_t \qquad for\ t = 1, 2 \tag{3.3}$$

where $\{X_t\}$ is a stationary process. The process $\{Y_t\}$ is nonstationary if the coefficient α = 1, but it is stationary if $|\alpha|<$ 1. Suppose that $\{X_t\}$ is an AR(k) process:

$$X_t = \phi_1 X_{t-1} + ... + \phi_k X_{t-k} + e_t \tag{3.4}$$

Under the null hypothesis that $a = 1, \quad X_t = Y_t - Y_{t-1}, \ letting\ a = \alpha - 1$, we have

$$\begin{aligned} Y_t - Y_{t-1} &= \left(\alpha - 1\right) Y_{t-1} + X_t \\ &= aY_{t-1} + \phi_1\left(Y_{t-1} - Y_{t-2}\right) + ... + \phi_k (Y_{t-k} - Y_{t-k-1}) + e_t \end{aligned} \tag{3.5}$$

3.4 Regime Switching Models

3.4.1 Assumptions of Regime Switching Models

i) Regime classification is probabilistic and determined by the data.

ii) The increasing and decreasing propriety of a future transition depends on only the last transition.

iii) The regime was not observed directly, but, inferred its operation through the observed behaviour of y_t .

iv) The Markovian variables s_t is as a result of random and frequent changes.

v) Persistence of each regime depends on the transition probabilities.

3.4.2 Simple Switching Mixture (SSM) Model

Simple switching mixture model as viewed by Andrew and Allan (2011), is a simple two-regime switching model which was given as

$$y_t = \mu_{s_t} + \sigma_{s_t}^2 \varepsilon_t \qquad \varepsilon_t \sim N\left(0, \sigma_{s_t}^2\right) \tag{3.6}$$

where s_t is the index of N possible regimes 1, 2, . . . , N and $\varepsilon_t \sim N\left(\mu_1, \sigma^2\right)$

This they viewed as the process where the multiplicative coefficients ϕ_i's are zero. Hamilton (1994), states that a given process is in at date **"t"** be indexed by an unobserved random variable s_t, where there are **N** possible regimes (s_t = 1, 2, . . . , or N). When the process is in regime 1, the observed variable y_t is presumed to have been drawn from a $N\left(\mu_1, \sigma_1{}^2\right)$ distribution. If the process is in regime 2, then y_t is drawn from a $N\left(\mu_2, \sigma_2{}^2\right)$ distribution, and so on. Hence, the density of y_t conditional on the random variable s_t taking on the value j is

$$f\left(y_t \mid s_t = j; \theta\right) = \frac{1}{\sqrt{2\pi}\sigma_j} \exp\left[-\left(\frac{\left(y_t - \mu_j\right)^2}{2\sigma_j^2}\right)\right] \tag{3.7}$$

for $j = 1, 2, \ldots, N$, different regimes, θ is a vector of parameters that includes $\mu_1, \mu_2, \ldots, \mu_N$, $\sigma_1^2, \sigma_2^2, \ldots, \sigma_N^2$ and $\pi_1, \pi_2, \ldots, \pi_N$. Therefore;

$$p\left(s_t = j; \theta\right) \quad = \quad \pi_j \text{ for } j = 1, 2, \ldots, N \tag{3.8}$$

This is the probability that the unobserved regime was generated by some probability distribution. It is referred to as the unconditional probability s_t which takes the value π_j

For two regimes, Hamilton (2005) viewed it as;

$$p\left(s_t = j\right) \quad = \quad \frac{1 - p_{jj}}{2 - p_{ii} - p_{jj}} = \pi_j \tag{3.9}$$

$$\pi = \begin{bmatrix} \left(1 - p_{22}\right) / \left(2 - p_{11} - p_{22}\right) \\ \left(1 - p_{11}\right) / \left(2 - p_{11} - p_{22}\right) \end{bmatrix}$$

The joint density-distribution functions of y_t and s_t is given as

$$p\left(y_t, s_t = j; \theta\right) \quad = \quad f\left(y_t \mid s_t = j; \theta\right) \;.\; p\left(s_t = j; \theta\right)$$

Therefore, we have

$$p\left(y_t, s_t = j; \theta\right) = \frac{\pi_j}{\sqrt{2\pi}\sigma_j} \exp\left[-\left(\frac{\left(y_t - \mu_j\right)^2}{2\sigma_j^2}\right)\right] \tag{3.10}$$

The unconditional density is the sum of the joint density-distribution function shown as

$$f\left(y_t, \theta\right) = \sum_{i=1}^{N} p\left(y_t, s_t = j; \theta\right) \tag{3.11}$$

The log likelihood is

$$L(\theta) = \sum_{t=1}^{T} \log f\left(y_t; \theta\right) \tag{3.12}$$

Therefore, the inference is

$$p\left(s_t = j \mid y_t; \theta\right) = \frac{p\left(y_t, s_t = j; \theta\right)}{f\left(y_t; \theta\right)} = \frac{\pi_j f\left(y_t \mid s_t = j; \theta\right)}{f\left(y_t; \theta\right)} \tag{3.13}$$

This is the inference that the probability given the observed data, that the unobserved regime responsible for observation **'t'** was regime j.

3.4.2.1 Maximum Likelihood Estimates of the Mean, Variance and unconditional probability

Mean ($\hat{\mu}_j$): This is a weighted average of all the observations in the sample.

$$\hat{\mu}_j = \frac{\sum_{t=1}^{T} y_t . p\left(s_t = j \mid y_t\right)}{\sum_{t=1}^{T} p\left(s_t = j \mid y_t\right)} \tag{3.14}$$

Variance: This is the weighted average of the squared deviations.

$$\hat{\sigma}_j^2 = \frac{\sum_{t=1}^{T} \left(y_t - \mu\right) . p\left(s_t = j \mid y_t\right)}{\sum_{t=1}^{T} p\left(s_t = j \mid y_t\right)} \tag{3.15}$$

3.4.2.2 Forecast using the matrix of m-period ahead Transition Probability for an ergodic two-state Markov chain

Hamidreza and Maryam (2012), presented an ergodic two state Markov chain, the transition probability as;

$$p^m = \begin{bmatrix} \dfrac{(1-p_{22})+\lambda_2^m(1-p_{11})}{2-p_{11}-p_{22}} & \dfrac{(1-p_{11})-\lambda_2^m(1-p_{11})}{2-p_{11}-p_{22}} \\ \dfrac{(1-p_{22})-\lambda_2^m(1-p_{22})}{2-p_{11}-p_{22}} & \dfrac{(1-p_{11})+\lambda_2^m(1-p_{22})}{2-p_{11}-p_{22}} \end{bmatrix}$$

$$= \begin{bmatrix} p_{11}^m & p_{12}^m \\ p_{21}^m & p_{22}^m \end{bmatrix} \tag{3.16}$$

$\lambda_2 = -1 + p_{11} + p_{22}$ (recall that, $\lambda_1 = 1$ and λ_2 is the second eigen value) . The difference in work here is that Hamilton has the matrix columns being summed to 1, but here and in many other literatures the rows are summed to be equal to 1.

The steady-state probability which is given as

$$\pi = \begin{bmatrix} \dfrac{1-p_{22}}{2-p_{11}-p_{22}} \\ \dfrac{1-p_{11}}{2-p_{11}-p_{22}} \end{bmatrix} = \begin{bmatrix} \pi_1 \\ \pi_2 \end{bmatrix}$$

With these, it can be seen that

$$\pi^m p^m = \begin{bmatrix} \pi_1 & \pi_2 \end{bmatrix} \begin{bmatrix} p_{11}^m & p_{12}^m \\ p_{21}^m & p_{22}^m \end{bmatrix}$$

$$= \begin{bmatrix} \pi_1 p_{11}^m + \pi_2 p_{21}^m & \pi_1 p_{12}^m + \pi_2 p_{22}^m \end{bmatrix} \tag{3.17}$$

The first element of the matrix is

$$\pi_1 p_{11}^m + \pi_2 p_{21}^m = \frac{1-p_{11}}{2-p_{11}-p_{22}} \times \frac{(1-p_{11})+\lambda_2^m(1-p_{11})}{2-p_{11}-p_{22}} + \frac{1-p_{11}}{2-p_{11}-p_{22}} \times \frac{(1-p_{22})+\lambda_2^m(1-p_{22})}{2-p_{11}-p_{22}}$$

$$\frac{(1-p_{11})\left((1-p_{22})+\lambda_2^m(1-p_{11})\right)+(1-p_{22})\left((1-p_{11})+\lambda_2^m(1-p_{22})\right)}{2-p_{11}-p_{22}}$$

$$\frac{(1-p_{11})^2+(1-p_{11})(1-p_{22})}{2-p_{11}-p_{22}}$$

$$\frac{(1-p_{22})(1-p_{11})+(1-p_{22})}{2-p_{11}-p_{22}}$$

$$\frac{1-p_{22}}{2-p_{11}-p_{22}}=\pi_1 \tag{3.18}$$

In the same manner, the second element of equation is calculated producing

$$\frac{1-p_{11}}{2-p_{11}-p_{22}}=\pi_2 \tag{3.19}$$

3.4.3 Markov Switching Autoregressive (MS-AR) Model

The Markov switching autoregressive model, assumes that a regime shift may be as a result of an unobservable stochastic process. This implies that the movement between regimes or regime shifts are unrelated to the past observations of the process but only the recent past. Hence, this enables probabilistic statements to be made about series being in a particular regime at a given time, Hamilton (2005).

