ALGORITHM DEVELOPMENT FOR FINGERPRINT IMAGE ENHANCEMENT USING WAVELET PROCESSING

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DEDICATION

This work is dedicated to God Almighty who is the source of all wisdom and giver of life, for making it possible for me to successfully complete this thesis. To my family, for their invaluable love, moral and financial support throughout my education.

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ABSTRACT

Biometrics is a physical, electronic and biological means of providing dependable and accurate identification of people. It entails data acquisition, storage and subsequent matching at points of access or identification. Biometrics is an emerging technology that identifies people by their physical and/or behavioral characteristics and practically requires the person to be identified to be physically present at the point of identification. In this thesis, efforts to improve biometrics were made in developing an algorithm that will enhance fingerprint images for effective matching and rapid identification. The algorithm was based on wavelet processing which employed Daubechiesô wavelets for decomposition as well as reconstruction. Wavelet smoothing and denoising were also included in the algorithm.

LIST OF FIGURES

LIST OF TABLES

TABLE OF CONTENTS

CHAPTER ONE

INTRODUCTION

1.1 PREAMBLE

Todayós world has been greatly modified by technology in the efforts of man to deal with his environment in order to tame it to suit his needs; with particular reference to electronics. Virtually, all aspects of our modern civilization depend on electronic [including hardware and software] systems for smooth and effective operations of daily activities. Ranging from military, communications, government, education, banking, automobiles and manufacturing to the space industry; all rely on electronic systems for various applications like process automation, control and monitoring systems, security, automated parking control, 24 hours surveillance systems, traffic collision avoidance system (TCAS) of aircrafts and on board data handling subsystems of satellites (OBDH).

With the dynamic growth of the worldós population, global economic meltdown, food scarcity, and unemployment; all these factors have led to significant increase in violent crimes globally making security a great challenge to all and sundry.

Governments, business owners, the organized private sector and educational institutions all require a robust, rugged and dependable means of recognizing and verifying peoplesô identities at various points of access to specific areas physically and electronically.

In order to ensure that intruders, impersonators are promptly fished out prior to breaching the security of these valued resources, information, assets and vaults; a robust and effective means of identifying and verifying peoplesô identities is paramount. The search for such systems culminated recently in *biometrics*.

In the late nineteenth century, it was discovered by Johannes Evangelista Purkinje a Czechoslovakian physiologist [1] that no two human beings share the same fingerprint even Siamese twins! This was a revolutionary breakthrough in the world of law enforcement as investigators now had a valid and reliable means of hunting down as well as precisely identify criminals [1].

Biometrics is an emerging technology that is used to identify people by their physical and/or behavioural characteristics and practically requires the person to be identified to be physically present at the point of identification [2].

Biometric identification relies on those characteristic features of humans that are inherently unique to individuals such as fingerprints, palmprints, retina, iris, hand geometry, facial features, and ears. Another category of biometrics utilize behavioural characteristics like speech [or voiceprint], lip movement, keystroke dynamics, gait, gesture, signature [2] and thermal emissions. These features are so distinct that no two humans on earth even Siamese twins possess exactly the same replicas! A physiological or behavioural characteristic that is unique to an individual is referred to as *biometric measurement* [e.g. voiceprints and fingerprints] which has the capability to reliably distinguish between an authorized person and an imposter [3].

Biometric systems capture these features as digital data using various devices like digital cameras and scanners, store them on a database where they serve as templates, retrieve the templates as quickly as possible when required for verification or authentication at various points of access. For a fingerprint based biometric system, a typical verification phase involves two stages: enrolment and verification.

Enrolment: this stage captureôs the userôs fingerprint; its distinctive features [*minutiae*] are extracted and stored as a template.

Verification: during the verification stage, a new fingerprint is acquired and compared to the stored template to verify the userôs claimed identity [4].

1.2 DEFINITIONS

Fingerprint: A fingerprint is the reproduction of a fingertip epidermis, which is generated when a finger is pressed against a flat surface or any other surface. The main structural characteristic of a fingerprint is the pattern of interleaved ridges or ridgelines and valleys which often run in parallel. [5] Defined fingerprint as a unique pattern of ridges and valleys on the surface of fingers of an individual.

Fingerprint patterns exhibit one or more regions where the ridgelines assume particular shapes. Figure 1 shows a fingerprint image taken from FVC 2002 indicating minutiae, core and delta points. These regions are called *singularities or singular regions* and they form the bases upon which fingerprints are classified into three different categories:

i. Loop
ii. Delta
iii. Whorl

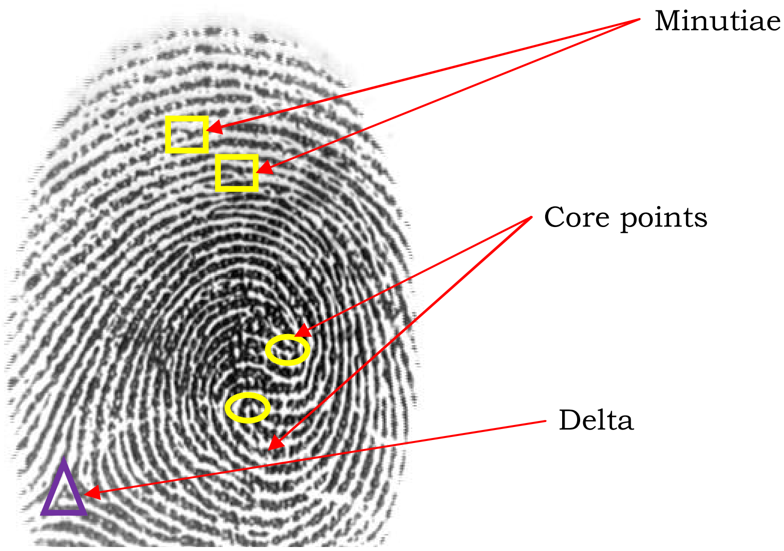

Figure 1: A whorl type fingerprint showing core points, Delta point and minutiae- source [6]

The singular regions are generally denoted by ʧ, æ and ʆ shapes respectively. Singular regions are commonly used for fingerprint classification, which is assigning a given fingerprint to a class among a set of distinct classes with the sole aim of simplifying search and retrieval.

Algorithm: Algorithm is defined as a step by step approach of executing a specific task. For any given task, several algorithms may exist but the results of such algorithms must be the same.

Algorithm can be thought of as a limited set of precise instructions which when executed in a predefined manner attains its objective. óAn algorithm is an effective method expressed; as a finite list of well defined instructions for calculating a functionô[7].

In computer science, an algorithm is defined as a logical, systematic, computational approach that is adopted to solve programming tasks. It is a step by step process that software engineers apply in the design of series of instructions a computer must execute to produce the desired output.

Enhancement: It is a process that improves the quality, attribute, features or effectiveness of any object. To enhance simply means to add value in order to increase perception.

Centre Point Core: Is defined as the position of the north-most loop singularity or as the point of maximum ridgeline curvature for fingerprints belonging to the arch class [*see figure 1*].

Orientation Image: Is a discrete matrix whose elements denote local orientation of the ridgeline pattern of a fingerprint.

Local Ridgeline Frequency: Is defined as the number of ridges per unit length.

Minutiae: Are ridgeline discontinuities, local characteristics of the pattern that are stable and robust to fingerprint impression conditions [8]. Minutiae are distinctive features adopted by most fingerprint based biometric systems for identification and verification; they may be classified into several types such as:

i. Termination- the point where a ridgeline suddenly ends.
ii. Bifurcation- the point where a ridgeline divides into two.
iii. Island
iv. Dot
v. Lake

A minutia point may be defined by its type, the x- and y- coordinates and the direction ɗ.

The ISO/IEC 19794-2:2005 standard specifies the data formats for minutiae based fingerprint representation. The fingerprint minutiae record format defines the fundamental data elements used for minutiae based representation of a fingerprint and optional extended data formats for including additional data such as ridge counts and singularities location.

Ridge: óA ridge is defined as a single curved segmentô of a fingerprintøs pattern [2].

Multimodal Systems: As the name indicates, they are biometric systems that utilize two or more biometric characteristics for recognition. Multimodal systems generally require integration schemes as they employ characteristics for identification and verification of claimed identity.

Unimodal Systems: These are biometric systems that are based on a single biometric feature such as fingerprint or facial feature for recognition. They are generally less accurate than multimodal systems and also more susceptible violation.

1.3 BRIEF HISTORY OF BIOMETRICS

Research efforts made by various scholars with sponsors from different stakeholders to provide reliable answers to questions such as these:

i. Does Mr. Obi have administrator privileges on this network?
ii. Is Chima authorized to access this facility?
iii. Has Musa been convicted of a crime prior to this interview?
iv. Is Iniobong who he says he is?

Culminated in Biometrics

Traditionally, organizations and governments have answered questions such as these by relying on the information the individuals possessed such as passwords or what they had like identity cards. These, have major setbacks as they can be easily stolen, manipulated or forged! These limitations provided an opportunity for improvements as well as total replacements; which led to researches on robust automated systems that can effectively identify people, verify identities as well as pose a great barrier to any one that attempts to falsify his/her identity. A need for a system that will make personal recognition based on who people were was paramount.

Humans have for generations identified each other with various body characteristics like voice, face, stance, mannerisms and gaits. These characteristics are particularly unique to each individual; hence any means of identification that can incorporate any of these features is likely to be foolproof and failsafe.

In the mid 19th century, Alphonse Bertillion, chief of the criminal identification division of the police department in Paris, developed and practiced the idea of using various body measurements (for example: height, length of arms, feet, and fingers) to identify criminals [9].

It was discovered in the late 19th century that no two human beings had the same fingers; just as Alphonse Bertillonôs practice of body parts measurement was gaining popularity.

This revolutionary breakthrough discovery became the prince of law enforcement as investigators now had a definitive means of identification at their disposal [1].

After this discovery, many major law enforcement departments adopted the concept of ○bookingô criminalsô fingerprints and storing them in databases or card-files. As the technology grew, police acquired the ability to ○liftô leftover, typically fragmentary fingerprints from crime scenes (commonly called latents) and match them with fingerprints in the database to determine criminalsô identities [9].

The English word *biometric* originated from two Greek words *bios and metrikos* that mean life and measure respectively. Biometrics is an emerging technology that automatically recognizes people based on their physiological and/or behavioural characteristics.

In 1892, Francesca Rojas an Argentine woman made history as the first killer to be caught using fingerprint evidence [1]. A fingerprint is potentially worthless unless it can be linked either to a suspect or a database entry. This implied long hours of manually searching card indexes in order to find a match for the crime scene fingerprint; fortunately, computerization techniques [image processing algorithms and DSP techniques] have changed this practice for good. An example of such algorithm is indexing fingerprints using minutiae as presented in [10]. The *integrated automated fingerprint identification system* (IAFIS) is software that contains a huge and expanding database of fingerprint records, and manual indexing which may have taken weeks or even months to match a fingerprint is now accomplished within seconds or minutes depending on the size of the database.

Each fingerprint is unique; they develop on a foetus at about six months and remain unchanged until the body decomposes after death! Johannes Evangelista Purkinje a Czechoslovakian physiologist (1787-1869) was the first to recognize and describe the nine basic patterns of fingerprints [1].

The earliest recorded practical application of fingerprints as a means of identification was in 1858, when a young administrative officer in India William Herschel, adopted the local *Bengali* practice whereby illiterate workers ○*signed*ô for their pay by leaving a thumbprint [1]. This was done to stamp out pension frauds.

In 1879, a Scottish physician working in Japan, Henry Faulds became involved in what is generally considered to be the first crime solved through the use of fingerprints; when a burglar making his escape from a Tokyo house left a dirty handprint on a white washed wall. When the police quickly arrested a suspect, Faulds inspected the manȏs handprint and declared him wholly innocent; much to the amusement of sceptical investigators. Three days later Faulds was vindicated, as another felon admitted to the burglary and his handprint also matched exactly the one left on the wall [1].

Faulds published his findings in Nature, the British scientific journal in 1880, sparking a long running feud with Herschel; who still regarded himself as the father of fingerprinting [1].

In 1892, Sir Francis Galton another English scientist, turned theory into practical reality when he laid out the ground rules for a basic classification system for fingerprints. He separated the most commonly observed feature into three categories:

i. Arches
ii. Loops
iii. Whorls

His iconic work, Finger Prints (1892), lit the beacon but it was another Indian police officer Edward Henry that completed the task of fingerprint classification.

Biometrics initially came into wide usage for law enforcement and legal purposes such as identification of criminals and illegal aliens, security clearances for employees in sensitive jobs, paternity determinations, forensics, positive identification of convicts, and prisoners. However, in recent times, many civil and private establishments apply biometrics to establish personal recognition and prevent identity thefts.

In 1956, Alphonse Bertillon conceptualized the idea and commercially implemented the procedure of applying biometrics [body measurements] in crime investigation and prosecution.

1.4 BIOMETRIC SYSTEMS

These are systems that identify, verify, authenticate and grant access to restricted resources or facilities based on distinctive human characteristics such as voiceprints, fingerprints, facial features and iris recognition. Generally, biometric systems are of two categories viz: unimodal [are biometric systems that are based on single biometric characteristic such as fingerprint recognition systems] and multimodal [refer to biometric systems that rely on multiple biometric features e.g. voiceprint & iris recognition systems] biometric systems.

A biometric system is primarily a pattern recognition system that recognizes an individual based on a feature vector obtained from a specific and unique physiological or behavioural characteristics that the individual possesses [1].

Figure 2 shows a generic architecture of a biometric system. It can be seen from the diagram that a biometric system can function in either of two modes namely:

i. Verification mode
ii. Identification mode

Verification mode: In this mode, the biometric system authenticates a userôs claimed identity by matching the captured biometric characteristic against his/her biometric template which has been previously enrolled. In this mode, a biometric system is required to answer the question, is Mr. A truly Mr. A. It provides identity verification using positive recognition in which the system performs a one to one comparison to determine whether the claimed identity is true or false. An example of this type of biometric system is electronic employee attendance register.

Identification mode: Recognition of an individual in identification mode is performed through a one to many searches. The biometric system searches its entire database of templates for a match of the individualôs biometric information. If the individualôs biometric data has not been enrolled previously, the system fails to identify him/her. The question that is answered in this mode is who is Mr. B? This is a form of negative recognition whereby the system establishes if Mr. B is who he implicitly is or explicitly denies being. An example of a biometric system that functions in negative recognition mode is fingerprint based access door control. The essence of negative recognition is to prevent a single individual from using multiple identities as evident in Nigeriaôs 2011 voter registration exercise. It is also possible to apply identification in positive recognition for convenience when the user is not required to make an identity claim [9].

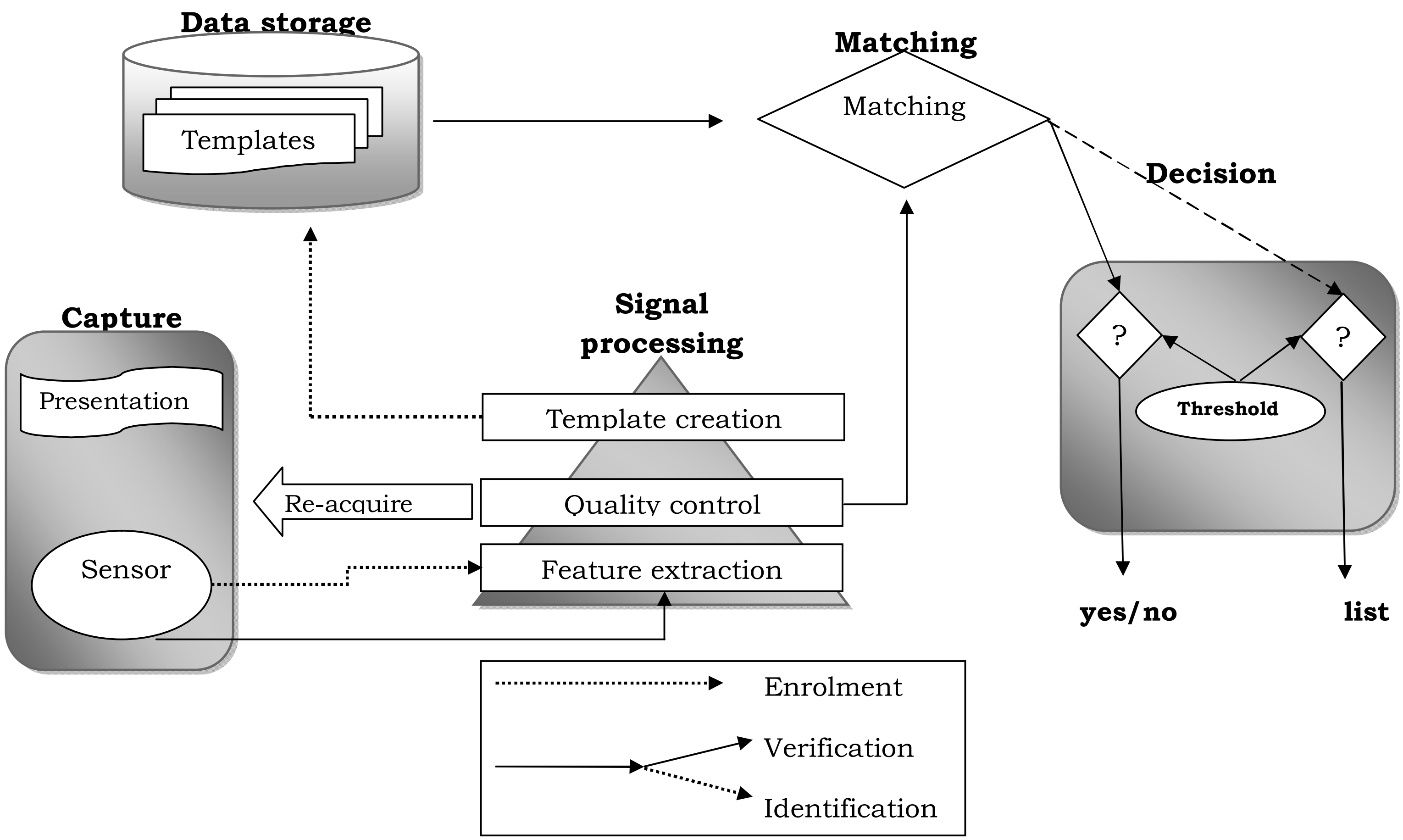

Figure 2: Generic architecture of a biometric system: source [11].

1.4.1 IMPORTANCE OF BIOMETRIC IDENTIFICATION

Conventional methods applied in personal identification like tokens, pins, keys and passwords provide only positive recognition whereas biometrics can provide both negative and positive recognition. The benefits of biometric identification cannot be overemphasized, let us highlight just a handful as an insight into the immense features that characterise biometrics:

i. **Rapid Identification:** Biometric systems provide rapid as well as reliable identification of individuals at various points of access that are sensitive and require restricted access.

ii. **Employee Attendance Register:** Biometric based staff attendance register provides a robust and dependable means of taking staffsô attendance and does not give room for manipulation. It can also serve as a reference for awards/punishments of staff based on punctuality and dedication to duty as a sequential order of attendance is logged and stored in a database.

iii. **Verification:** Biometric systems provide reliable means of verifying peoplesô identities of particular reference is at points of bank transactions; where account holders will be required to be distinguished from third parties.

iv. **Forensics:** It has become imperative to preserve the integrity of crime scenes in order to accurately acquire evidence(s) that assist in tracking down and convicting criminals. Such evidences when garnered will require a robust, reliable and accurate system for analysis to provide failsafe identification. Biometric systems provide such platforms.

v. **Archaeological Discoveries:** Biometric systems are applied in the analysis of prehistoric manuscripts for handwriting recognition and also traceability with respect to the era and cultures in which such documents were written.

vi. **Security:** Biometric systems are generally less susceptible to hackers as is the case with traditional identification technologies. Biometric data are unique and since they require the physical presence of the individual to be identified, security risks are minimized. Biometrics cannot be easily shared, duplicated, lost, or stolen as is the case with the conventional technologies.

vii. **Convenience:** They are very convenient to use as they cannot be forgotten or lost, forged, repudiated without compromising their accuracy or integrity as they ensure that the user is always present at the point and time of recognition.

1.4.2 APPLICATIONS OF BIOMETRICS

Biometrics can be applied broadly in four major categories:

i. **Government applications:** This category of application lists various functions/responsibilities a government should offer its citizens; for instance: issuance of national identity cards, votersô registration, issuance of driversô licenses, prison services, border monitoring and control, passport control and immigration services as well as administering aid services.

ii. **Military applications:** Areas that biometrics can be deployed in the military include: personnel identification, clearance level or rank regulated resources (such as access to Amory, clandestine mission authorization, command and control centres for active operations), hardware control and authorization, missile launch and facility control.

iii. **Commercial and civil applications:** Numerous areas of biometricsô application abound commercially, examples include: computer access control, electronic attendance systems, computer network access control, automated teller machines, e-commerce, internet access, physical access control, mobile phones, personal digital assistants, electronic data management systems, debit and credit cards authentication, medical and official records management and distance learning and so on.

iv. **Forensic applications:** This category generally deals with medical and scientific methods of crime solving; examples include identification of corpse, criminalsô and terroristsô identification, missing children, crime scene investigation.

(a) **(b)** **(c)** **(d)**

Figure 3: Examples of biometric systems application (a) Digital Personaøs fingerprint verification system provides personal recognition for computer and network login. (b) Indivos manufactures a fingerprintbased point-of-sale (POS) terminal that verifies customers before charging their credit cards. (c) BioThenticaøs fingerprint-based door lock restricts access to premises. (d) The Inspass immigration system, developed by Recognition Systems and installed at major airports in the US, uses hand geometry verification technology: source-[9].

CHAPTER TWO

LITERATURE REVIEW

2.1 PRELIMINARY

An image is defined as a signal of two dimensions that has its intensity specified by a function of two variables in space at any given point [12].

Fingerprint is an impression that is created when the finger(s) is pressed against a flat [or any] surface. They are inherently unique to each human; fingerprints are formed on the fingers of the foetus at about six months and remain unchanged until decomposition after death.

Image enhancement techniques abound under which fingerprint image enhancement has its context. The objective is to improve recognition rate while reducing the rejection rate of enrolled users. Literature abound on image enhancement techniques, in this chapter we will review some state of the art technologies that have been employed to enhance fingerprint images. The main focus will be on wavelet processing techniques applied in fingerprint images enhancement, as this is the primary goal of this research work.

As stated earlier, biometric systems automatically identify and/or verify humans based on their unique inherent characteristics of which fingerprint is the most popular. Methods and techniques that extract these unique characteristics from fingerprint images such minutiae extraction form the foundation of the biometric system.

This review will provide insight on the methodology, overview of some fingerprint image enhancement techniques, summarize works that have been done in this area, evaluate such works as well as provide the context of this thesis. It also aims to identify gaps where there are rooms for improvements and develop understanding of the theories and concepts associated with fingerprint image enhancement algorithms.

Image enhancement techniques select a particular characteristic of an image to adjust the appearance of any given image. The overall objective is to improve the image quality by manipulating its features such as: enhancement of contrast, normalization of illumination, zooming, detection of edges, red eye reduction, and manipulation of shadows. An enhanced image provides detailed information from the original image signals by removing or reducing noise and its effects, this is usually done by applying digital signal processing techniques such as filtering and decomposition. Other methods employed in image quality enhancement include: spectral methods, wave-front or shock methods, use of non-Albelian group operations [i.e. logical image operations and connected operations].

Wavelet processing is the application of mathematical functions that cut up data [for instance an image signal] into different frequency components, and investigates individual parts of the frequency with a corresponding resolution to its own standard or scale for image processing. In order to define a given function as a wavelet system, it must consist of the following three characteristics as stipulated in [13]:

i. General functions were derived with wavelets as basic components or blocks i.e. a particular function or signal is denoted in the wavelet domain as an average of infinite series of wavelets.
ii. There is a space-frequency localization of wavelets implying that majority or a significant portion of the energy in a wavelet is restricted in a fixed range or interval and also frequency components of the transform is within a particular frequency band.
iii. The wavelets are capable of handling fast as well as efficient transform algorithms [13].

Since there are numerous image enhancement techniques, this review will be restricted to and organized as follows:

i. Fingerprint image enhancement techniques.
ii. Wavelet processing.
iii. Image enhancement using wavelet processing.
iv. Fingerprint image enhancement using wavelet processing.

2.2 FINGERPRINT IMAGE ENHANCEMENT TECHNIQUES

In [14], a fingerprint was defined as "a characteristic decorative pattern in a person's fingers and skins, which consists of rise lines of skins. The start point, end point, bifurcation, and bonding point, of these rise lines is referred to as fingerprint's specific character".

[13] Described fingerprint image enhancement as the next phase that succeeds image segmentation [this process distinguishes image foreground from its background] that is employed to improve image quality. This process clearly separates ridges and valleys of the fingerprint image by enhancing the visual qualities of the image through texture filtering, as well as improvement of the local features of the fingerprint image such as rate and direction.

From [15], we learn that most algorithms deployed towards fingerprint image enhancement that reduce noise while effectively sharpening the contrast of valleys and ridges; rely on gray scale images. These algorithms can be broadly classified into óspatial domain filtering enhancement and transformed domain enhancement techniquesô [15]. Examples include Fourier and wavelet domains respectively. According to Yuan et al [15], it is computationally cumbersome to implement spatial domain enhancement techniques as they apply spatial convolution of images during filtering.

[16] And [17] both show that Gabor filters are widely adopted in the areas of fingerprint classification and verification to improve the applicationsô performance. Gabor filter being bandpass filters possess frequency as well as orientation selective properties, making them suitable in both frequency and spatial domains for optimal joint resolution [15]. A technique was proposed in [15] in which super resolution is incorporated as a pre-processing step in the enhancement algorithm particularly for low quality images usually obtained from a camera. óSuper resolution is a technique of enhancing image resolution by combining information from multiple imagesô [15]. Recent methods applied in super resolution enhancement of fingerprint images are primarily based on spatial domain. [18] Put forward a proposal that recommended the use of a method that generates high resolution images through a combination of both space and time domain information. [19] Suggested a technique called *Projection Onto Convex Set (POCS)* for super resolution which is currently one of the most applied super resolution techniques. In [20], an extension on the discussion of POCS with modifications aimed at its improvements was made. Another conventional super resolution technique is the *stochastic* based super resolution, such as that presented in [21] advocated the use of *Bayesian estimation* in super resolution techniques. *Maxima-a-Posteriori (MAP)* a common stochastic process was employed in the technique put forward by [22].