The difference between the two models is the introduction of an autoregressive element to MS-AR model..

3.4.3.1 Properties of Markov Switching Autoregressive Model

According to Hamilton (2005), Markov switching autoregressive model with an AR process of order one is given as:

$$y_t = c_{s_t} + \phi\left(y_{t-1}\right) + \varepsilon_t \tag{3.20}$$

where,óy_tô is the dependent variable, ócô is a constant which switches across the regimes alongside with its variance, óϕô is the autoregressive coefficient, $\varepsilon_t \sim N\left(0,\sigma^2\right)$.ó$s_t$ô refers to the regimes taking values as either $s_t = i$, or $s_t = j$. These s_t and s_{t-1} represent unobserved regime variables which in this context is used as up and down trend, taking the values of 1 and 2 respectively. These are basically observed through the behaviour of the dependent variable (y_t). Markov switching autoregressive model is applied to the variables using the conventional Hamiltonôs model. We used a one-step model (i.e., given the regime only Y_{t-1} enters the model which is the lag of the variable Y_t). In this study, the movement will be with focus on one time regime shifts, allowing the mean and variance to shift simultaneously across the regime as in Mohd and Zaidi (2008).

Transition Probability Matrix

A one-step Markov chain of an AR (1) process that is conditioned on y_{t-1} having conditional probability for state s_t, depends on the past only through the most recent s_{t-1}. This implies that prediction about the future behaviour of a system, only its present state and not its past histories is considered. The specification here is that s_t evolve according to two State Markov chain with the transition probability given as $p(s_t = j \mid s_{t-1} = 1, s_{t-2} = 2, ..., y_{t-1}, y_{t-2}, ...)$ which turns out to be;

$$p(s_t = j \mid s_{t-1} = i) = p_{ij} \tag{3.21}$$

p_{ij} represents the probability of a transition from state i to j. Each row of this transition matrix represents the probability distribution of s_t, conditional on the indicated value of s_{t-1} Hamilton (1994).

Therefore;

$$p(s_t = 1 \mid s_{t-1} = 1) = p_{11}$$

$$p(s_t = 1 \mid s_{t-1} = 2) = p_{12} \equiv 1 - p_{11}$$

$$p(s_t = 2 \mid s_{t-1} = 1) = p_{21} \equiv 1 - p_{22}$$

$$p(s_t = 2 \mid s_{t-1} = 2) = p_{22}$$

The above produces

$$p = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} \tag{3.22}$$

This is the probability of switch from one regime to another having; in this study, our row probabilities summing to one (1). i.e. $p_{11} + p_{22} = 1 \; and \; p_{21} + p_{22} = 1$

Mean of an AR (1) Process

$$\mu = \frac{c}{1-\phi} \tag{3.23}$$

Variance of an AR(1) process

$$\sigma^2 = \frac{\sigma^2}{1-\phi^2} \tag{3.24}$$

Autoregressive Coefficient of an AR(1) process

The autoregressive coefficient for two states according to Hamilton (1994), is seen as the second eigen value that is;

$$\phi = -1 + p_{11} + p_{22} \tag{3.25}$$

Expected Duration of the Regimes:

The expected time spent in each regime before a transition takes place as stated by Mohd and Zaida (2008) is given as;

$$ED = \frac{1}{1-p_{ij}} \tag{3.26}$$

3.4.3.2 Estimation of Markov Switching Autoregressive Model using Hamilton's Filter Algorithm

The algorithm known as Hamilton Filter as introduced by James D. Hamilton, was used to make inference about the current regime s_t in the model in equation (3.22) above. This inference entails the analyst assigning a conditional probability that the yth observation was generated by regime j.

The inference is stated as follows:

$$\xi_{t\backslash t} = p\left(s_t = j \mid Y_t;\theta\right) \tag{3.27}$$

This is can be written as

$$\xi_{t\backslash t} = \frac{\eta_t . p\left(s_t = j, Y_{t-1};\theta\right)}{f\left(y \mid Y_{t-1};\theta\right)}$$

$$\xi_{t\backslash t} = \frac{f\left(y_t \mid s_t = j, Y_{t-1};\theta\right).p\left(s_t = j \mid Y_{t-1};\theta\right)}{f\left(y_t \mid Y_{t-1};\theta\right)}$$

$$\xi_{t\backslash t} = \frac{p\left(y_t, s_t = j \mid Y_{t-1};\theta\right)}{f\left(y_t \mid Y_{t-1};\theta\right)} \tag{3.28}$$

Notations:

a) p_{ij} : This is the transition Probability between regimes.

b) θ : This is the vector of parameters which includes,

$\theta = \{\phi, c_1, c_2, \sigma^2, p_{11}, p_{22}\}$

c) Y_t : Vector of historical observations up to time t

$Y_t = \{y_t, ..., y_0\}$

d) η_t: Conditional probability density functions of y_t given the current regime s_t and past observations up to time t-1.

Where; $\eta_t = f(y_t \mid s_t = j, Y_{t-1}; \theta)$

$$= \frac{1}{\sqrt{2\pi}\sigma_j} \exp\left[-\left(\frac{y_t - c_j - \phi(y_{t-1})}{2\sigma_j^2} \right) \right] \tag{3.29}$$

e) $\hat{\xi}_{t\backslash t}$: Is the conditional regime probability distribution given all observations up to time **t**.

$\hat{\xi}_{t\backslash t} = p(s_t = j \mid Y_t; \theta)$

The log likelihood function of the regime is given as:

$$L(\theta) = \sum_{t=1}^{T} \log f(y_t \mid Y_T; \theta) \tag{3.30}$$

One of the advantages of Hamiltonôs filter algorithm is that the log likelihood can be derived as a by-product in the calculation. Hamilton (1994).

3.4.3.3 Prediction of Future Regime Probabilities

Hamiltonôs filter of future forecast relies on the use of information concerning events that have occurred in the past. This is the estimation of the probability of being in a particular regime given the observation obtained. In this study, forecasts of one step ahead i.e., how likely the process is to be in regime (j) in period t + 1 given observations obtained through date ótô. This is given as

$$\hat{\xi}_{t+1\backslash t} = P\hat{\xi}_{t\backslash t} \tag{3.31}$$

from above, **ót'** is the period of time where $\hat{\varepsilon}_{t\backslash t}$ is the estimated regime conditional probability and P is the transition probabilities.

3.4.4 Model Comparison and Forecast Evaluation Measure

Criteria performance evaluation procedures applied in this study are Mean Absolute Percentage Error (MAPE) and Root Mean Squared Error (RMSE) according to Muhammad (2012) is given as :

1) **Mean Absolute percentage Error (MAPE)**

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\frac{|Y_i - F_i|}{Y_i} \tag{3.32}$$

2) **Root Mean Squared Error (RMSE)**

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - F_i)^2} \tag{3.33}$$

where, Y_i = Observed

F_i = Forecasted

CHAPTER FOUR

APPLICATION OF THE MODELS TO FINANCIAL SERIES

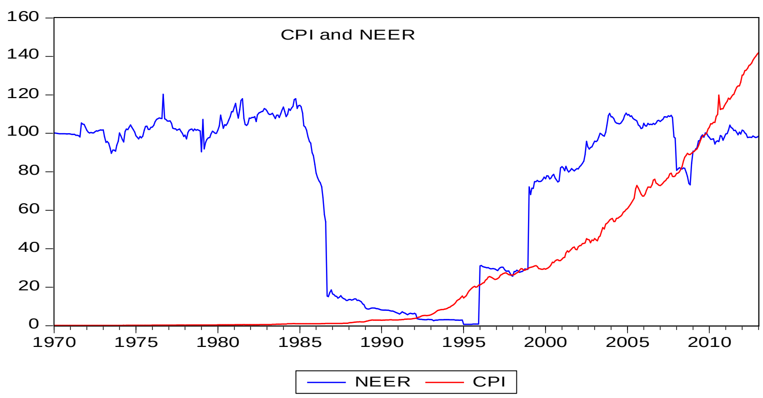
4.1 Introduction

The criteria performance evaluation procedures MAPE and RMSE were used as the yardstick for comparison between the Simple Switching Mixture (SSM) model and Markov Switching Autoregressive (MS-AR) model.