Super Resolution consequently results in an increase in the density of pixel, causing the super resolution image to accommodate frequencies outside the *Nyquist boundary* of the input images. [23] States that conventional fingerprint image enhancement techniques perform satisfactorily for high quality images but are not suitable for low quality ones. Its argument being that these techniques rely on the information concerning the local ridge structure with regards to the estimation of its orientation from the fingerprint images that form the input; hence are seriously affected by noises like holes, greases, and smudges that characterise low quality fingerprint images. [22] Proposed a four step approach towards the enhancement of fingerprint images that will remove redundant information while boosting the relevant information contained in the fingerprint image. These steps include:

i. **Normalization:** A conventional approach that is adopted in contrasting images through transformation of their intensity values as shown in [24] is the histogram equalization. Due to the amplification of noise to a greater value than the input image by histogram equalization, [23] applied *contrast limited adaptive histogram equalization (CLAHE)* was developed. CLAHE supports enhancement of the contrasts of small segments and subsequent combination of adjacent segments of the image through bilinear interpolation. This approach eradicates synthetically introduced boundaries. Application of the ˛clip limitˆ factor prevents over-saturation of images particularly at homogenous sections which show high peaks in the histogram of certain segments of the image as a lot of pixels fall within the same gray level boundary [24]. [23] Filters the image in both spatial and frequency domain to produce an adequately enhanced image.

ii. **Binarization:** This process converts the image from its gray scale to binary image through calculation of its mean value of 32 x 32 block matrix of the input and consequently changing pixel values to 1 if greater than the mean or to 0 if less than the mean [23].

iii. **Quality Markup:** The task in this step entails the elimination of irrelevant data in order to dissociate the background from the foreground prior to enhancement; an algorithm formulated on distinct block processing is used to obtain the standard deviation of the image array. Removal of the background yields boundaries for the required regions of each matrixôs subset with a value that is greater than the threshold.

iv. **Thinning:** Here, each pixel magnitude is found in order to characterise each feature. The pixel neighbourhood thinning which yields a maximum value prior to filtering in the last stage to remove false lonely points and break, form the foundations of some thinning methods [8] [25].

[23] Posited another algorithm that removes the false information through slide neighbourhood processing in the initial step succeeded by thinning and no other filtration. This isolates the fingerprint image finally from the background with the local minutiae positioned on the thinned binary image.

[26] Proposed an algorithm for fingerprint image enhancement, extraction of features like singularity points and minutiae dependably through the application of a *Hierarchical Markov Random Field Model (HMRF)*. [26] Categorizes methods that have been adopted to enhance fingerprint images into two main groups: those that model the noise in order to eliminate it like median filtering, Weiner filtering, contrast enhancement and others that model the fingerprint structure. [26] Argues that the effectiveness of these techniques lie on the viability of the assumptions on which the models are based and also its capability to include contextual information of the image being enhanced.

Conventionally, these approaches that enhance fingerprint images apply contextual filters. These filters are designed according to restrictions posed by the functions indicated by local ridge orientation, reliability, and spatial components. From [27] pioneer efforts that incorporated elongated filters used on the ridgeline direction can be seen. Others performed filtering exhaustively in the frequency domain as documented in [28]. In the case of [29], a combination of both frequency and spatial domain like Short Time Fourier Transform were investigated. [30] Adopted Gabor filters as the most common approach used in spatial domain filtration. Gabor filters have been observed to possess optimal joint resolution both in spatial and frequency domains. [31] And [32] all made efforts to alter as well as extend selective application of Gabor filtering. The HMRF model is based on the assumption that the signal at any point in the local region is a set of equally spaced parallel ridges or a pattern that is sinusoidal having a unit direction as well as unit frequency; [26] states that this assumption is contravened within elevated curvature regions like singularities, and bifurcation points. It was also recorded in [26] that filter accuracy drops at points with unreliable contextual information like areas smeared with creases, cuts and smudges. [33] Categorized filters regulated by parameters into isotropic and anisotropic; isotropic filters retain the information but fail to retrieve a lot of regions while anisotropic filters manufacture significant instantaneous peak structures. Consequently, compromise is always reached between the magnitude of image enhancement and the production of instantaneous peak feature of the fingerprint.

HMRF model came about as a result of two factors:

i. Trade-off between enhancement and spurious structures is adjustable by adopting a tight model for fingerprints [26] and
ii. Regions with severe degradation resulting to minimal information can be boosted through firm application of context that narrows the model in such regions.

These objectives were tackled by firstly modelling the fingerprint with a group of image patches acquired from neat fingerprint images. Secondly, contextual information was included with constraints that were restricted with two close patches. Observation was made that the outcome can be rapidly synthesized with a Markov Random Field, where the outcomes that were made are image patches. The MRF model was expanded to the HMRF format in order to account for the contextual constraints; each section of the model handles a unique constraint like ridgeline continuity, and singularity. MRF and Bayesian paradigms had been previously designed to dependably isolate orientation fields from noise infected fingerprint images. Quality as well as smoothness combination were applied initially before isolating orientation fields by [34].

The HMRF yields a better, robust forecast of orientation field through continuous variation of the orientation field about the singularities. Despite this, recovery of orientation field from big patches is unattainable. To solve this challenge, [35] adopted samples training of different orientation that functioned as priors. [26] Argued that only the orientation field is not solely capable of enhancing fingerprint images particularly at regions that are severely smudged with most of its information mutilated. [26] Presented an integrated technique which enhances fingerprint images, extracts orientation field, classification, ridge frequency forecasting (or prediction) as well as minutiae detection through the application of a multi-layer MRF paradigm. Incorporation of regional geometry and spatial interaction are popular features of Hierarchical MRF paradigms as discussed in [36]. It is evident from [26] that improved restoration of noisy areas can be achieved if two different sized patches are obtained from two layers and applying available good quality regions in the fingerprint image. Figure 4 shows the enhancement outcome of the technique presented in [26] using HMRF and multi-stage Gabor filtering.

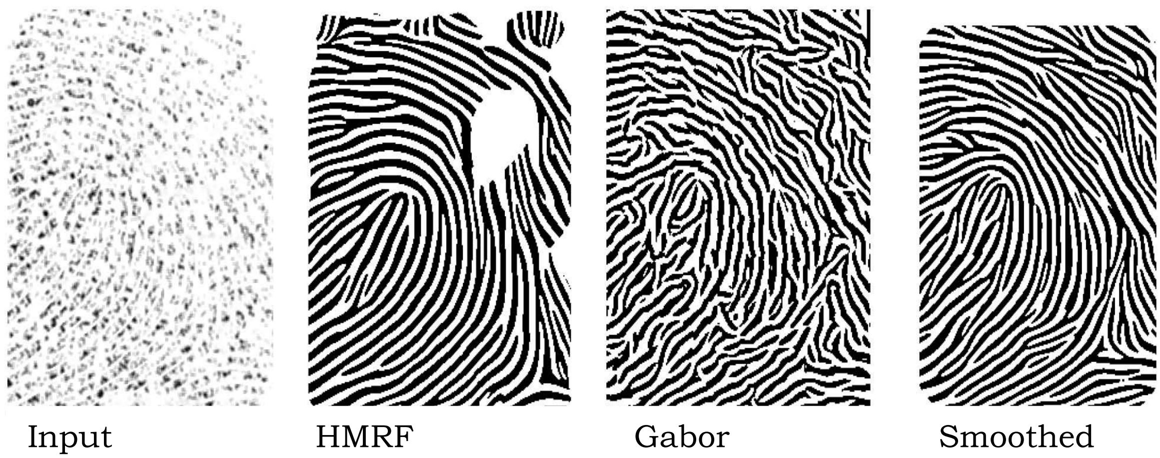

Figure 4: Enhancement result using HMRF and multi-stage Gabor filtering and its smoothed version: source - [26]

From figure 4, it is evident that the input fingerprint image is of low quality, poor orientation and invisible ridgeline structure. Although the output fingerprint image of the HMRF is of improved quality, with better orientation of the ridges and visible ridgeline structure; the output fingerprint image of the Gabor filter and its smoothed version offer a better estimation of the orientation of the ridgeline pattern as visible from the region without information.

[37] put forward a proposal for Orientation Field Estimation for Latent Fingerprint Enhancement; this approach argues that although conventional orientation field estimation techniques perform satisfactorily for live scan and inked fingerprint images, they do not function well for most latent prints. This poor performance of the conventional algorithms can be traced to the very low quality, uneven contrast, blurred ridgeline pattern and crisscross patterns that characterise most latent prints as they are usually impressed unintentionally. The proposed orientation field estimation algorithm of

[37] is composed of an off-line dictionary construction stage, as well as an on-line orientation field estimation stage.

The off-line stage: selects manually a good quality group of the different classes of fingerprints [arch, loop, and whorl], their orientation fields form the foundation of the fabrication of a dictionary of orientation patches.

The on-line stage: this stage accepts a given fingerprint and estimates its orientation field automatically by going through the following three phases:

i. **Initial estimation:** This phase produces a preliminary orientation field estimation by applying any local orientation estimation technique like local Fourier analysis of [38].
ii. **Dictionary look up:** This phase breaks up the initial orientation field into overlapping patches. Six closest patches of each initial orientation patch are seen as a candidate for substituting the noisy orientation patch.
iii. **Context based correction:** Finally in this phase, an optimal integration of candidate orientation patches is obtained by putting compatibility of close orientation patches into account.

[39] Championed an enhancement technique that is based on a novel filter algorithm design whose foundation lies on the stability of the orientation in order to boost the performance of Gabor filter so as to attain higher performance in distinguishing high curvature ridges in the noisy fingerprint image(s). This approach effectively selects adaptively the window dimension of Gabor filter relying on the consistency of the orientation.

The proposed technique tunes the filter window size by itself as against the application of the two fix-regions comprising a distinct threshold that selects to and fro the two window sizes; through application of orientations within a region that is concordant with the prevailing orientation [39].

[40] Advocated an algorithm that segments fingerprint images using the Fuzzy C-Means (FCM) with statistical features, which is an unsupervised clustering method. Unlike conventional segmentation techniques that use features such as gray variance, frequency elements or directional orientation to segment fingerprint images. To enhance fingerprint

images, [40] applied frame difference between original image, its variance and the mean of the fingerprint image which is obtained by the sliding window.

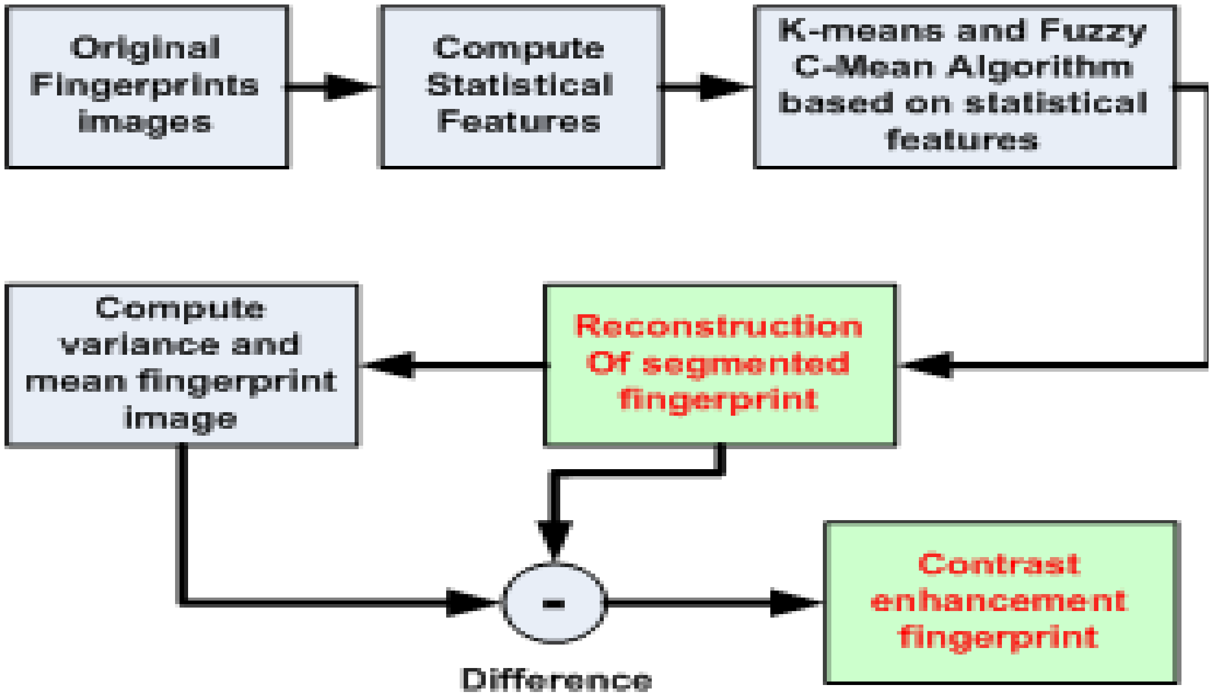

Figure 5: Flowchart of fingerprint segmentation and enhancement: source-[40]

Figure 5 shows the flowchart of fingerprint segmentation and enhancement as proposed by [40]. Their algorithm is based on three major processes for fingerprint image segmentation and enhancement:

i. Calculation of the statistical features: The mean, the variance, 3rd and 4th order moments, applying fuzzy c-mean algorithm proposed by [41].
ii. Input the region of interest acquired from the original fingerprint image that has been segmented.
iii. Finally enhance the fingerprint regions using frame difference of the original image, variance and mean fingerprint image through sliding window.

[5] Posited the effectiveness of using Type-2 fuzzy logic in fingerprint enhancement when Hongᅆs algorithm has been used to pre-process the input. They proposed a technique that enhances a fingerprint image by contrast enhancement with type-2 fuzzy

set. The Fuzzy logic procedure for image enhancement utilizes either one of these methods:

i. Application of IF-THEN rules presented by [42].
ii. Modelling parts of pixel property using type-1 fuzzy set and finally performing some manipulations to alter it.

[5] Proposed the use of type-2 fuzzy logic system due to the limitation of type-1 fuzzy logic systems in management of uncertainties. The approach of [5] preliminarily applied the contrast intensification operator NINT whose foundation was cast on type-1 fuzzy set, after which type-2 fuzzy logic was applied to tune up the enhancement of the image.