4.2 Numerical Diagnostics Tests

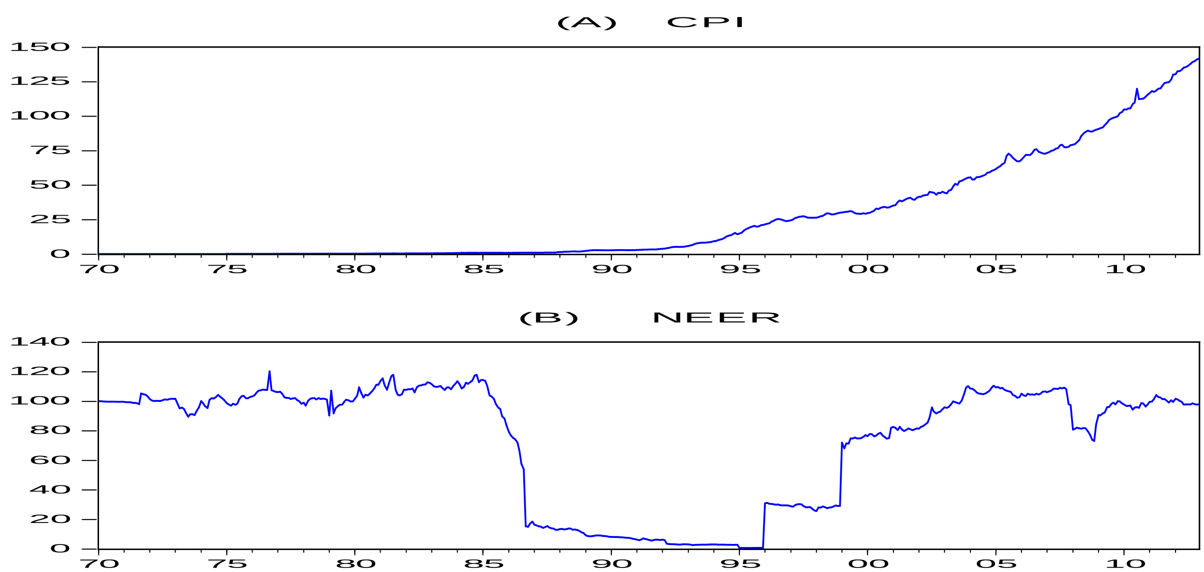
4.2.1 Graphical Representations

Figure 1: Joint plot of CPI and NEER



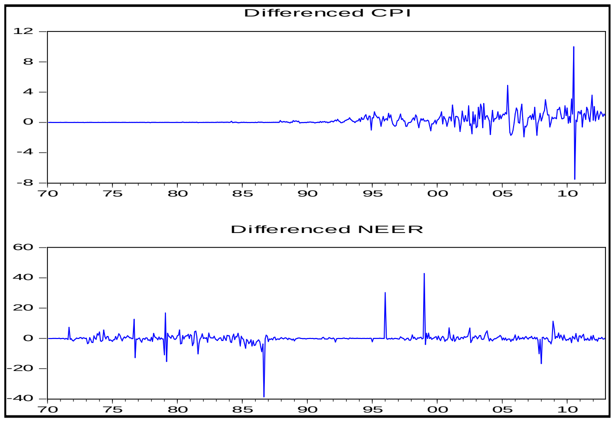
Different plots of the financial series of interest were shown and some tests of linearity carried out.

Figure 2: Individual Plots of CPI and NEER



The time trends of the series shown in Figure (2) and their first differences in Figure (3) showed non-stationary attributes in these plots. In order not to rely on these alone, the CUSUM test was presented in Figure (4).

Figure 3: Differenced Series of CPI and NEER

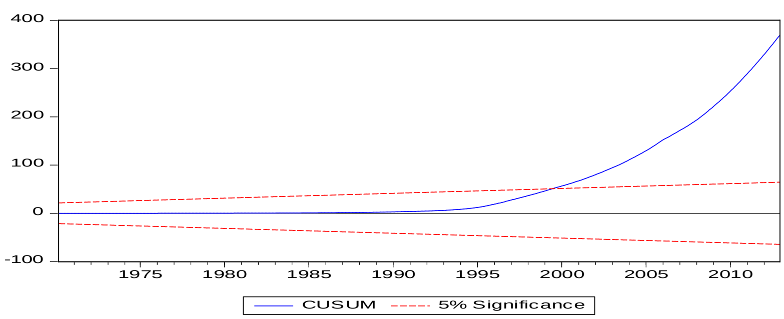


4.2.2 CUSUM Test

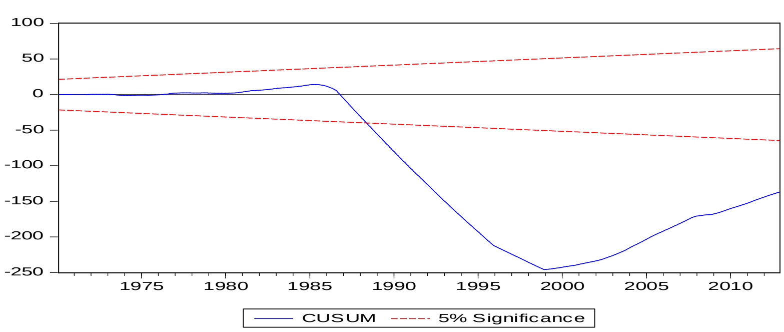
The movement of figure 4(A) and 4(B) outside the intervals is also, a clear indication of non-stationary attributes of the series. This is also a proof of the non-linear model application in this study

Figure 4: CUSUM Test Plot

(A) CPI



(B) NEER



4.2.3 Unit Root Test

We carried out a unit root test to ascertain if there exists a unit root in the series or not.

Table 2: Summary Table of Augmented Dickey Fuller Test

Variable	**Levels**	**Critical Values**		**Order of Integration**	**Remarks**
CPI	5.442845	**1%**	-3.442845	**I(1)**	Non-Stationary at Levels
		5%	-2.866943		
		10%	-2.569709		
NEER	-1.658873	**1%**	-3.442820	**I(1)**	Non-Stationary at Levels
		5%	-2.866943		
		10%	-2.569703		

The ADF test was carried out using the series in levels. The ADF statistic value is 5.443 for CPI and -1.6589 for NEER and the associated one-sided p-value (for a test with 516 observations) is 0.9617 and 0.7725 respectively . The critical values were reported at the 1%, 5% and 10% levels. The t_α is greater than the critical values so that we do not reject the null hypothesis of the existence of unit root in the two series being considered. This goes further to suggest that at levels the series are not stationary.

4.3 Parameter Estimation

In this study, we modelled log of the series instead of the actual values themselves letting $Y_t = \log X_t$ as in Hamidreza and Maryann (2012). Significantly, there was a switch in the levels of the series in 1986. This might be as a result of some policies such as the introduction of ñSAPò policy in 1985. Prediction of future regimes for Simple Switching Mixture (SSM) model was estimated using two-state Markov chain ergodic m-period ahead matrix, while the Markov Switching Autoregressive (MS-AR) model was performed by Hamiltonôs filter algorithm.

4.3.1 Simple Switching Mixture (SSM) Model

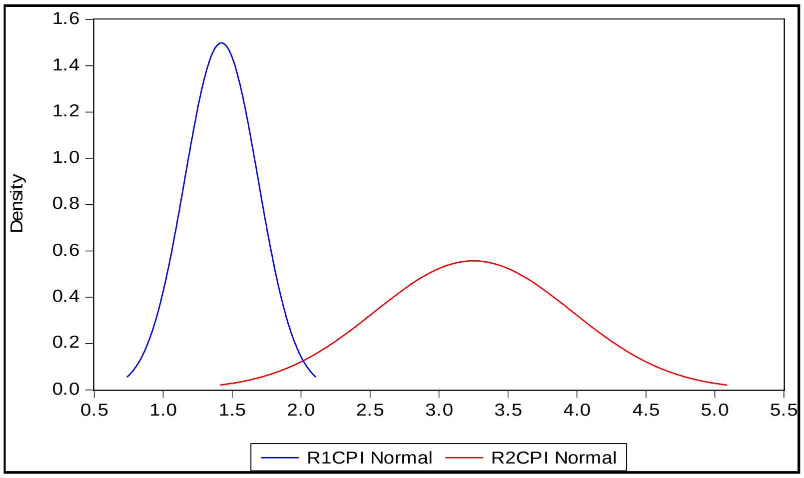
The inference using SSM model has the mean and variance varying across the regimes. This as stated by Andrew and Allan (2011) is given as

$$y_t = \mu_{s_t} + \sigma^2_{s_t}\varepsilon_t \qquad \varepsilon_t \sim N\left(0, \sigma^2_{s_t}\right) \tag{4.1}$$

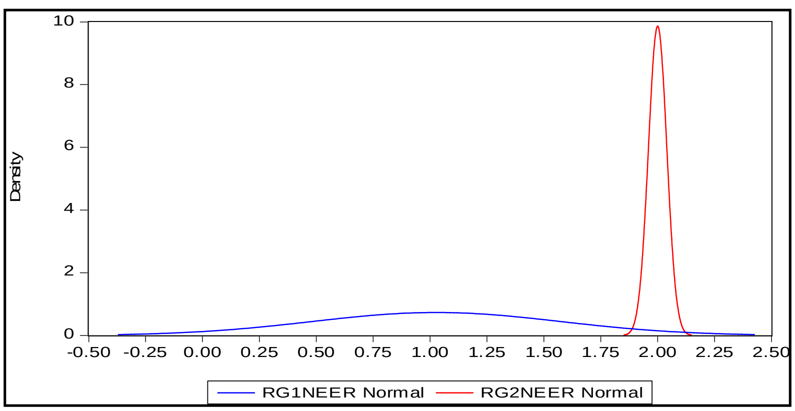
The essence of the series being subjected to Simple Switching Mixture (SSM) model is to ascertain if the up and down regimes can be attributed to be the mixture of two Gaussian distributions, Hamilton (1994).