[43] Extended the capability of Gabor filters in the enhancement of low quality images by proposing a model in which the local image is non-stationary as well as applying Short Time Fourier Transform (STFT) to it. This is because Gabor filtersô function are limited in low quality images as conventional methods are adopted in the estimation of frequency map as well as unreliable orientation. [43] Approach advances mathematical probability theory to obtain both orientation and frequency concurrently.

[44] Proposed a copyright protection scheme that applies human fingerprint images as watermarks. The algorithm consists of extraction of fingerprint image features and the creation of the share image applied to retrieve the fingerprint image. The fingerprint image had to be enhanced before feature extraction can be carried out so as to get clearer and accurate estimates of feature locations.

The scheme of [43] adopted fingerprint image enhancement to remove various kinds of noise that may distort the fingerprint image structures (valleys and ridges) to be used as watermarks and also normalize intensity variations. The scheme adopted filtering to enhance the fingerprint image; its phase of enhancement as shown in figure 6 involves the following five processes:

i. Application of normalization (this makes even the illumination on the fingerprint image) transformation in order to eliminate the effects of the sensor noise and/or finger pressure variation.
ii. Estimate the orientation (the angular direction of the ridgelines) of the fingerprint image for the Gabor filter transformation.

iii. Employ the Gabor filter (essentially bandpass filters with orientation and frequency selective properties) depending on the orientation as well as the frequency to boost the fingerprint image structure.

iv. Select the histogram equalization to enhance the fingerprint image structure and ultimately boost the properties of the image.

v. Binarization of the fingerprint to obtain binary image.

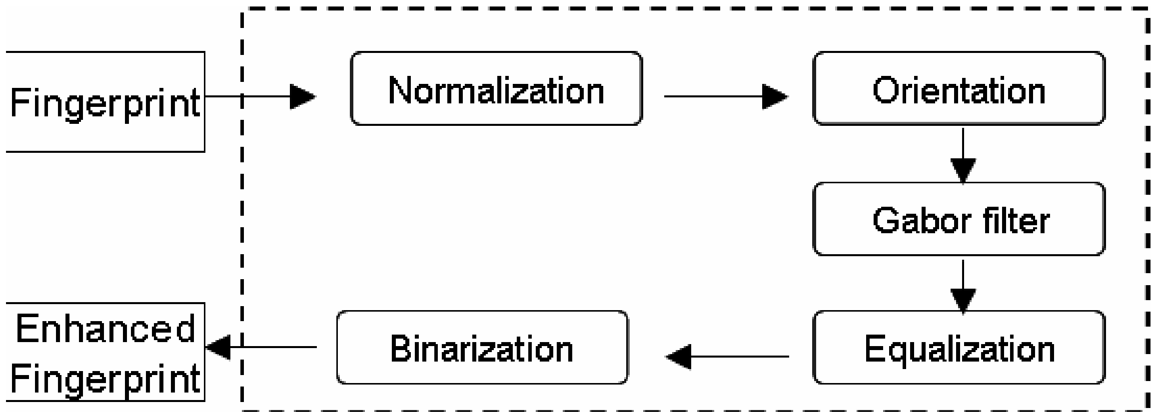

Figure 6: the process of the fingerprint image enhancement: source-[43]

[45] Presented a new, improved method for enhancing fingerprint images on the basis of the image normalization and Gabor filtering. The input fingerprint image is first adaptively normalized using block processing. The image is partitioned into sub-blocks with k x l size and then the **region of interest** (ROI) of the fingerprint image is extracted. The ridge direction and frequency parameters of the Gabor filters are selected using the probabilistic approach and directional projection respectively.

A block diagram of the proposed enhancement algorithm of [45] is shown in figure 7:

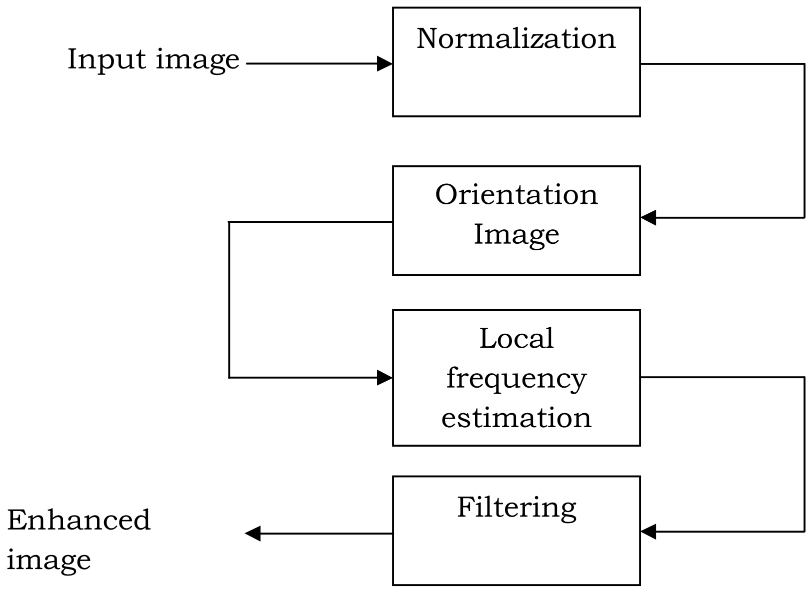

Figure 7: Block diagram of fingerprint image enhancement: source [45]

2.3 WAVELET PROCESSING

Wavelets are regarded as mathematical functions that slice up data [signal to be processed] into various frequency components so as to investigate each component with the purpose of matching the componentôs resolution to its scale [46].

The wavelet transform has been explained by many mathematicians as a mathematical method that analyzes or synthesizes a specific signal in the time domain through the application of different aspects of a contracted and translated basis expression known as *the wavelet paradigm or the mother wavelet.*

[47] Defined wavelet transforms as efficient mathematical kit or approach applied in multiresolution and local computation of non-stationary and rapid transient signals. Wavelets offer another method of attaining localization of Fourier transforms [i.e. signals consisting of components x-spatial component and u-frequency component] like Gabor transform which minimizes the localization uncertainty product of two functions æxæu.

$$F(u) = \int_{-\infty}^{\infty} f(x)\, e^{(-i2\pi ux)}\, dx \quad \text{------------------------} \quad (1)$$

Wavelets achieve this through the operation of a constant Q referred to as the *quality factor*.

$$Q = \frac{|u|}{|\Delta u|} \quad \text{------------------------------} \quad (2)$$

Where u is the frequency component of the signal being processed, Δu is the change in u. This implies that wavelet transforms offer an inverse relation between frequency and resolution; wavelet transforms are invertible. An integration of these properties crowns wavelet transform kings in comparison to Short Time Fourier Transform (STFT) and Wigner distributions. In order to obtain invertability of wavelet transform, we need to express that:

$$C_{\psi} = \int_{0}^{\infty} \frac{|\Psi(u)|^{2}}{u}\, du < \infty \text{ -------------------------- } (3)$$

Where Ψ (u) is the Fourier transform of function ψ(x); any ψ(x) that meets this *"admissibility condition"* is suitable as a *"mother wavelet"*. Consequently, we can obtain *"daughter wavelets"* through a mother wavelet as follows:

$$\psi_{a,b}(x) = \frac{1}{\sqrt{a}}\, \psi\left(\frac{x-b}{a}\right) \text{ -------------------------- } (4)$$

Where a = magnification and b = shift. We can then state the wavelet transform as:

$$WT[f(x)] = W(a,b) \text{ ---------------------------- } (5)$$

$$= \int_{-\infty}^{\infty} \psi_{a,b}^{*}(x) f(x)\, dx \text{ ---------------------- } (6)$$

$$= \sqrt{a} \int_{-\infty}^{\infty} F(u)\, \Psi^{*}(au) \exp(iub)\, du \text{ ---------------- } (7)$$

With the inverse wavelet transform defined as:

$WT^{-1}[W (a, b)] = f(x)$

$$= \frac{1}{C_{\psi}} \int_{-\infty}^{\infty} \int_{0}^{\infty} W(a,b)\, \psi_{a,b}(x) \frac{da\, db}{a^{2}} \text{ --------------- } (8)$$

But we have that:

$$\psi_{a,b}(x) = \sqrt{a} \int_{-\infty}^{\infty} \Psi(au') \exp(iu'(x-b))\, du' \text{ ---------------- } (9)$$

Substituting equation 7 into 9 we get that:

$$WT^{-1}[W(a,b)] = f(x)$$

$$= \int_{-\infty}^{\infty} du\, F(u) \left\{ \frac{1}{C_{\psi}} \int_{-\infty}^{\infty} \Psi^{*}(au)\, \Psi(au') \times \int_{-\infty}^{\infty} \exp(ib(u - u'))\, db \frac{da}{a} \right\} \times \exp(iux) \text{ --------- } (10)$$

Observe that in equation 10,

$$\int_{-\infty}^{\infty} \exp(j\omega(t - t'))\,d\omega = 2\pi\delta(t - t') \qquad (11)$$

So as to make the contents of {} equal unity to enable equation 10 denote the inverse Fourier transform, it is pertinent to find:

$$C_{\psi} = \int_{-\infty}^{\infty} \frac{|\psi(au)|^{2}}{u}\,du < \infty \qquad (12)$$

Equation 12 is identical to equation 3 which is the wavelet admissibility condition. From the above mathematical derivation, it has been shown that the inverse wavelet transform exists in a continuous form. Taking a close look at the admissibility condition, we see that: $\Psi(0) = 0$ for all admissible $\psi(t)$. Hence $\psi(t)$ cannot possess a '*dc*' component. Therefore a localized function without a dc component is called '*little wave or wavelet*'.

[48] Shows that for digital computation, it is conventional to apply wavelets of the form:

$$\psi_{j,m}(x) = 2^{-\frac{j}{2}}\psi[2^{-j}x - m] \qquad (13)$$

This type of wavelet is referred to as '*dyadic*' [i.e. an integer of second order]. The beauty of dyadic wavelets lies in the fast wavelet transform with unparalleled simplicity. The complexity of its calculation is just an order of N, precisely *$0(NLog_2L)$*, where N is the length of the data vector and L is the length of the digital filter [49]. In comparing wavelets to the ultra efficient fast Fourier transform (FFT) we find that its complexity is of *$0(NLog_2N)$* form.

[50] Presented a new technique of wavelet transform algorithms with FFT as its foundation after the analysis of non-compact-supported orthogonal wavelet. [50] Describes wavelet as being a small wave and as a result feeble. '*Wave*' in this context describes its volatility. When scaled and translated, a function ψ yields another function of cluster [ψ_{ab}]: One function is denoted as the basic or mother wavelet while that function cluster [ψ_{ab}] determined from the function ψ is denoted as daughter wavelet or simply wavelet, a is the scale parameter; which varies the bandwidth and decides the wavelet transform's frequency information. The location parameter is denoted as b; gives the outcome of the transformation of the time domain information. This relation portrays the wavelet function as a function that is composed of both time domain and frequency domain position features [50].

Currently, the so called wavelet basis or wavelet function contains triangular hermite wavelet, orthogonal periodic quasi wavelet, Beylkin wavelets, Coifam wavelet, Vaidyanathan wavelet, vector wavelet and other high dimensional wavelet basis [50].

Wavelet processing can be viewed as a deliverable in the history of the evolution of Fourier analysis as presented by [51].

[50] Observed that though wavelet processing stemmed from Fourier analysis, it has surpassed Fourier analysis. He goes further to comment that the wavelet transform of a signal is normally concluded by the convolution of the signal with its wavelet functions. On the contrary, the analysis of a non-compact- supported (or part of the compactly supported) spectral characteristics of orthogonal wavelets and wavelet transform properties discovered that the wavelet transform and its inverse can be obtained through Fourier transform. This shows that fast wavelet decomposition and synthesis can be done using FFT. A detailed derivation of the above relationships is given in [49].

[52] Presented wavelet transform as a technique or process that translates a given function through its integration with altered models of some *kernel function*. [53] Described the kernel function as the mother wavelet, and the alterations as translations and compressions (or dilations) of the mother wavelet.

For a function to qualify as a mother wavelet, it must satisfy the admissibility condition which describes a function **g** = $\in L^2(R)$ as admissible if [54]:

$$C_g \equiv \int_{-\infty}^{\infty} \frac{|G(\omega)|^2}{|\omega|} d\omega < \infty - (14)$$

Where G(ω) is the Fourier transform of g(t), L^2(R) is the set of all square integrable or finite energy signals and R represents the real numbers. C_g is the admissibility constant of the function g(t), and the constraint that it is finite permits inversion of the wavelet transform. From equation 14, it can be noted that any admissible function may be a wavelet. [50] Listed properties of wavelet transforms as follows:

i. **Interchanging f(t) and g(t):** this property reverses the roles of f(t) and g(t), where f(t) and g(t) are admissible.

$$W(a, b) = C_g * W\left(\frac{1}{a}, \frac{-b}{a}\right) - (15)$$

ii. **The correlation structure:** let $\psi_a(t) = \frac{1}{\sqrt{|a|}}\psi^*\left(\frac{-t}{a}\right)$. Then the wavelet transform $W_f(a,b)$ is a correlation between f(t) and $\psi_a(t)$, i. e. let $\oplus$ denote correlation. Then, $f(t)\oplus\psi_a^*(b) = \int_{-\infty}^{\infty} f(t)\psi_a(b-t)\,dt$ =

$$\frac{1}{\sqrt{|a|}}\int_{-\infty}^{\infty} f(t)\psi^*\left(\frac{t-b}{a}\right)dt - (16a)$$

$$= W_f(a,b) - (16b)$$

iii. **The resolution of identity [54]:** this characteristic notes that the wavelet transform is an isometry [- a transformation that is invariant with respect to distance] from one Hilbert space to another. Let $f_1, f_2 \in L^2(R)$, $\psi(t)$ be admissible, while $a \neq 0$, then

$$< f_1, f_2 > = \frac{1}{C_\psi}\int_{-\infty}^{\infty} < f_1, \psi_{ab} > \int_{-\infty}^{\infty} < \psi_{ab}, f_2 > \frac{da\,db}{a^2} - - - - - - - - - - (17)$$

A form of Parsevalôs identity yields for the exceptional case where $f_1 = f_2 = f$; the identity resolution states that the energy contained in time domain and transform domain are the same [53]:

$$\int_{-\infty}^{\infty}|f(t)|^2 dt = \frac{1}{C_\psi}\iint_{-\infty}^{\infty}|W_f(a,b)|^2\frac{da\,db}{a^2} -------------------------------- (18)$$

on the condition that the integration is over $a \neq 0$

iv. **The inversion formula** [55]**:** let f, $\psi \in L^2(R)$, $\psi(t)$ be admissible, and $a \neq 0$. Then the inverse wavelet transform is:

$$f(t) = W_\psi^{-1}[W_f(a,b)] = \frac{1}{C_\psi}\iint_{-\infty}^{\infty} W_f(a,b)\frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right)\frac{da\,db}{a^2} - - - - - (19)$$

v. **The cross wavelet transform** [56]**:** the cross wavelet transform maps $W_\psi f(a,b)$ to $W_\phi f(a,b)$ and is expressed as:

$$W_\phi f(a,b) = \frac{1}{C_\psi}\int_{-\infty}^{\infty}\int W_\psi f(x,y)\,\psi_\phi^*\left(\frac{x}{a},\frac{y-b}{a}\right)\frac{dx\,dy}{x^2} - - - - - - - - - - (20)$$

vi. **The wideband ambiguity function as a wavelet transform** [57] [58]**:** gave the wideband cross ambiguity function of two signals as :

$$WB_{fg}(a,b) = \sqrt{|a|}\int_{-\infty}^{\infty} f(t)g^*(a(t-b))dt - - - - - - - - - - - - - (21)$$

Through changing variables in the definition of the wavelet transform, we obtain the wideband cross ambiguity function as a wavelet transform:

$$\square\ \square_{\square\square}\square\frac{1}{\square},\square\square = |\square|\,\square\int_{\square\infty}^{\infty}\square\square\square\square * \square\square\square - \square\square\square\square - - - - - - - - - - - - - (22\square)$$

$$= \square_{\square}\square\square\square,\square\square - (22\square)$$

vii. **The cross wavelet transform, group convolution and group correlation:** property E indicates a group theoretic correlation, where the group is the affine [or dilation- is a transformation that translates a parallel line to another parallel line] (ax+b) group [59] [60]. The cross wavelet transform is a convolution on the affine group, written as:

$$\square_{\square}\square\square\square,\square\square = \square_{\square}\square\square\square,\square\square * \square_{\square}\square\ \square\square,\square\square - - - - - - - - - - - - - - - - - (23\square)$$

Where $*w_b$ represents wideband (affine) convolution. Applying property I, the cross wavelet transform is also a correlation on the affine group. If $\oplus$ represents correlation on the affine group, then the cross wavelet transform can be expressed as:

$$\square_{\square}\square\square\square,\square\square = \square_{\square}\square\square\square,\square\square \oplus \square_{\square} * \square\square\square,\square\square - - - - - - - - - - - - - - (23\square)$$

Equation 23a differs from equation 23b in that the second wavelet transform $\square_{\square}\square$, is conjugated and the function, h, and mother wavelet, g, were reversed using proper i.

[61] presented wavelet transform as a solution to the dilemma that confronted the Short Time Fourier Transform which even though proffered a path that permitted extraction of time information as well as localization of various frequency components from a signal; the problem was how to cut the signal. The answer provided by wavelets lies in provision of a fully adjustable modulated window that does not require anymore cutting of signals. [60] recorded that towards the start of the 20th century, Haar started the first wavelet transform named after him. The Haar wavelet basis function contains integrated support in addition to integer coefficients. Mallat, Meyer and Daubechiesô research works attained lofty heights in signal processing with the application of wavelet transforms. Based on the pioneer works of Mallat about the relationships of the Quadrature Mirror Filters (QMF); pyramid algorithms as well as orthonormal wavelets [62] Meyer implemented a set of wavelet orthonormal basis functions, which has formed the backbone in a lot of applications [63].

A couple of years later, [64] proposed a set of wavelet biorthogonal basis function which found relevance in various fields particularly in image coding. Currently, a biorthogonal wavelet-based compression technique is widely used as the new standard for image compression; known as JPEG 2000 [65]. [61] Showed how to arrive at the wavelet transform equation from the traditional Fourier Transform (FT) as follows:

Given a signal f(t), it can be represented in the frequency domain using FT as:

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt - (24)$$

It is also possible to obtain the original signal f(t) in the time domain through the application of inverse Fourier transform under specific conditions as:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega - (25)$$

Similarly, discrete-time versions of direct as well as inverse forms of the Fourier transform can also be found. The exponential term of equation 25 shows the non-locality and the time-independence basis functions of Fourier analysis. This limits Fourier transform to ‘*time dependent*’ statistical characteristics signals. This implied that methods that can provide many applications are both time and frequency dependent. The Short Time Fourier Transform (STFT) was developed to solve this setback and also accommodate the analysis of aperiodic signals. The STFT is a windowed version of FT in which a signal f(t) is defined with a time interval (τ) through a sliding window w (centered at time τ) with a frequency ω as documented in [66], [67], [68], [69, and [70]:

$$STFT(\tau, \omega) = \int_{-\infty}^{\infty} f(t)w(t-\tau)e^{-j\omega t}dt - - - - - - - - - - - - - - - - - - (26)$$

From equation 26, it is evident that although the limit of integration is from $-\infty$ to $+\infty$, the analysis is always restricted to a section of the signal, contained in the range $[-\tau, \tau]$ of the sliding window.

Even though the STFT provides time and frequency information of a part of the signal, the choice of the window size is a major constraint. This is so because the smaller the window size, the better the time-resolution.

However, ‘the smaller the window size also, the more the number of discrete frequencies which can be represented in the frequency domain will be reduced, and therefore the more weakened will be the discrimination potential among frequencies” as stated by [71]. This limitation as to how to select the window size led to the wavelet transform.

Hence, analyzing the convolution of the sliding window with the exponential ($e^{-j\omega t}$) which is time dependent within equation 26’s integral yields:

$$F_{\omega,\tau}(t) = \int f(t)\, w(t-\tau)\, e^{-j\omega t} \tag{27}$$

If we substitute the frequency ω with a scaling factor a, and shift the bound window τ by a shifting factor b, we obtain the initial step that takes us to the *continuous wavelet transform* (CWT), as written in equation 27.

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\,\psi^{*}\left(\frac{t-b}{a}\right) f(t)\, e^{-j\omega t},\ a \in R \tag{28}$$

If we combine equations 26 and 28, we find the CWT as given by [72]:

$$W(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t)\, \psi^{*}\left(\frac{t-b}{a}\right) dt \tag{29}$$

Where f(t) is within the square integral functions space, $L^2(R)$. Similarly, the inverse CWT can be defined as [68]:

$$f(t) = \frac{1}{C_\psi} \int_{0}^{\infty} \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} W(a,b)\, \psi\left(\frac{t-b}{a}\right) \frac{da\, db}{a^2} \tag{30}$$

C_ψ factor is required for reconstruction purposes. The reconstruction can only be achieved if this factor is specified. This is referred to as the admissibility condition.

Generally, we can replace ψ(t) with X(t), permitting a wide range of choices, which can be applied to enhance specific properties of some applications [71], [73], and [74].

Note that the CWT form of equation 29 is very redundant; hence its implementation is not a priority.

According to [75] “The CWT technique expands the signal onto basis functions created by expanding, shrinking and shifting a single prototype function, which is named as mother wavelet, specially selected for the signal under consideration. This transformation decomposes the signal into different scales with different levels of resolution. Since a scale parameter shrinking or expanding the mother wavelet in CWT, the result of the transform is a time-scale representation. The scale parameter is indirectly related to frequency, when considered, the centre frequency of mother wavelet”.

To ensure that the wavelet transform succinctly supports localization in time, with the capability of capturing localization in time, and with the capability of capturing short duration spikes, a mother wavelet must meet the *zero mean value condition* as required of the transformation kernel [75].

A signal expressed as an orthogonal collection of real-value generated by the use of a compatible transformation of the original wavelet represents a wavelet expansion. Features and benefits of a family of wavelets are dependent on the mother waveletôs features. Hence, an expansion generated using two dimensional expansions of a signal yields a time-frequency localization of the input signal. Consequently, the majority of the signalôs energy is captured with a few coefficients. Scaling and translation procedures generate the basis functions of a wavelet transform [75].

A *dyadic-orthonormal wavelet transform* is generated when the scaling is of the second order; this provides a path to discrete wavelet transform [76], [77]. There is no redundancy in the discrete wavelet transform as long as there is an orthonormal property in the selected mother wavelet. This also yields a multi-resolution algorithm that decomposes a signal into scales with distinct time and frequency resolution [78], [79].

The primary notion in multiresolution requires the determination of average features and details of the signal using scalar products in addition to scaling signals and wavelets. Noise as well as spikes normally occupies high frequency bands, making it possible to eliminate them by decomposition through multiresolution into various layers. Mother wavelet functions are differentiated in the manner in which signal scaling and wavelets are defined [80].

[80] Documented that óThe continuous wavelet transform is computed by changing the scale of the mother wavelet, shifting the scaled wavelet in time, multiplying by the signal and integrating over all timesö. [80] Further discusses that ówhen the signal to be analyzed and wavelet function are discredited, the CWT can be realized on computer and the computation time can be significantly reduced if the redundant samples removed respect sampling theoremö.

The advent of the applicability of discrete wavelet transform to signal decomposition into frequency bands gave rise to its rapid deployment to signal analysis, and image compression [81].

2.4 IMAGE ENHANCEMENT USING WAVELET PROCESSING

[82] Presented a new approach that integrated wavelet transform with Gabor filtering in order to provide a more efficient technique for fingerprint image enhancement. [82] Described fingerprint as an impression of the pattern of ridges on a finger. In order to investigate the statistics of fingerprint minutiae, it is very vital to dependably extract minutiae from images of fingerprints. Hence, it is imperative to adopt image enhancement methods before extracting minutiae so as to get a better estimate of minutiae position. The technique proposed by [82] for image enhancement using wavelet transform and Gabor filtering applies wavelet transform to denoise and enhance the contrast between the ridge and background by using a map function to the wavelet coefficient set and then application of Gabor filtering to boost the ridge using orientation and frequency information. This approach consists of two main phases:

A) WAVELET DECOMPOSITION: [82] States that a discrete wavelet transform is applicable to the computation of multi-resolution modelling of signals. To determine the approximation of a function [or signal] $C(A_{\square\square\ \square})$, the coefficient of $(A_{\square\square\ \square})$ otherwise called the *trend;* can be found through the convolution of the input function cA_j with a low pass filter. To down sample, a column or a row is left out of two but since the signal or function in this instance is an image, we talk about retaining either a column pixel or a row pixel.

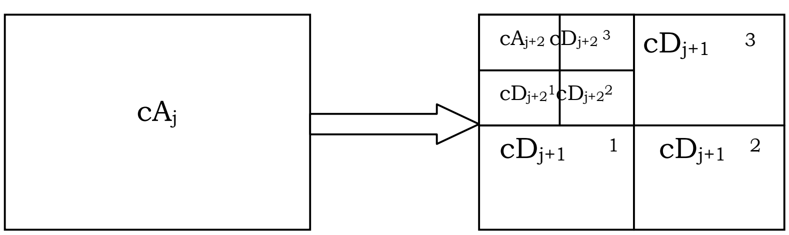

Figure 8: Level 2 decomposition of an image: source [82]

Iterating the process yields all the approximations, cA_{j+1}, $1 < j < J$

Where J = is the maximum scale.

cD_{j+1} = the difference of the input signal also referred to as the details at scale J.

To calculate cD_{j+1}, cA_j is convolved with the high pass filter and then retrieve every other sample of output. The effectiveness of wavelet transforms of a digital signal cA_j can be determined through continuous decomposition of cA_{j+1} into cA_{j+2} and cD_{j+2} for $0 \leq j < J$. This model informs us about approximation as well as the details of signals at various scales. It is simple to adapt this multi-resolution analysis to two dimensions through separate transformation in column and row directions.

The wavelet transform of any image is composed of four sub-images with each having a quarter of the original area of the image. The first sub-image is the approximation sub-detail while the three left over images with high frequency components are referred to as vertical, horizontal and diagonal detail images. This form of wavelet transform of two-dimensions precedes decomposition of coefficientôs approximation at j level in four components viz: the approximation coefficients and the other details of the three orientations (horizontal, vertical and diagonal) at j+1 level [83]. Figure 9 shows a level 1 decomposition of an image as illustrated in [81]. It shows the combined action of low pass and high pass filters on the initial image to generate the four sub images.