Figure 5: Guassian Mixture for CPI and NEER

A: CPI



B: NEER



4.3.1.1 Inference for Consumer Price Index (CPI)

$$f\left(y_t \mid s_t = j; \theta\right) = \frac{1}{\sqrt{2\pi}\sigma_j} \exp\left[-\left(\frac{\left(y_t - \mu_j\right)^2}{2\sigma_j^2}\right)\right] \tag{4.2}$$

Regime 1

$$f\left(y_1 \mid s_1 = 1; \theta\right) = \frac{1}{\sqrt{2 \times 3.142 \times 0.088305}} \exp\left[-\left(\frac{(1.01 - 1.4658)^2}{2 \times 0.08806}\right)\right]$$

$$= 0.99098$$

Regime 2

$$f\left(y_1 \mid s_t = 2;\theta\right) = \frac{1}{\sqrt{2\times 3.142\times 0.46442}} \exp\left[-\left(\frac{\left(2-3.3032\right)^2}{2\times 0.46442}\right)\right]$$

$$= 0.3945$$

The unconditional probabilities are given as

$$\pi = \begin{bmatrix} \left(1-p_{22}\right)/\left(2-p_{11}-p_{22}\right) \\ \left(1-p_{11}\right)/\left(2-p_{11}-p_{22}\right) \end{bmatrix} \tag{4.3}$$

$$\pi = \begin{bmatrix} \left(1-0.9868\right)/\left(2-0.9868-0.9767\right) \\ \left(1-0.9767\right)/\left(2-0.9868-0.9767\right) \end{bmatrix}$$

$$= \begin{bmatrix} 0.3616 \\ 0.6384 \end{bmatrix}$$

Also, the joint density-distribution function is given as

$$p\left(y_t, s_t = j;\theta\right) = \frac{\pi_j}{\sqrt{2\pi}\sigma_j} \exp\left[-\left(\frac{\left(y_t-\mu_j\right)^2}{2\sigma_j^2}\right)\right] \tag{4.4}$$

Regime 1:

$$p\left(y_t, s_t = j;\theta\right) = 0.3616\times 1.33031$$

$$= 0.3583$$

Regime 2:

$$p\left(y_t, s_t = j;\theta\right) = 0.6384\times 0.3946$$

$$= 0.2598$$

Therefore, the unconditional density being the sum of the regime joint density-distribution functions. Table 3 of Appendix D shows the calculation with

$$f\left(y_t, \theta\right) = \sum_{i=1}^{N} p\left(y_t, s_t = j;\theta\right) \tag{4.5}$$

$$f\left(y_t, \theta\right) = 74.22 + 115.18 = 189.4$$

Hence, the inference is given as

$$p\left(s_t = j \mid y_t;\theta\right) = \frac{p\left(y_t, s_t = j;\theta\right)}{f\left(y_t;\theta\right)} = \frac{\pi_j f\left(y_t \mid s_t = j;\theta\right)}{f\left(y_t;\theta\right)} \tag{4.6}$$

Regime 1

$$p\left(s_t = j \mid y_t;\theta\right) = \frac{74.22}{189.4} = 0.3918$$

Regime 2

$$p\left(s_t = j \mid y_t;\theta\right) = \frac{115.18}{189.4} = 0.6081$$

4.3.1.2 Prediction of future Regimes using M-Period ahead Transition Probabilities Matrix for CPI

Let $m = 2$, representing two states and λ_1, λ_2 be the eigen values. "λ_2" is equivalent to the autoregressive coefficient **"ϕ",** hence, using transition probability matrix in equation (4.7) below and steady state probabilities in equation (4.3) above;

$$\text{TPM} = \begin{bmatrix} 0.9767 & 0.0233 \\ 0.0132 & 0.9868 \end{bmatrix} \tag{4.7}$$

The unconditional probabilities as in equation (4.3) are:

$$\begin{bmatrix} \pi_1 \\ \pi_2 \end{bmatrix} = \begin{bmatrix} 0.3616 \\ 0.6384 \end{bmatrix}$$

$$\phi = -1 + 0.9767 + 0.9868 = 0.9636$$

Therefore, substituting the parameters to solve **2**-period ahead transition matrix probabilities forecast of the regime variables we have;

$$\check{P}^2 = \begin{bmatrix} \dfrac{(1-p_{22}) + \lambda^2 (1-p_{11})}{2-p_{11}-p_{22}} & \dfrac{(1-p_{11}) - \lambda^2 (1-p_{11})}{2-p_{11}-p_{22}} \\ \dfrac{(1-p_{22}) - \lambda^2 (1-p_{22})}{2-p_{11}-p_{22}} & \dfrac{(1-p_{11}) + \lambda^2 (1-p_{22})}{2-p_{11}-p_{22}} \end{bmatrix}$$

$$\check{P}^2 = \begin{bmatrix} \dfrac{(1-0.9868) + 0.9635^2 (1-0.9767)}{2-0.9767-0.9868} & \dfrac{(1-0.9767) - 0.9635^2 (1-0.9767)}{2-0.9767-0.9868} \\ \dfrac{(1-0.9868) - 0.9635^2 (1-0.9868)}{2-0.9767-0.9868} & \dfrac{(1-0.9767) + 0.9635^2 (1-0.9868)}{2-0.9767-0.9868} \end{bmatrix}$$

$$\check{P}^2 = \begin{bmatrix} \dfrac{0.03483}{0.0365} & \dfrac{0.00167}{0.0365} \\ \dfrac{0.000946}{0.0365} & \dfrac{0.035554}{0.0365} \end{bmatrix}$$

$$\hat{P}^2 = \begin{bmatrix} 0.9542 & 0.0458 \\ 0.0259 & 0.9741 \end{bmatrix}$$

With these, it can be seen that, the two periods ahead regime probabilities are;

$$\hat{\pi}\hat{P}^2 = \begin{bmatrix} 0.3616 & 0.6384 \end{bmatrix} \begin{bmatrix} 0.9542 & 0.0458 \\ 0.0259 & 0.9741 \end{bmatrix}$$

$$= \begin{bmatrix} 0.3616 & 0.6384 \end{bmatrix}$$

For three periods ahead the same result holds

`

$$\hat{\pi}\hat{P}^3 = \begin{bmatrix} 0.3616 & 0.6384 \end{bmatrix} \begin{bmatrix} 0.9326 & 0.0674 \\ 0.0382 & 0.9618 \end{bmatrix}$$

$$= \begin{bmatrix} 0.3616 & 0.6384 \end{bmatrix}$$

4.3.1.3 Inference for Nominal Effective Exchange Rate (NEER)

Regime 1

$$f\left(y_1 \mid s_1 = 1; \theta\right) = \frac{1}{\sqrt{2 \times 3.142 \times 0.29724}} \exp\left[-\left(\frac{(0.968 - 1.0276)^2}{2 \times 0.29724}\right)\right]$$

$$= 0.732$$

Regime 2

$$f\left(y_1 \mid s_1 = 1; \theta\right) = \frac{1}{\sqrt{2 \times 3.142 \times 0.0011635}} \exp\left[-\left(\frac{(2 - 2.00011)^2}{2 \times 0.0011635}\right)\right]$$

$$= 9.8657$$

The unconditional probabilities as in equation (4.3), is given as;

$$\pi = \begin{bmatrix} (1 - 0.9972)/(2 - 0.9056 - 0.9972) \\ (1 - 0.9056)/(2 - 0.9056 - 0.9972) \end{bmatrix}$$

$$= \begin{bmatrix} 0.9712 \\ 0.0288 \end{bmatrix}$$

Regime 1:

$$p\left(y_t, s_t = j; \theta\right) = 0.9712 \times 0.7575$$

$$= 0.7358$$

Regime 2

$$p\left(y_t, s_t = j;\theta\right) = 0.0288 \times 9.8657$$

$$= 0.2841$$

$$f\left(y_t, \theta\right) = 45.81 + 2195.70$$

$$= 2241.5$$

Likewise, the inference is given as;

Regime 1

$$p\left(s_t = j \mid y_t;\theta\right) = \frac{45.81}{2241.5}$$

$$= 0.0204$$

Regime 2

$$p\left(s_t = j \mid y_t;\theta\right) = \frac{2195.7}{2241.5}$$

$$= 0.9796$$

Simple Switching Mixture model for both series showed similar inference. The step-by-step full calculation of the above SSM model is shown in table 3 (Appendix **D**).

4.3.1.4 Prediction of future Regimes using two- state m-Period ahead Transition Probabilities Matrix for NEER

Given the transition probability matrix as

$$\text{TPM} = \begin{bmatrix} 0.9056 & 0.0944 \\ 0.00281 & 0.9972 \end{bmatrix}$$

The m-period ahead transition probabilities for NEER are:

$$\breve{E}^2 = \begin{bmatrix} 0.8204 & 0.1796 \\ 0.0053 & 0.9947 \end{bmatrix}$$

Two periods ahead later will be

$$\breve{E}^2\breve{E}^2 = \begin{bmatrix} 0.0288 & 0.9712 \end{bmatrix}\begin{bmatrix} 0.8204 & 0.1796 \\ 0.0053 & 0.9947 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0288 & 0.9712 \end{bmatrix}$$

$$\breve{E}^3\breve{E}^3 = \begin{bmatrix} 0.0288 & 0.9712 \end{bmatrix}\begin{bmatrix} 0.7434 & 0.2566 \\ 0.0076 & 0.9924 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0288 & 0.9712 \end{bmatrix}$$

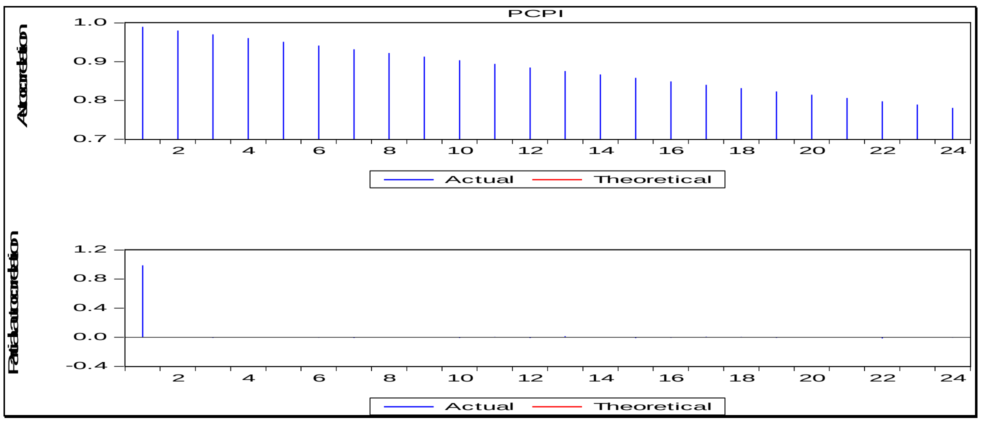
The SSM model exhibits steady state probabilities which had been shown by the m-period ahead transition matrix calculations.

4.3.2 Markov Switching Autoregressive Model

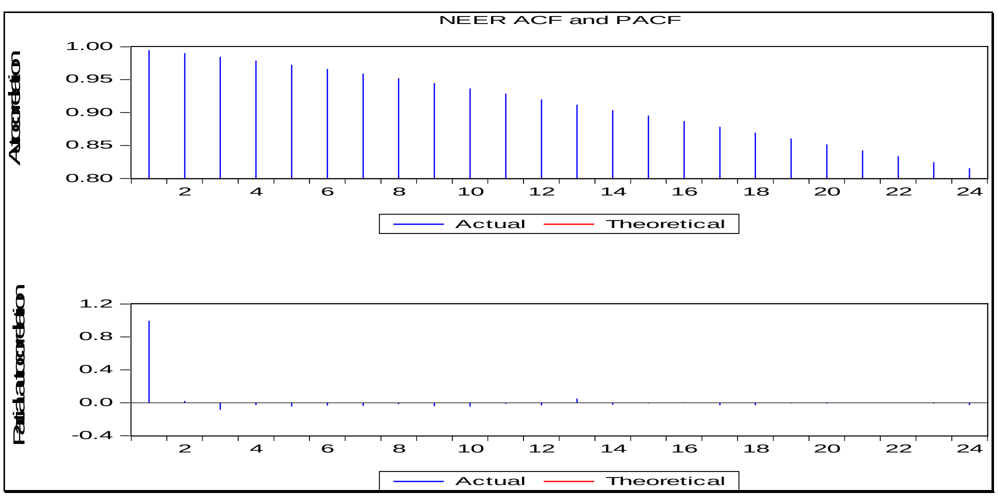
The two state regimes of the variable were selected by visual observation of the up and down movement of the trend plot.

Figure 6: The ACF and PACF of CPI and NEER

(A)



(B)



The ACF decays slowly, showing that the variable is non-stationary. The PACF shows a positive spike at lag 1. This is the sign of the first order autoregressive AR(1). The approximate linear decay of the sample ACF is often taken as a symptom that the underlying time series is non-stationary.

4.3.2.2 Expected Duration in a Particular Regime

The expected time spent in each regime before a transition takes place is given as;

$$ED = \frac{1}{1-p_{ij}} \tag{4.8}$$

Whereby, the first regime is given as

$$ED = \frac{1}{1-0.9767}$$

$$= 42.97$$

Also, for the second regime,

$$ED = \frac{1}{1-0.9868}$$

$$= 76.02$$

NEER:

Regime 1:

$$ED = \frac{1}{1-0.9056}$$

$$= 10.59$$

Regime 2:

$$ED = \frac{1}{1-0.9972}$$

$$= 356.48$$

From the calculation of expected duration, it is seen that regime 2 persisted longer than that of regime 1 for the two variables. This implies that the upward regime of both variables have stayed longer than the downward regime.

4.3.2.3 The Constant, Variance and the Autoregressive Coefficient

The Constant term and the variance switch across the regimes (as seen in table 2) below, while the autoregressive coefficient is non-switching. The autoregressive coefficient for two states according to Hamilton (1994), is seen as the second eigen value that is; $\phi = -1 + p_{11} + p_{22}$

Constant (c) of an AR (1) Process

$$\mu = \frac{c}{1-\phi} = E\left(Y_T\right) \tag{4.9}$$

CPI;

Regime 1:

$$c = 1.4658\left(1-0.9636\right)$$
$$= 0.05187$$

Regime 2:

$$c = 1.0276\left(1-0.9636\right)$$
$$= 0.1184$$

Variance of an AR (1) process

$$\sigma^2 = \frac{\sigma^2}{1-\phi^2} = \tag{4.10}$$

Regime 1 CPI:

$$\sigma_1^2 = \frac{0.07086}{0.07152} = 0.9908$$

Regime 2 CPI:

$$\sigma_2^2 = \frac{0.46442}{0.07152} = 6.4936$$

$\phi = -1 + p_{11} + p_{22}$

For CPI

$$= -1 + 0.9767 + 0.9868$$

$$=0.9635$$

While for NEER is

$$= -1 + 0.9972 + 0.9056$$

$$= \quad 0.9028$$

NEER:

Regime 1:

$$c = 1.0276\left(1-0.9028\right)$$
$$= 0.09914$$

Regime 2:

$$c = 2.00011(1-0.9028)$$
$$= 0.19441$$

Regime 1 :

$$\sigma_1^2 = \frac{0.29723}{0.18495} = 1.6071$$

Regime 2 :

$$\sigma_2^2 = \frac{0.00163}{0.18495} = 0.0088$$

4.3.2.4 Inference using Hamilton's Filter Algorithm

Given that a Markov switching autoregressive model of the form

$$y_t = c_{s_t} + \phi(y_{t-1}) + \varepsilon_t \tag{4.11}$$

Using the above model, we have the autoregressive coefficient not switching across the regimes, while the mean, the variance and the probabilities are switching.

The inferences of the regimes were as follows;

$$\hat{\xi}_{t|t} = \frac{\hat{\xi}_{t|t-1} \odot \eta_t}{I'\left(\hat{\xi}_{t|t-1} \odot \eta_t\right)} \tag{4.12}$$

Parameter Estimation of Consumer Price Index (CPI);

Using the algorithm introduced by Hamilton

The conditional density is given by

$$\eta_t = \begin{bmatrix} 76.34 \\ 49.49 \end{bmatrix}$$

The joint density function is given as:

$$F(y_t \mid Y_{t-1};\theta) = 76.34 + 49.49$$
$$= 125.83$$

Then,

$$\hat{\xi}_{t|t} = \frac{\eta_j p(s_t = j \mid Y_{t-1};\theta)}{F(y_t \mid Y_{t-1};\theta)}$$

$$\hat{\xi}_{t|t} = \frac{f(y_t \mid s_t = j, y_{t-1};\theta).p(s_t = j \mid Y_{t-1};\theta)}{F(y_t \mid Y_{t-1};\theta)}$$

This gives

$$\hat{\xi}_{t|t} = \left(\frac{75 \times 0.9767}{123.8} \right)$$

$$\hat{\xi}_{t|t} = 0.6058$$

Also, for the second regime, we have

$$\hat{\xi}_{t|t} = \left(\frac{48.8 \times 0.9868}{123.8} \right)$$

$$\hat{\xi}_{t|t} = 0.3887$$

Bringing it together, the inference for Consumer Price Index is given as

$$\hat{\xi}_{t|t} = \begin{bmatrix} 0.6058 \\ 0.3887 \end{bmatrix}$$

Parameter Estimation of Nominal Effective Exchange Rate (NEER)

and the conditional density is

$$\eta_t = \begin{bmatrix} 6.89 \\ 2445.76 \end{bmatrix}$$

The joint density is then given as:

$$F\left(y_t \mid Y_{t-1};\theta\right) = 6.89 + 2445.76$$

$$= 2452.65$$

This gives us

$$\hat{\xi}_{t|t} = \left(\frac{6.869}{2452.65} \right)$$

$$= 0.0028$$

Also, for the second regime, we have

$$\hat{\xi}_{t|t} = \left(\frac{2215.2}{2452.65} \right)$$

$$= 0.9032$$

Bringing it together, the inference for Consumer Price Index is given as

$$\hat{\xi}_{t|t} = \begin{bmatrix} 0.0028 \\ 0.9032 \end{bmatrix}$$

All the calculations for MS-AR and SSM models are shown in Appendix (D)