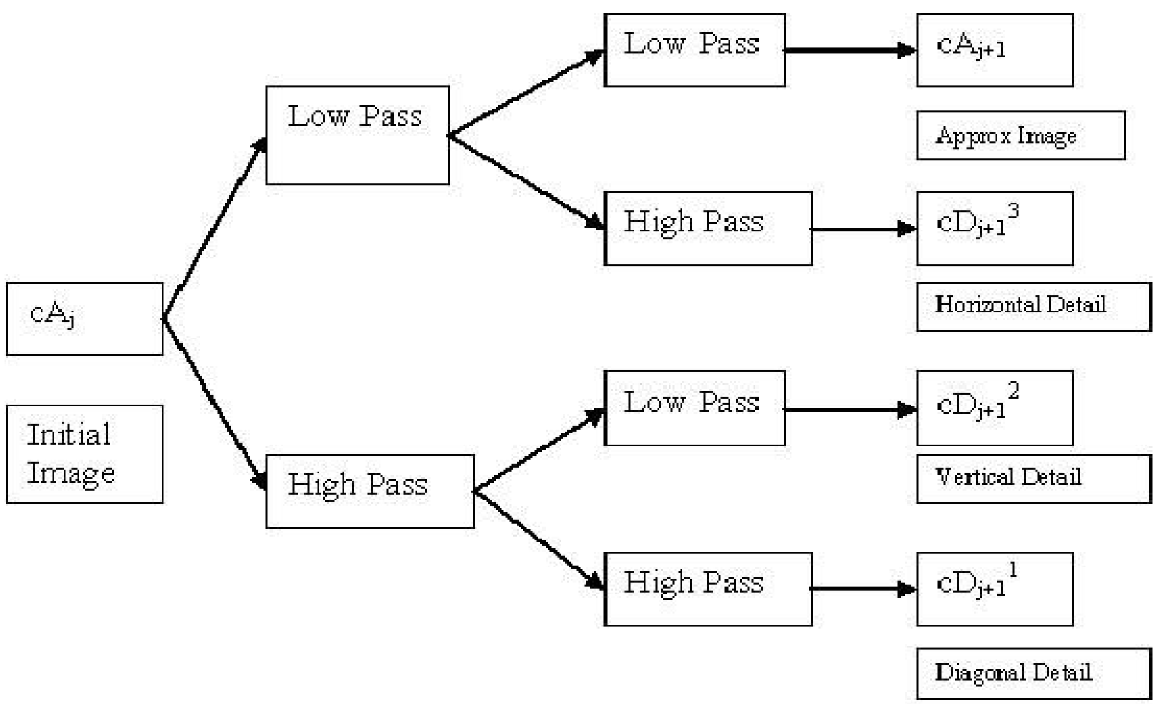

Figure 9: Level 1 decomposition of an image: source [81].

B) WAVELET COEFFICIENTS: The technique presented by [81] was based on the ability of wavelets as an effective approach in image denoising. It is evident that the application of wavelet transform to an image, generates in the transform a compact group of coefficients that can easily be applied in computation or calculation. Since wavelets rely on nested function spaces, they can be adopted in signal analysis at multi-scales.

From literature that has been discussed previously, we can observe that wavelet coefficients consist of time as well as frequency information while the basis functions change in both scale and location.

Discrete wavelet transform (DWT) is used to generate these coefficients at the output of a low pass filter.

i. **Normalisation:** This phase is employed to minimize the grey level value differences between the ridges and the valleys so as to obtain specific mean and variance of the pixel values contained in an image. The initial step is to calculate the mean (M) and variance (V) before applying equation 31 below to obtain the normalized values of the image.

$$G(i,j) = \begin{cases} M_0 + \sqrt{\dfrac{V_0 (I(i,j) - M)^2}{V_0}} & \text{if } I(i,j) > M \\ M_0 - \sqrt{\dfrac{V_0 (I(i,j) - M)^2}{V_0}} & \text{otherwise} \end{cases} \quad ----------------------(31)$$

Given that: I(i,j) = gray-scale value at pixel (i,j)

M_0 = desired mean

V_0 = desired variance

G = normalised image

The desired mean and variance can be assumed to be 0 and 1 respectively, in order to obtain a normalized image whose new intensities of the pixels lie between -1 and 1 (i.e. making standard deviation 1 on both sides of the mean), simplifying further computations.

ii. **Wavelet Decomposition:** In this step, the image is decomposed through the application of wavelet transform in several resolution representations. When the input signal is convolved with the different base functions, various results are generated in different resolutions. From the viewpoint of theory, it is attainable to decompose an image into sub images regardless of the level. On the contrary, it is not practically convenient to do so at too low resolution as too much down sampling of the signal may result in the disappearance of the ridge structure's orientation characteristic.

Conventionally, either one or two decomposition levels are chosen. The advantage of this choice is not just in retaining the ridge structures feature for matching to the filter, it also enhances the speed of the computation [84].

iii. **Wavelet Coefficient Adjustment:** A group of wavelets result when decomposition is completed with each group of the wavelet coefficient traceable to a resolution level of the dyadic scale. Application of gain to the group of wavelet coefficients is done through a non mapping function. This gain that is applied to the coefficients has a direct impact on the image enhancement after inverse transform computation. Consequently, the application of the gain should be restricted to the image coefficients without the noise elements because this will lead to a magnified noise visibility in the resultant image. A practicable trick is to adopt a threshold value for the coefficient values, discriminately boost the gain for values above the threshold while reducing the gain for those below the threshold.

iv. **Gabor Filtering:** This step eliminates noise and shields the ridge structures. The enhanced image is filtered by an array of Gabor filters – *band pass filters*, set to the specific local frequency and orientation for each group of coefficients. A Gaussian envelope is used to modulate a sinusoidal wave of a Gabor filter in spatial domain. Gabor filters possess both orientation and frequency selection characteristics which utilize the orientation and the frequency of the image in filtering the enhanced (normalized) image.

Gabor filter is generally denoted in a form that has δ_x and δ_y as standard deviations of the Gaussian envelope within x and y axes simultaneously. $\emptyset$ Represents the filter's orientation; f denotes the sinusoidal wave's frequency. The filter's equation is given below:

$$G(x, y, \emptyset, f) = \exp\left[-\frac{1}{2}\left(\frac{x_{\emptyset}^{2}}{\delta_x^{2}} + \frac{y_{\emptyset}^{2}}{\delta_y^{2}}\right)\right]\cos(2\pi f x_{\emptyset}) \quad (32)$$

Where $x_{\emptyset} = x\cos\emptyset + y\sin\emptyset$

$y_{\emptyset} = -x\sin\emptyset + y\cos\emptyset$

v. **Reconstruction:** This last phase adjusts the estimation sub image using Gabor filters and reconstructs the last enhanced image. It is common practice to apply the same wavelets previously used for the decomposition during reconstruction [84].

[75] Recorded that noise at varying levels affect digital images which can be traced to corruption during acquisition as well as transmission errors. He specifically stated that all medical image devices are noisy with particular reference to Magnetic Resonance Imaging (MRI) and Computed Tomography (CT) machines. The presence of noise in these images reduces significantly the image quality invariably making diagnosis as well as treatment complex.

The wavelet denoising method is a very efficient approach as the wavelet transform is capable of capturing the energy of a signal in scanty energy transform valves. Decomposing an image by wavelet transform generates four sub-images as previously stated; approximation, vertical, horizontal details as well as diagonal details. After decomposition at best suitable level, threshold gain application is done to selectively boost the required image information while eliminating the noise components. The denoised image can then be reconstructed using a convenient technique like Gabor Filtering.

[75] Concluded that wavelet denoising methods provide high quality and flexible approach to sort out the noise problem of signals and images. His research work examined several denoising techniques using several variations such as thresholding rules and wavelet type so as to establish effective denoising outcomes of wavelet processing. The experiments revealed that decomposition level is the principal factor in wavelet denoising and not the type of wavelet, threshold type or the estimation of threshold value [75]. He also stated that ñthe wavelet type is not very important if the oscillation number is not very low, the decomposition level absolutely depends on the frequency band of the signal to be analyzed and its sampling frequencyôö [75].

[85] Presented a proposal in which a new technique for magnetic image resonance (MIR) enhancement was discussed. This approach as they termed it is óa new wavelet based

adaptive enhancement algorithm for MRIô, was developed based on two non-linear adaptive rules that were applied to improve lower and higher frequency wavelet coefficients independently. The algorithm of [85] is a new self-adapting enhancement algorithm that relies on denoising over enhancement. Their algorithm consists of five main steps:

i. **Step 1:** Input an image; perform 2-dimensional discrete wavelet transform at 2 scales.

ii. **Step 2:** Calculate the lower and higher threshold parameters using: [illegible] = [illegible](2[illegible])[illegible] – (33)

Where [illegible]= lower threshold of low frequency after wavelet transform at scale L.

[illegible] = 0.25 is the modify factor

N = size of MRI

[illegible] = Median (M) / 0.675 is the variance of noise for MRI

[illegible] = 0.25 [illegible]

[illegible] = [illegible] $0 <$ [illegible] < 1 – (34)

Obtain adaptive parameter k through the formula

[illegible] = [illegible] > [illegible]

[illegible] = [illegible]

This step pre-removes noise using the wavelet coefficient.

iii. **Step 3:** Enhancement of wavelet coefficients through adaptive parameters.

iv. **Step 4:** Perform inverse two-dimensional discrete wavelet transform to obtain enhanced images.

v. **Step 5:** Post-remove noise after enhancement of wavelet coefficients [86].

[87] Proposed a technique for edge detection which was founded on, improved Gabor wavelet-based (IGWs) edge detector with the capability of detecting edges that have eight directions. The paper stated that Morphological operations are suitable for both image pre- or post- processing and such operations included: morphological filtering, pruning and thinning. The algorithm that they developed applied morphological operations to thin the edge images extracted using IGWs. Morphology as defined in [87] is óa tool for

extracting image components that are useful in the representation and description of region shape, such as boundaries, skeletons, and the convex hullö.

In the research paper titled; óA New Iris Recognition Method based on Gabor Wavelet Neural Networkô [88], presented a new technique for iris recognition founded on Gabor wavelet neural network as well as 2-dimensional principal components analysis (2DPCA). Gabor Wavelet Neural Networkôs (GWNN) algorithm extraction layer was applied in choosing the feature extraction approach and determining the optimum basal function parameter values of the wavelet. Their process tuned Gabor parameters adaptively using Gabor wavelet atomic transform function. As soon as it was established, wavelet methods in conjunction with Gabor filtering were then employed to detach the iris texture characteristics. The outcome of this process is an efficient and compact feature vector. The succeeding verification phase, which is the 2D principal component analysis (2DPCA) method in addition to the classification layer structure; percept row of GWNN that trail path layers of network were adopted for classification as well as reduction of dimensionality. Gabor wavelet neural networks were applied in the terminal network simulation investigations for classification. Simulation findings indicate an effective iris recognition performance.

[88] Documented that numerous correct iris recognition techniques applied multiscale and wavelet methods, which give adequate modelling for iris recognition.

The extracted iris image properties are required to contain [88]:

i. Reliable indicators showing that the extracted iris image features accurately denote the original iris image.
ii. Rapid computation of compact for the similarity measurement.
iii. Low redundancy that ensures rapid data storage and recognition.

Gabor Wavelet Neural Network as explained by [88] is óa perception with Gabor-node function as a pre-process unit for extraction of features. Network includes two parts: the first part of the feature extraction layer and the follow-up classification part of the structureâ.

The input layer $\emptyset$[illegible] consists of k nodes; K 1, 2. . . K, with $\emptyset$[illegible] indicating the Kth node of the Gabor basal function with respect to i group target signal vector.

$\emptyset_{[illegible]}$, k = 1, 2 . . . k, is the absolute value of the inner-product of the basal function vector [illegible] = [illegible], [illegible], [illegible] and objectives vector i.e.

$$\emptyset_{[illegible]} = |< [illegible], [illegible] >| = [illegible] \frac{1}{[illegible]} [illegible]^{*} \frac{[illegible] - [illegible]}{[illegible]} [illegible] \quad ----------- (35)$$

Explains the relationship between k node basis function as:

[illegible] = [illegible], [illegible], [illegible] and the i signal [illegible]

Gabor node:

$$[illegible] = [illegible] \left[\frac{[illegible]}{[illegible]}\right] [illegible] \quad -------------------------- (36)$$

It is allowed to tune the component basal function cluster, which is the deformation of Gabor basal function g(t) during the network training period.

Where: [illegible] = Time lapse parameters

[illegible] = Scaling parameters

[illegible] = The frequency modulation parameters

Gabor transform function is to be related to these variable parameters. "Sigmoid function is the exciting function of the hidden layer" [88].

Improved steepest descent algorithm was employed to obtain the minimal sum of square deviations obtained between the predicted output and the actual output of the input target vector.

At the study phase, it is imperative to adaptively tune the weight coefficients w of the hidden layer; its output layer and the parameters of Gabor basal function. Equation (37) below shows how to set the objective function;

$$[illegible] = \frac{1}{2} [illegible] - [illegible]^{2} \quad ------------------------- (37)$$

Where [illegible] = Vector comprised of the parameters of w_i, d, u, [illegible]

[illegible] = Network output defined by parameters for the vector [illegible]

Adjustment of [illegible] parameters is along the negative side of the gradient through steepest descent technique in each repetitive phase of the training period pending the convergence of the objective function. The determined Gabor atomic nodes form "the optimal-time-frequency-scale space for all the training samples" [88]. The next classification phase is performed by the Gabor basal function with decision-making through the parameters as they are the most dependable.

óOptimizated scale and translation parameters of the standards Wavelet transform (Ð vector), complete the effect to feature extraction and classification, improves the classification efficiency as well as removes the redundantô [88].

[89] Proposed an integration technique of speckle suppression and edge detection in ultrasonic images applying Gabor filters. The technique initially applies the logarithm to the ultrasonic images converting the multiplicative noise to additive noise in this instance. The speckle noises contained in the ultrasonic images are then eliminated through wavelet denoising method.