4.3.2.5 Forecast of future regimes using Hamilton's Filter Algorithm

One major objective of time series analysis is the creation of suitable models for prediction.The forecast about the regime for some future period is about how likely the process is to be in regime j in period $j+1$ given observations obtained through date ***t***. This is given as;

$$\hat{\varepsilon}_{t+1|t} = P\hat{\varepsilon}_{t|t}$$

Recalling that $\hat{\varepsilon}_{t|t}$ is an (N ×1) vector from equation (4.9) whose jth element is given as $p\left(s_t = j \mid y_{t-1};\theta\right)$

$$p\left(s_t = j \mid y_{t-1};\theta\right) = \left[\begin{bmatrix} 0.9767 & 0.0233 \\ 0.0132 & 0.9868 \end{bmatrix}\begin{bmatrix} 0.6058 \\ 0.3887 \end{bmatrix}\right] \tag{4.13}$$

$$= \begin{bmatrix} 0.6007 \\ 0.3916 \end{bmatrix}$$

The above is the forecast when t =1

When t = 2,

$$\hat{\varepsilon}_{2|1} = \left[\begin{bmatrix} 0.9767 & 0.0233 \\ 0.0132 & 0.9868 \end{bmatrix}^2 \begin{bmatrix} 0.6058 \\ 0.3887 \end{bmatrix}\right]$$

$$= \begin{bmatrix} 0.5959 \\ 0.3943 \end{bmatrix}$$

The forecast will proceed in this manner. The optimal m-period ahead forecast of ε_{t+m} using Hamiltonӗs filter, can be formed taking expectation of both sides of the equation. Therefore,

$$\hat{\varepsilon}_{t+m} = p^m \varepsilon_t$$

Using the above, the m-forecast for CPI is given as

$$\hat{\varepsilon}_{t+m} = \left[\begin{bmatrix} 0.9767 & 0.0233 \\ 0.0132 & 0.9868 \end{bmatrix}^m \begin{bmatrix} 0.3616 \\ 0.6384 \end{bmatrix}\right]$$

Also, the m-period ahead forecast for NEER is given as

$$\hat{\varepsilon}_{t+m} = \left[\begin{bmatrix} 0.9056 & 0.0944 \\ 0.0028 & 0.9972 \end{bmatrix}^m \begin{bmatrix} 0.0028 \\ 0.9032 \end{bmatrix}\right]$$

Table 3: Empirical Results of the Model Specifications

	SSM		MS-AR	
	CPI	**NEER**	**CPI**	**NEER**
μ_1	1.4228	1.0276	0.05187	0.09914
μ_2	3.2501	2.00011	0.118394	0.1944
σ_1^2	0.07086	0.29724	0.9908	3.776
σ_2^2	0.46442	0.001635	7.1671	0.00313
ϕ	0	0	0.9635	0.9028
π_1	0.3616	0.0288	0.3616	0.00288
π_2	0.6384	0.9712	0.6384	0.9712
p_{11}	0.6582	0.65004	0.9767	0.9056
p_{22}	0.3417	0.34996	0.9868	0.9972
ED_1	2.93	2.86	42.97	10.59
ED_2	1.52	1.54	76.02	356.49
# Param	10	10	9	9
Log Like	-296.63	-148.72	1599.12	848.24
RMSE	0.8943	1.0509	4.9073	0.56114
MAPE	32.83	113.982	144.77	70.7109

Table 4: One Step ahead Regime Probabilities of the two economic variables

	MS-AR Model				**SSM Model**			
DATE	**CPI** P(S(t)= 1)	**CPI** P(S(t)= 2)	**NEER** P(S(t)= 1)	**NEER** P(S(t)= 2)	**CPI** P(S(t)= 1)	**CPI R2** P(S(t)= 2)	**NEER** P(S(t)= 1)	**NEER** P(S(t)= 2)
1970M02	0.3610	0.63890	0.02885	0.97114	0.65829	0.34170	0.00280	0.99719
1970M03	0.3592	0.64072	0.02885	0.97114	0.65829	0.34170	0.00280	0.99719
1970M04	0.3568	0.64317	0.02885	0.97114	0.65829	0.34170	0.00280	0.99719

1970M05	0.3532	0.64679	0.00280	0.99719	0.65829	0.34170	0.00280	0.99719
1970M06	0.3502	0.64977	0.00280	0.99719	0.65829	0.34170	0.00280	0.99719
1970M07	0.3478	0.65215	0.00280	0.99719	0.65829	0.34170	0.00280	0.99719
1970M08	0.3460	0.65393	0.00280	0.99719	0.65829	0.34170	0.00280	0.99719
1970M09	0.3442	0.65572	0.00280	0.99719	0.65829	0.34170	0.00280	0.99719
1970M10	0.3413	0.65863	0.00280	0.99719	0.65829	0.34170	0.00280	0.99719

4.4 Discussion of Results

The regime 1 signify (the downward regime) while regime 2 is (the upward regime). CPI is characterized with the SSM model having its variances $\sigma_1^2 = 0.07\ and\ \sigma_2^2 = 0.46$ and the MS-AR model as $\sigma_1^2 = 0.99\ ,and\ \sigma_2^2 = 7.16$. The means are **SSM** model $\mu_1 = 1.42\ ,and\ \mu_2 = 3.25$ and **MS-AR** model $\mu_1 = 0.05\ ,and\ \mu_2 = 0.118$. Likewise, nominal effective exchange rate, the SSM model has $\sigma_1^2 = 0.297\ and\ \sigma_2^2 = 0.0016$ and its **MS-AR** have $\sigma_1^2 = 3.776\ ,and\ \sigma_2^2 = 0.0031$. The means are **SSM** model $\mu_1 = 1.027,and\ \mu_2 = 2.0001$ and for **MS-AR** model $\mu_1 = 0.09914\ and\ \mu_2 = 0.1944$.

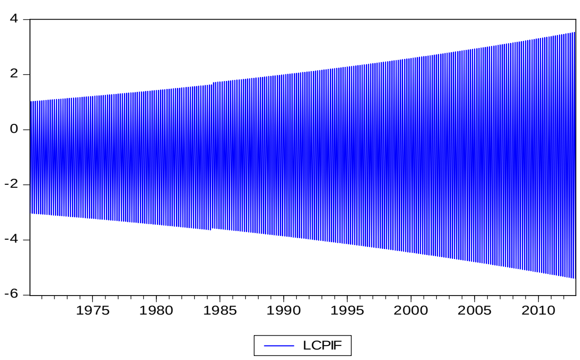
Each of the two regimes identified also have economic interpretation. The average time spent in each of the regimes μ_1 were lower than μ_2 for all the regimes in the two series. The variances showed high volatility in regime two for CPI and regime 1 for NEER. The P_{22} and P_{11} have higher probabilities, this implies that the regime is persistent and will not transit easily to the next regime. This mean that switching from regime two back to regime one will take a long time. It was shown by the expected duration of the regime two for the two series.

The expected duration of the upward regime is longer than that of the downward regime for the two variables. For CPI, the downward regime will last for 43 months before moving to the next regime and when it goes into the second regime it will last for 76 months. Also, for NEER, it will be in the first regime for 11 months before moving into the second regime where it will remain for 357 months before going back to the first regime. This is very significant to the incident rise in these financial series.

The steady state transition probabilities for CPI was obtained to be $p(s_t = 1 | s_{t-1} = 1) = 0.3616$ and $p(s_t = 2 | s_{t-1} = 2) = 0.6384$ for NEER is $p(s_t = 1 | s_{t-1} = 1) = 0.0028$ and $p(s_t = 2 | s_{t-1} = 2) = 0.9712$. When the process is in regime 1 there is a low probability while, regime 2 has a high probability. This also points to the longer duration of regime 2 of the two series.

Figure 7: MS-AR and SSM Forecast Diagrams

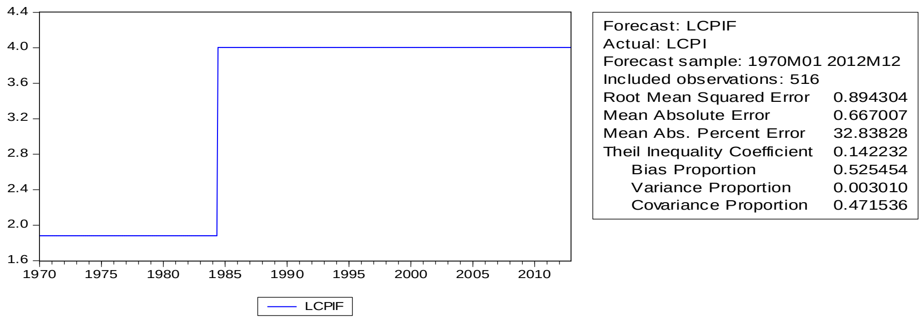
MS-AR Forecast for CPI



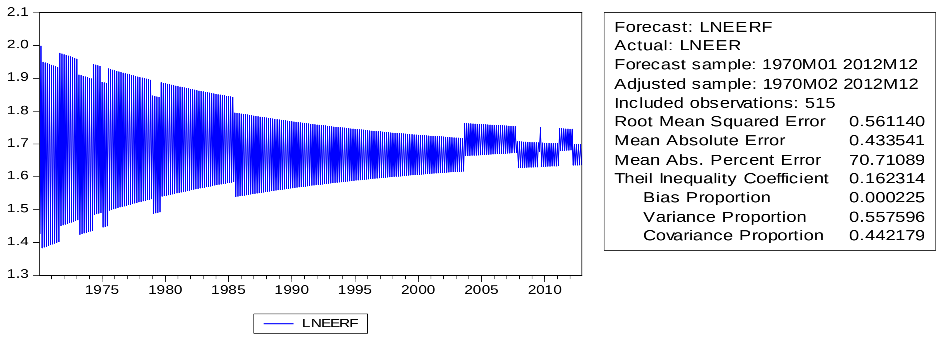
Forecast: LCPIF
Actual: LCPI
Forecast sample: 1970M01 2012M12
Adjusted sample: 1970M02 2012M12
Included observations: 515

Root Mean Squared Error	4.907361
Mean Absolute Error	3.609659
Mean Abs. Percent Error	144.7742
Theil Inequality Coefficient	0.797374
Bias Proportion	0.540869
Variance Proportion	0.185514
Covariance Proportion	0.273617

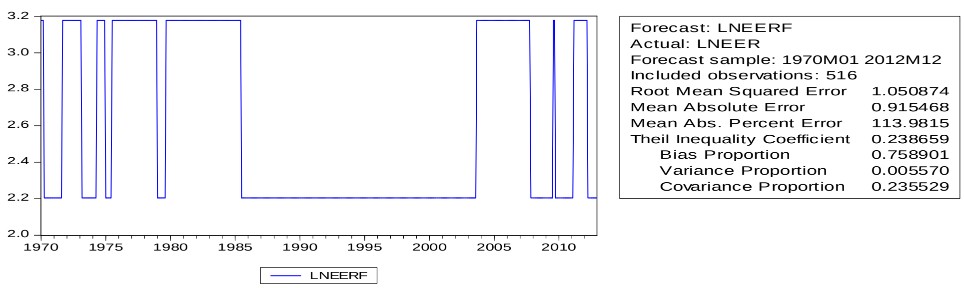
SSM Forecast for CPI



MS-AR Forecast for NEER



SSM Forecast for NEER



CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATION

5.1 SUMMARY

The study proposed a non-linear modelling of regime switching models comprising of Simple Switching Mixture (SSM) and Markov Switching Autoregressive (MS-AR) models, these were applied to two financial series in Nigeria. The Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE), were used for the selection of the best fit model for two of the series. It was observed that the SSM model had a better forecast for

Consumer Price Index (CPI) series of Nigeria, while for the Nominal Effective Exchange Rate (NEER), the MS-AR had a better forecast

5.2 CONCLUSION

This study had outlined techniques for forecasting Regime Switching Models using Simple Switching Mixture **(SSM)** and Markov Switching Autoregressive **(MS-AR)** model. In a case of two regimes, m-step-ahead transition probability matrix was used for **SSM** model while, Hamiltonǿs filter algorithm was used for **MS-AR**. Our application focused on fluctuations of Consumer Price Index and Nominal Effective Naira Exchange Rate in Nigeria. These variables have fluctuations and jumps giving rise to the application of Regime Switching models. Hence, these models can be useful for modelling and forecasting these data and other financial time series data.

Some of the things that trigger regime shifts are events such as wars, insurgences (as being experienced in the country presently), financial panics, or significant changes in government policies. Therefore we concluded that major events happening in Nigeria will have some effects on the behaviour of consumer price index and nominal effective exchange rate. These are to be checked with the study of regime switching for better planning purposes.

5.3 RECOMMENDATION

Based on the research findings, both SSM and MSAR model which are called regime switching models can be used for financial series forecasting. However, I recommend that: to control hyper-inflation, regime switching model using the one step ahead forecast of each regime is very essential to enable the analyst predict the future from its present stand.

Finally, it is recommended that the application of regime switching model with switching in the mean and the variance will help to monitor and create stable, effective and efficient policies for exchange market of our country.

REFERENCES

Adeoye, B.W. and Saibu, O. M. (2014). Monetary Policy Shocks and Exchange Volatility in Nigeria. *Asian Economic and Financial Review* 4(4), 544-562.

Adetiloye, K. A. (2010). Exchange Rates and the Consumer Price Index in Nigeria; A Causality Approach. *Journal of Emerging Trends in Economics and Management Sciences (JETEMS)* 1(2): 114-120.

Andrew, A. and Allan T. (2011). Regime Changes and Financial Markets. Columbia Business School, 3022 Broadway 413 Uris, Columbia..

Box, G. E. P., Jenkins, G. M. and Reinsel, G. C. (1994). *Time Series Analysis; Forecasting and Control.* Third Edition. Prentice Hall.

Brown, R. L. Durbin, J. and Evans J. M (1975). Techniques for Testing the Constancy of Regression Relationships over Time. *Journal of the Royal Statistical Society*. Series B Vol. 37, No. 2(1975), pp. 149-192.

Cox, D.R. and Miller, H.D. (1965). *The Theory of Stochastic Processes*. Glasgow New York Tokyo Melbourne-Madras, Chapman and Hall London.

Gianna, A., and Roberta, C. (2013). Money Growth and Inflation- Evidence from a Markov Switching Bayesian VAR . Politic EconPapers.

Goldfeld, S. M., and Quandt, R. E. (1973). A Markov Model for Switching Regressions. *Journal of Econometrics*. 1: 3ï 16.

Hamidreza, M. and Maryam, S. (2012). Point Forecast Markov Switching Model for U.S. Dollar/ Euro Exchange Rate. *Sains Malaysiana 41(4)(2012)*: pp- 481-488.

Hamilton, J. D. (1989). A New Approach to the Economic Analysis of Nonstationary Time series and the Business Cycle. Econometrica, 57: 357-384.

Hamilton, J. D. (1990). Analysis of Time Series Subject to changes in Regimes. *Journal of Econometrics* 45: 39-70.

Hamilton, J. D. (1994). *Time Series Analysis*. Princeton University Press, Princeton New Jersey.

Hamilton, J. D (2005). Regime Switching Models. *Palgrave Dictionary of Economics.* Department of Economics University of California, San Diego, USA.

Ike, D.N. (1988). The Structural Adjustment Programme as it relates to Development/Adaptation of Technology in Nigeria. *A paper presented at the workshop on SAP, NIPSS Kuru, Jos Plateau State.*

Ismail O. S. (2009), Modelling Exchange Rate in Nigeria in the Presence of Financial and Political Instability, An Intervention Analysis Approach. *Euro-journals Issue 5 Publishing.*

Jeremy, P. (2007), Models of Regime Changes. *Springer Encyclopaedia of Complexities and System Science.*

Jonathan, D. C. and Kung-sik, C. (2008), *Time Series Analysis with Application in R*. Texts in Statistics. Springer 2nd Ed., Newyork.

Kim, C., Piger, J. and Startz, R. (2008), Estimation of Markov Regime-Switching Regression Models with Endogenous Switching. *Journal of Econometrics* 143, 263-273

Kuan C., (2002), Lecture on the Markov Switching Model. *Institute of Economics Academia Sinica*, Taipei 115, Taiwan.

Lee, L., Maddala, G. S. and Trost, R. P. (1982), *Testing for Structural Change by D-Methods in Switching Simultaneous Equations Models*. Naval Studies Group, Alexanria, Virginia 22311.

Lingyun, Y., (2012), Markov Regime Switching Models. Department of Mathematics and Statistics Dalhousie University Halifax, Nova Scotia.

Maddala, G. S. and Nelson F. (1975), Switching Regression Models with exogenous and endogenous switching. *Proceedings of the American Statistical Association*, 423-426

Masoud, Y., Hamidreza, M. and Maryam, S. (2012), Markov Switching Models for Time Series Data with Dramatic Jumps. *Sains Malaysiana* 41(3) 371ï 377

Michael, P. C. and Korlzig H. (1998), A Comparison of the Forecast Performance of Markov-Switching and Threshold Autoregressive models of US GNP. *Econometrics Journal*, Volume1, pp.C47-C75.

Mohd, T. I. and Zaidi, I. (2008), Identifying regime shifts in Malaysian stock market returns. *International research journal of finance and economics*, ISSN 1450-2887, issue 15.

Ozge, K. K (2013). An Empirical Investigation of the U.S. GDP Growth: A Markov Switching Approach. *A thesis submitted to the University of Sheeld for the Degree of Doctor of Philosophy in the Department of Economics.*

.

Ronald, H. and Ronald, M. (2006). Revisiting Uncovered Interest Rate Parity Switching between the UIP and the Random Walk.

Williams, W. S. (2006). *Time series Analysis, Univariate and Multivariate Methods*. 2nd Ed. Boston: Pearson Education Inc.