After this, the technique performs computation of the inverse wavelet transform after which inverse logarithm is performed. The edge detection process is carried out using the wavelet transform through the application of third and fourth scales Gabor filters. Combination of the wavelet transform coefficients of the edges candidates is done through the alternative to product in conjunction with shift technique in which the shifting edge locations by multiple scales are conceived for noise, presents robust edge detection.

The last step is the generation of the output through non maxima suppression.

Waveletôs capability to extract high frequency components contained in an image endeared them to edge detection application [89].

óGabor filters are complex exponential functions modulated by a Gaussian functionô [89]

A Gabor odd filter can be utilized as an edge detector. It can be defined as [90]:

$$h(x) = e^{-\frac{1}{2}\left[\frac{x}{\sigma}\right]^{2}} \sin(2\pi\omega_{0}x) - (38)$$

Where: σ = the special spread

ω_0 = the centre frequency

Wavelet processing techniques have been applied in image compression schemes since their initial acceptance as a compression standard based on wavelets referred to as the FBI fingerprint standard [91]. Its application areas have expanded from the FBI fingerprint compression standard to the recent JPEG 2000 standard.

Robust, rugged as well as powerful compression potentials of wavelets have been tapped in many applications such as off- and on-line, single images and image sequences compression, computer vision, robot vision and security systems. Their capability to denoise and compress images is significantly dependent on waveletsô coefficients [92].

The concept of applying wavelets in image resolution enhancement was derived from waveletsô compression capability, which entails the reversal of the process to obtain better resolutions. Super-resolution finds tremendous relevance in the military, security services, police, and the scientific community.

[93] Stated that ócontrast enhance is a very important task in image processing and that contrast is produced through the difference in the reflected luminance from two adjacent surfacesö. Rephrasing this definition implies that contrast is the visible difference that separates an object from its background as well as other objects. Contrast can be found through the colour and brightness difference of the *object of interest* (OOI) in comparison to other objects.

[93] Proposed an image equalization method that provides an even illumination of an image so as to prevent loss of information due to unbalanced contrast.

This technique comprises of three main sections:

i. **Application of SVD:** the determined singular value method through SVD contains illumination information.

ii. **Application of DWT:** illumination information is integrated in the low-low frequency sub-band with the edges located in low- high, high- low and high-high sub-band. Therefore dividing the high frequency sub-bands and by using illumination enhancement in low-low sub-band alone preserves the edge information from likely corruption. [94].

iii. **Reconstruction:** the inverse discrete wavelet transform (IDWT) is applied to reconstruct the final image which is enhanced with regards to illumination and also appears sharper.

To apply this proposed technique, this general algorithm can be adopted:

i. Process the input image x through the application of general histogram equalization (GHE) to obtain ▯

ii. Transform both images x and ▯ with discrete wavelet transform into four sub-band images.

iii. Use this expression for correction coefficient determination for singular value matrix:

$$\xi = \frac{\max(\Sigma_{LL_{\hat{A}}})}{\max(\Sigma_{LL_{A}})} \text{ ------------------------------ } (39)$$

Where $\Sigma_{LL_{A}}$ = the low low frequency sub-band singular value matrix of the input image

$\Sigma_{LL_{\hat{A}}}$ = the LL singular value matrix of the output of the GHE

The resultant LL image is comprised of:

$$\overline{\Sigma}_{LL_{A}} = \xi \Sigma_{LL_{A}}$$

$$\overline{LL}_{A} = U_{LL_{A}} \overline{\Sigma}_{LL_{A}} V_{LL_{A}} \text{ ------------------------------ } (40)$$

The $\overline{LL}_{A}$ and LH_x , HL_x, and HH_x subband images of the original input image are recombined using IDWT to produce the output equalized image $\overline{A}$.

Where LL_x = low-low frequency subband image

LH_x = low-high frequency subband image

HL_x = high-low frequency subband image

HH_x = high-high frequency subband image

$\overline{A}$ = IDWT [$\overline{LL}_{A}$, LH_x, HL_x, HH_x]

The paper chose Daubechies (db .9/7) wavelet function as the DWT mother function.

[95] Emphasized the importance of image denoising as a pre processing step in many image applications ranging from minor image segmentation to complex robotic visions. The purpose of denoising images is to attenuate noise to the minimum level without loss of important characteristics like edges and fine details.

The fundamental wavelet denoising challenge comprises: a noisy input image, separation of the entire wavelet coefficient into important (if larger than a threshold value) or unimportant (if smaller than a threshold value) and finally the manipulation of these coefficients emanating from each group using specific rules. Soft or hard thresholding are adopted in most wavelet denoising. The idea is to provide a filtration approach in which the wavelet coefficients are compared to a chosen threshold and those with smaller magnitudes than the threshold are discarded else it is left undisturbed-hard thresholding rule or shrinked-soft thresholding rule.

[96] Research results indicate that the optimal soft-thresholding estimator provides a smaller estimation error than the optimal hard-thresholding estimator.

[96] Applied a sub-optimal combinational optimization algorithm in the approximation of the optimal Maxima-a-Posteriori (MAP- is a popular stochastic process used in super resolution analysis of images) solution, Game Strategy Approach (GSA) Shrink narrows to a local maximum making their approach suitable for various initializations. Combination of Bilateral filtering and GSA Shrink indicate that the MAP-Markov Random Field (MRF- is a fingerprint image enhancement technique that extracts singularity points and minutiae from the fingerprint image prior to enhancement) technique researched by [96] can reduce noise as well as protect the most important image features and prevents the presence of visible artefacts. Mathematically speaking, the wavelet transform óis an expansion that decomposes a given signal into a basis of orthogonal functionsô [95]. The wavelet transform applies non-periodic, smooth and finite support basis function *(like Haar and Daubechies)* permitting a better representation via multiresolution computation; unlike the Fourier transform that adopts periodic, smooth and unlimited basis functions *(e.g. cosine and sine functions)*. [96] tackled the primary limitation of MAP-MRF which is the absence an analytical solution for MAP estimation through the application of a modified version of an optional algorithm; Game Strategy Approach (GSA) referred to as GSA Shrink. GSA is founded on non co-operative game theory ideas and initially presented for finding solution to MRF image labelling challenges.

2.5 FINGERPRINT IMAGE ENHANCEMENT USING WAVELET PROCESSING

[97] Discussed a new fingerprint image enhancement technique founded on the integration of Directional Gaussian Filtering and Non-Sub-sampled Contourlet Transform (NSCT). The method applied bandpass filtering to remove non uniform illumination that usually characterized fingerprint images and also to generate the frequency of the ridge image. Elimination of directional noise was performed using directional Gaussian filter prior to the decomposition of the image into Non-Subsample directional images through

NSCT. The construction of the enhanced image was carried out through, block by block process which selects the image with maximum energy through comparison of all directional imagesô energies. The enhanced image was binarized and thinned to make it shaper.

In [97], Gaussian filtering was employed to smooth the image while directional analysis was performed using the Non-subsampled Contourlet Transform on the smoothened image; reconstruction of the enhanced image was then implemented with directional energies.

Binarization and thinning were then carried out on the enhanced image.

The proposed technique of [97] consists of the following steps:

i. **Non-Uniform Illumination Correction:** In this step, non ideal Butterworth bandpass filter was applied to not only filter through the frequencies but also to stop the non-uniform illumination that is concentrated at the low frequency region, while its upper stop-band region effectively removed the high frequency noise. The bandpass was realized through Discrete Fourier Transform (DFT) of the input image. Inverse DFT was performed after filteration to convert the filtered image to its original spatial domain.

ii. **Segmentation:** This step separates the ridges and valleys from the background as it does not have any valid fingerprint information. This was done using variance thresholding based on non-overlapping blocks where the computation of variance of non-overlapping block is performed. The variance if smaller than the global threshold is discarded as background but classified to foreground if greater than the global threshold. This non-overlapping block thresholding is plagued by the blocky effects; hence the need to remove the introduced blocky effects.

iii. **Noise Removal Using Directional Gaussian Filtering:** This step smoothens as well as reduces noise through the application of Gaussian filtering. The concept of directional Gaussian filtering involves the division of the image into non-overlapping blocks with each non-overlapping block having equidistant direction and number of directions k, as chosen by the user is passed through a Gaussian Filter. The general equation of the Gaussian Filter is [97]:

$$G(x, y) = \frac{1}{2\pi\sigma_{x}\sigma_{y}} \exp\left\{-0.5\left(\frac{x^{2}}{\sigma_{x}^{2}} + \frac{y^{2}}{\sigma_{y}^{2}}\right)\right\} - - - - - - - - - - (41)$$

Lastly, the direction ($\propto$) with maximum variance is adopted for smoothing, and the filter in $\propto$ direction is written as:

$$f(x, y, \alpha) = \frac{(x\cos\alpha + y\sin\alpha)^{2}}{\sigma_{x}^{2}} + \frac{(-x\sin\alpha + y\cos\alpha)^{2}}{\sigma_{y}^{2}}$$

$$G(x, y) = \frac{1}{2\pi\sigma_{x}\sigma_{y}} \exp\left[-0.5 f(x, y, \alpha)\right] - - - - - - - - - - (42)$$

iv. **Blocky Effect Removal:** The introduced blocky effects from non-overlapping block thresholding is removed in this step through the use of a high pass filter at the output of the preceding step and finally subtracts it from its weighted version through the application of equation (41) [97].

High = high-pass filter (output smooth, cut-off, order)

Output = 2 x output smooth ï high---------------------- (43)

v. **Creation of Non-Sub-samples Directional Images:** Non-Subsampled Contourlet Transform (NSCT) was applied to produce directional images as its output is equal in size to that of the input mage; hence there isn't need for interpolation since there is a one-to-one pixel matching of all the directional images. NSCT was conceived on the idea of Non-Subsampled Laplacian pyramid [98] as well as Decimation-Free Directional Filter Banks (DDFB), [99] alternatively called Non-Subsampled Laplacian Pyramid. Non-Subsampled Laplacian Pyramid is adopted in decomposing an image into low- and high- pass components, there after the application of Non-Subsampled Directional Filter Bank is done only to the high pass images so as to extend the decomposition of the frequency spectrum.

When these steps have been performed, the enhanced image was then reconstructed using directional energy of the images. Binarization was applied to the enhanced image through global thresholding which compared the grey level of each pixel in the enhanced image to the global threshold. The final step is thinning of the image prior to extraction of minutiae.

[100] Proposed a technique to enhance fingerprint images founded on a rule-base approach. They pointed out that false estimation of local ridge direction causing poor enhancement, as the major limitation of most enhancement algorithms that rely on local ridge direction.

The rules as proposed will be formed on human expertsô knowledge of fingerprints, as well as mimic what the expert does to improve the fingerprint image. The algorithm further integrates the binary image, the skeleton image of ridges and valleys.

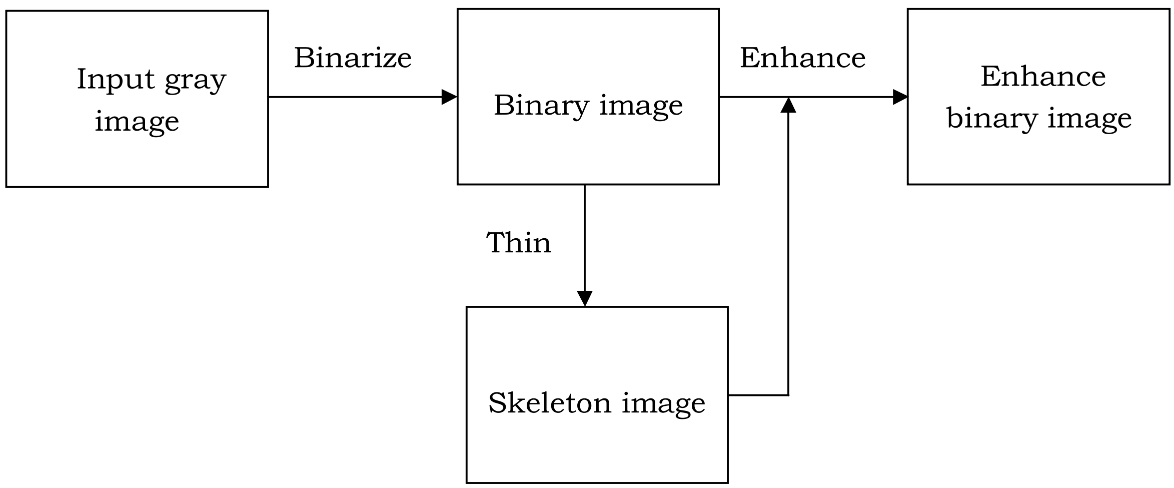

Figure 10: Block diagram of knowledge based fingerprint image enhancement: source [100]

An overview of the proposed system is given in figure 10 above. The fingerprint is acquired as an input gray scale image. This input image is converted to binary image through binarization which is then thinned to obtain the skeleton image. The skeleton image contains ridge connection pattern which is then utilized in the enhancement of the binary image [100].

Fingerprint image enhancement is a crucial step in any automatic fingerprint based biometric system that performs identity verification [101].