Yaqub, J. O. (2010), Exchange Rate Changes and Output Performance in Nigeria, A Sectorial Analysis. *Pakistan Journal of Social Sciences,* Vol. 7 Issue: 5 pg 380-387,

APPENDICES

Appendix A : Critical Values for the Dickey-Fuller Unit Root t-Test Statistics

Model	Statistic	N	1%	2.5%	5%	10%	90%	95%	97.5%	99%
			Probability to the Right of Critical Value							
Model I (no constant, no trend)										
	ADF_{tr}	25	-2.66	-2.26	-1.95	-1.60	0.92	1.33	1.70	2.16
		50	-2.62	-2.25	-1.95	-1.61	0.91	1.31	1.66	2.08
		100	-2.60	-2.24	-1.95	-1.61	0.90	1.29	1.64	2.03
		250	-2.58	-2.23	-1.95	-1.61	0.89	1.29	1.63	2.01
		500	-2.58	-2.23	-1.95	-1.61	0.89	1.28	1.62	2.00
		>500	-2.58	-2.23	-1.95	-1.61	0.89	1.28	1.62	2.00
Model II (constant, no trend)										
	ADF_{tr}	25	-3.75	-3.33	-3.00	-2.62	-0.37	0.00	0.34	0.72
		50	-3.58	-3.22	-2.93	-2.60	-0.40	-0.03	0.29	0.66

100	-3.51	-3.17	-2.89	-2.58	-0.42	-0.05	0.26	0.63
250	-3.46	-3.14	-2.88	-2.57	-0.42	-0.06	0.24	0.62
500	-3.44	-3.13	-2.87	-2.57	-0.43	-0.07	0.24	0.61
>500	-3.43	-3.12	-2.86	-2.57	-0.44	-0.07	0.23	0.60

Model III (constant, trend)

ADF_{tr}	25	-4.38	-3.95	-3.60	-3.24	-1.14	-0.80	-0.50	-0.15
	50	-4.15	-3.80	-3.50	-3.18	-1.19	-0.87	-0.58	-0.24
	100	-4.04	-3.73	-3.45	-3.15	-1.22	-0.90	-0.62	-0.28
	250	-3.99	-3.69	-3.43	-3.13	-1.23	-0.92	-0.64	-0.31
	500	-3.98	-3.68	-3.42	-3.13	-1.24	-0.93	-0.65	-0.32
	>500	-3.96	-3.66	-3.41	-3.12	-1.25	-0.94	-0.66	-0.33

Print-out from EVIEW

Null Hypothesis: NEER has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.648170	0.7725
Test critical values:	1% level	-3.975976	
	5% level	-3.418570	
	10% level	-3.131798	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: CPI has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 2 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.823604	0.9617
Test critical values:	1% level	-3.976046	
	5% level	-3.418605	
	10% level	-3.131818	

*MacKinnon (1996) one-sided p-values.

Appendix B : MS-AR Models for CPI and NEER

Dependent Variable: LNEER
Method: Switching Regression (Markov Switching)
Date: 07/02/15 Time: 14:03
Sample (adjusted): 1970M02 2012M12
Included observations: 515 after adjustments
Number of states: 2
Initial probabilities obtained from ergodic solution
Ordinary standard errors & covariance using numeric Hessian
Random search: 25 starting values with 10 iterations using 1 standard
deviation (rng=kn, seed=1109493861)
Convergence achieved after 30 iterations

Variable	Coefficient	Std. Error	z-Statistic	Prob.
		Regime 1		
C	1.713474	0.464584	3.688188	0.0002
SIGMAF	-0.038055	0.031145	-1.221866	0.2218

Regime 2				
C	1.714827	0.467336	3.669366	0.0002
SIGMAF	-2.941660	0.162229	-18.13272	0.0000
Common				
AR(1)	0.995653	0.004143	240.3443	0.0000
LOG(SIGMA)	-3.086056	0.031404	-98.27058	0.0000
Transition Matrix Parameters				
P11-C	5.873499	1.018887	5.764621	0.0000
P21-C	-2.260930	0.962195	-2.349764	0.0188

Mean dependent var	1.676753	S.D. dependent var	0.556183
S.E. of regression	0.080559	Sum squared resid	3.303304
Durbin-Watson stat	1.966052	Log likelihood	848.2375
Akaike info criterion	-3.263058	Schwarz criterion	-3.197129
Hannan-Quinn criter.	-3.237221		

Inverted AR Roots	1.00

Equation: UNTITLED
Date: 07/02/15 Time: 14:04
Transition summary: Constant Markov transition
probabilities and expected durations
Sample (adjusted): 1970M02 2012M12
Included observations: 515 after adjustments

Constant transition probabilities:
P(i, k) = P(s(t) = k | s(t-1) = i)
(row = i / column = j)

	1	2
1	0.997195	0.002805
2	0.094411	0.905589

Constant expected durations:

	1	2
	356.4906	10.59201

MS-AR MODEL CPI
Dependent Variable: LCPI
Method: Switching Regression (Markov Switching)
Date: 07/02/15 Time: 11:40
Sample (adjusted): 1970M02 2012M12
Included observations: 515 after adjustments
Number of states: 2
Initial probabilities obtained from ergodic solution
Ordinary standard errors & covariance using numeric Hessian
Random search: 25 starting values with 10 iterations using 1 standard
deviation (rng=kn, seed=889534864)
Failure to improve objective (non-zero gradients) after 40 iterations

Variable	Coefficient	Std. Error	z-Statistic	Prob.
Regime 1				
C	-0.448970	0.428387	-1.048048	0.2946
SIGMA	0.112667	0.021642	5.205874	0.0000

		Regime 2		
C	-0.483204	0.428268	-1.128275	0.2592
SIGMA	0.027429	0.013332	2.057374	0.0397
		Common		
AR(1)	1.001779	0.000238	4203.976	0.0000
LOG(SIGMA)	-4.615833	0.032920	-140.2123	0.0000
		Transition Matrix Parameters		
P11-C	3.940729	0.434350	9.072696	0.0000
P21-C	-3.904604	0.493162	-7.917489	0.0000

Mean dependent var	2.647679	S.D. dependent var	1.050728
S.E. of regression	0.012645	Sum squared resid	0.081383
Durbin-Watson stat	1.531499	Log likelihood	1599.118
Akaike info criterion	-6.179100	Schwarz criterion	-6.113171
Hannan-Quinn criter.	-6.153263		

Inverted AR Roots 1.00

Estimated AR process is nonstationary

Equation: UNTITLED
Date: 07/02/15 Time: 11:41
Transition summary: Constant Markov transition
probabilities and expected durations
Sample (adjusted): 1970M02 2012M12

Constant transition probabilities:
P(i, k) = P(s(t) = k | s(t-1) = i)
(row = i / column = j)

	1	2
1	0.976726	0.023274
2	0.013154	0.986846

Constant expected durations:

1	2
42.96609	76.02204

Appendix C: SSM Models for CPI and NEER

Dependent Variable: LNEER
Method: Switching Regression (Simple Switching)
Date: 07/02/15 Time: 14:11
Sample: 1970M01 2012M12
Included observations: 516
Number of states: 2
Ordinary standard errors & covariance using numeric Hessian
Random search: 25 starting values with 10 iterations using 1 standard
deviation (rng=kn, seed=1109493861)
Convergence achieved after 6 iterations

Variable	Coefficient	Std. Error	z-Statistic	Prob.
		Regime 1		
C	2.025133	0.026292	77.02569	0.0000
SIGMAF	-3.479083	0.092951	-37.42933	0.0000

Regime 2

C	2.024065	0.019272	105.0258	0.0000
SIGMAF	-0.462430	0.067863	-6.814129	0.0000

Common

LOG(SIGMA)	-1.508426	0.032926	-45.81197	0.0000

Probabilities Parameters

P1-C	-0.619175	0.123636	-5.008057	0.0000

Mean dependent var	1.677381	S.D. dependent var	0.555826
S.E. of regression	0.480353	Sum squared resid	117.9078
Durbin-Watson stat	0.070010	Log likelihood	-143.7147
Akaike info criterion	0.580289	Schwarz criterion	0.629663
Hannan-Quinn criter.	0.599637		

Dependent Variable: LCPI
Method: Switching Regression (Simple Switching)
Date: 07/02/15 Time: 14:23
Sample: 1970M01 2012M12
Included observations: 516
Number of states: 2
Ordinary standard errors & covariance using numeric Hessian
Random search: 25 starting values with 10 iterations using 1 standard
deviation (rng=kn, seed=1736667780)
Convergence achieved after 13 iterations

Variable	Coefficient	Std. Error	z-Statistic	Prob.
Regime 1				
C	1.270699	0.094520	13.44364	0.0000
SIGMAFCPI	2.104192	0.195420	10.76756	0.0000

Regime 2

C	1.050419	0.054708	19.20060	0.0000
SIGMAFCPI	5.213042	0.116212	44.85810	0.0000
Common				
LOG(SIGMA)	-1.253701	0.032304	-38.80946	0.0000
Probabilities Parameters				
P1-C	-0.632969	0.117203	-5.400640	0.0000

Mean dependent var	2.644511	S.D. dependent var	1.052171
S.E. of regression	0.609555	Sum squared resid	189.8655
Durbin-Watson stat	0.017658	Log likelihood	-297.0192
Akaike info criterion	1.174493	Schwarz criterion	1.223866
Hannan-Quinn criter.	1.193841		
Inverted AR Roots	1.00		
Estimated AR process is nonstationary			