In [101], a new algorithm for fingerprint image enhancement was presented; that is capable of improving the clarity, continuity of ridge and valley structure. The technique entails decomposition of a normalized fingerprint into approximation and sub images, after which estimation of the approximation imagesô orientation is performed. Lastly, the Gabor filter was applied to enhance the image in the wavelet domain and then reconstruct the fingerprint. The enhancement algorithm in wavelet domain as proposed by [101] consists of the following major steps:

i. **Normalization:** This step modifies the input fingerprint imageǿs gray level values in order to provide a particular mean and variance values.

ii. **Wavelet decomposition:** Here, decomposition is performed on the normalized image yielding an approximation detail as well as three detail sub images.

iii. **Block orientation estimation:** Raoǿs technique was employed to determine the local orientation of all pixels contained in the approximation sub images. This local orientation of approximation is adjusted to local ridge orientation while ridge frequency is applied to the ridge and valley pixel in the approximation in addition to the sub-images to get enhanced images in the wavelet domain.

iv. **Wavelet reconstruction:** The enhanced images are in wavelet domain hence to obtain the outcome enhanced image, wavelet reconstruction is done.

v. **Gabor filter enhancement:** Used in wavelet domain to remove noise and preserve true ridge/valley structures due to their orientation- and frequency-selective properties with optimal joint resolution in spatial and frequency domains.

The above proposed method of [101] saves computational time and resources as well as improves visibly the quality of fingerprint image.

In [102], an algorithm that utilized directional wavelet transform in conjunction with second derivative Gaussian filter to enhance fingerprint images was showcased. An overview of the technique is discussed below.

An original fingerprint image is either acquired directly from a scanner or digital camera or from a database and then decomposed to obtain both approximation and detail sub-images. A second derivative of Gaussian filter is employed to adjust the image properties of each segment. Poincare method is then adopted to check its enhanced profile through the success of its core point identification.

[102] Described fingerprint as ǲthe pattern of ridges and valleys on the surface of a finger. Minutiae are local discontinuities in the fingerprint patternǳ. The major minutiae on a

fingerprint are the ridgeline endings and valleys together within a local region constitute a sinusoidal-patterned plane wave that is characterized by a clear frequency and orientation. The need for fingerprint image enhancement was also discussed by [102] as they are riddled with a lot of discontinuous ridges [like smudges, and scars] hence the need to enhance the images in order to determine valid minutiae and discard false ones.

The primary objective of fingerprint image enhancement algorithms is to aid recognition by increasing the clarity of the ridge patterns in fingerprint images in valid regions and eliminate those in invalid regions.

The algorithm of [102] applied a directional wavelet transform that decomposed the fingerprint image into its orientation model; directional filtering was then used to smooth each direction of the images prior to reconstruction of the image.

The second derivative of Gaussian filter which is a linear extension of the first Gaussian filter was adopted and it holds the properties that rely significantly on both the ridge frequency and orientation.

An algorithm that is based on wavelet application to fingerprint image enhancement was discussed in [103]; the work adopted wavelets for fingerprint image enhancement because of their spatial localization feature and their promise with oriented wavelets like Gabor wavelets for estimating orientation flow. The proposed technique is composed of two processing phases: smoothing and Gabor wavelet filtering.

The smoothing phase of the enhancement adopted Gaussian filtering to suppress noise to an acceptable level. Gaussian filters were selected due to its conformance to the uncertainty principle at its limits.

Gabor wavelet filtering was applied to improve the quality of the smoothed image in the algorithm due to its selective properties in both frequency and orientation estimation. The algorithm was stated to be independent of the orientation.

[102] Stated that "fingerprints are basically oriented texture fields of quasi periodic and smooth pattern of ridges and valleys having dominant frequency that reside in mid frequency range (1/2~1/4 pi). Ridge orientation, ridge spatial frequency and more significantly the structure of minutiae and their distribution in the fingerprint image, are the main intrinsic features of a given fingerprint".

The algorithm presented in [103] has two steps which are further subdivided as follows:

Step1:

- Application of Gaussian filters to the image in equidistant directions with the user specifying the number of directions k.
- A k x k block division of the image is performed, with k selected so as to preserve the ridgesô frequency.
- The filtered block contained within the orientation of the ridge is selected; reconstruction of the final image is performed by combining these blocks.

Step 2:

- Segment the image into a k x k block.
- Apply Gabor wavelet decomposition to the direction orthogonal to the direction that was selected in step 1 and subsequently at different scales; the final enhanced image was then reconstructed using the adjoining chosen filtered blocks.

[104] Developed an algorithm for image enhancement through a wavelet based nonlinear filtering scheme. It is common knowledge in digital signal processing that wavelet transform is applied in the decomposition of an image into a finite number of resolution scales which is very compatible or applicable to image analysis and image compression. This resolution decomposition is relevant also in enhancement of noisy images; this is feasible as generally, majority of visible noise lie in the low and high frequency region which are easily eliminated by bandpass filtering using wavelet transform.

The technique proposed by [104] applied wavelet transform to decompose and reconstruct images. Then non linear piecewise-quadratic [PWQ] filters were employed to process decomposed progressive high frequency wavelet components prior to reconstruction. [104] deliberated on image enhancement through the application of wavelet based non linear filtration. It has been established, that the decomposition of images into a limited number of resolution scales by wavelet transform is very important in image analysis as well as compression. This decomposition is also effective in noisy image enhancement as it practically isolates noise in the low- and high- frequency components of the wavelet transform. This filtration approach was shown to be effective for noise smoothing, preservation of details and also the visual quality of images.

The PWQ filter applied in this method, is a form of non linear filter that was deduced from canonical PWQ functions, which is an extension of canonical piecewise-linear [PWL] functions. The piecewise feature of the filter enables it realize accurately weak and strong non-linearity. In [105], it was documented that canonical PWL are universal approximants; since PWQ filters extend PWL filters, they are more robust in their capability to perform approximations [106].

[107] Developed a new algorithm that integrated the stationary wavelet transform (SWT) with morphological operations, quick mask, spatial filtering for enhancing fingerprint images.

The composite algorithm has five main phases organized as:

i. **Morphological Operation:** From biology, morphology refers to structure and shape of living things. Dilation and erosion morphological operations play a significant role with the structuring element. Morphology in mathematics is the comparison of the objects in an image with a reference object referred to as *structuring element (SE)*. Conventionally, in fingerprint image enhancement, the dilation ($\oplus$) and erosion (?) are commonly applied in conjunction with similar or different structuring element. This process is employed to segment the fingerprintôs foreground area from the background and also eliminate redundant information.

ii. **Decomposition with SWT:** Stationary wavelet transform was applied to the segmented image in order to decompose it into multiresolution representation. Daubechies wavelet at level 1 and 2 are used so as to preserve the orientation characteristics of the ridge structure and also obtain a better result.

iii. **Quick mask:** This phase detects the edges. The quick mask was performed on the sub diagonal image. The use of mask alternatively known as filters or kernels or templates or window is one of the primary techniques that are applied in edge detection. The two main rules that govern edge detector mask are ï the sum of the mask numbers is zero and the mask ought to round off differentiation or magnify the slope of the edge. Quick mask pioneered by Dwayne Philips is one of the most relevant masks [108]. Other popular masks that are used for edge detection include: Kirsch, Sobel, Laplacian, Roberts, Prewitt and Sobel. These entire masks require the application of eight different masks, in order to find all eight directional connectivity neighbours in an image except quick mask. The name quick mask, originated from its ability to detect all eight edges in one convolution. This offers a great speed benefit. [107] employed quick mask through spatial filtering on diagonal sub image.

iv. **Reconstruction:** The image was reconstructed after adjusting the diagonal coefficients sub image with quick mask. The inverse of SWT was computed from the processed data to obtain the reconstructed image.

v. **Spatial filtering:** This final phase provides enhancement of smaller regions and details by the application of spatial filter to the reconstructed image.

[109] Proposed a new algorithm for fingerprint image enhancement that is based on non-separable filter banks. The image is initially decomposed by non-separable filter banks, producing wavelet coefficients. The adaptive technique was applied to coefficients, so as to eliminate noise as well as boost the contrast of the ridges and valleys with regards to the geometric property of the images. The inverse wavelet transform maps the result with a two dimensional median filter employed to reduce noise and protect the edges.

According to [109], fingerprint image enhancement should perform the following functions:

i. Sharpen the contrast between the valleys and ridges.

ii. Magnify local ridges along the ridge orientation.

iii. Promote detection of points of interest [such as singularities, bifurcations and ridge endings.] as well as preserve genuine minutiae points after enhancement.

iv. Should not invert nor alter the ridge, valley structure and minutiae type.

v. Enhance the clarity of the ridge in the recoverable regions and highlight unrecoverable areas as irrelevant areas for spurious minutiae filtering.

The proposed new fingerprint enhancement algorithm of [109] primary objective was to obtain a clearer ridge structure in order to speed up feature extraction process. The algorithm consists of three main phases: decomposition, wavelet coefficient adjustment and reconstruction.

i. **Decomposition:** The non-separable wavelet filter bank is applied in this phase to decompose the fingerprint image into a multiresolution representation. N is determined by the image size and N pairs of parameter $(\square_{\square}, \square_{\square})$, k = 1, ..., N. It is convenient to choose either 1 or 2 decomposition level although theoretically, we can decompose at any level. This is to protect the orientation features of the ridge structure in order to suit the filtering and speed up computation. Decomposition using the non-separable filter bank generates four sub images A, H, V and D at each level.

ii. **Wavelet Coefficient Adjustment:** The trend coefficients (A) are undisturbed while (H, V, and D) fluctuation coefficients are modified. For a corresponding set (H, V, and D) in the decomposition at level j, the following manipulations are carried out:

   a) Normalization of coefficients and
   b) Adjust the normalized coefficients.

iii. **Non Separable Wavelet Reconstruction:** The output reconstructed image is generated when the modification of the sub image approximation with Gabor filters is finished. The same wavelets applied in the decomposition process are also used in reconstruction.

The research conducted by [110] produced a new algorithm for pre-processing fingerprint images based on morphological wavelet transformation. The technique featured a set of flat structuring elements generated from the morphological wavelet transformation and adopted to adjust high frequency coefficients. Consequently, filtering and enhancing the image without altering the fingerprintsôoverall pattern.

[110] Stated that wavelet transformation uses specific wavelet function to decompose an image at different resolution levels, which is able to remove noise, but the low pass filter adopted for wavelet decomposition will smooth the edge of the image.

[110] Presented morphological wavelet theory as part of orientations of non linear expanding research on wavelet theory. It is founded on the non linear features of mathematical morphology, the complete reconstruction and non-redundancy properties of mathematical morphology adapted to morphological decomposition of images.

Morphological wavelet theory integrates nonlinearity of wavelet transformation and nonlinearity of morphological operation. Hence it possesses both morphological filtering characteristics of mathematical morphology in addition to the multiresolution property of wavelet transform. This compensates for the shortfalls of wavelet analysis in nonlinear transformation yielding improved resolution and noise discrimination.

Filtering and enhancement of fingerprint image based on morphological wavelet transformation according to [110] involves the following:

i. Structure of morphological wavelet.
ii. Formation of flat multi-structuring elements and
iii. Filtering and enhancement and inverse transformation of morphological wavelet.

[111] Described an efficient fingerprint matching approach through the application of wavelet based features. It is a form of image based processing method in which the initial core points is sensed by hybrid method. The wavelet is done on a narrower area surrounding the core point. The technique adopts three properties that depend on energy, standard deviation and Fourier Mellin Transform. These three properties are retrieved from the particular cropped region uniquely.

Distance based classification within the four distances was employed for classification. The idea of voting was incorporated because of the use of the four distances; it is a new approach which also conserves a significant amount of calculation time and resources. In conclusion, authentication is done if needed at the end.

[112] Performed a research work titled óA wavelet based method for fingerprint image enhancementô. The work was founded on wavelet decomposition; it incorporated texture filter process with wavelet transform to generate a better, dependable and effective image enhancement.

[112] Defined fingerprint image as óthe pattern of ridges and furrows on the surface of fingertips. The local ridge characteristics and their relationship determine the uniqueness of a fingerprintô. [113] noted that the local ridges and valley descriptions of the fingerprint round up to a total of eighteen distinct kinds but not most of them are applied in matching. Generally, it is just the ridge ending and bifurcation together called *minutiae* are adopted for fingerprint representation.

The process proposed by [112] is an integration of Gabor filter and the wavelet transform with particular reference to multiresolution analysis. It comprises of three major steps:

i. **Phase 1:** This phase applies the wavelet transform to the fingerprint image in order to decompose it into sub-images.

ii. **Phase 2:** Gabor filter is adopted in this step to smooth the approximation sub-images.

iii. **Phase 3:** In this phase, the enhanced image is obtained by inverse wavelet transform of the sub-image.

2.6 SUMMARY OF WAVELET TRANSFORM

From the overview of wavelet transform of [112] a summary or wavelet transform is given as:

- Wavelet transform (WT) is a transformation that translates a signal to a multiresolution representation.
- Wavelets relate better to the human sense than other types of transformations e.g. Fourier transform.
- WT features make it indispensable in signal and image processing particularly in image compression and denoising applications.
- Some elegant techniques based on wavelet are capable of yielding effective outcomes than most traditional approaches.
- A wavelet can be seen as a timescale technique integrated with the property of frequency.

Mathematically, a continuous one dimensional wavelet transform (CWT) is the decomposition of a signal f(t) into a collection of basis functions $\psi_{a,b}(t)$ known as wavelets.

$$W(a,b) = \int f(t)\,\psi_{a,b}^{*}(t)\,dt - (44)$$

While the wavelets produced from a mother wavelet through dilation and translation are:

$$\psi_{a,b} = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) - (45)$$

Where a = the scale factor; b = the translation factor.

For discrete data (signal), we apply discrete wavelet transform (DWT). Hence, both scale and translation factors must be digitized. Scale and translation factors are chosen based on power of two in practice to avoid over down sampling ($a = 2^{j}$; $b = k2^{j}$). With this, we can write the equivalent DWT equation by substituting equation 45 into 44 [112]:

$$\psi_{j,k}[n] = 2^{\frac{j}{2}}\,\psi\left[2^{j}n - k\right] \quad j, k \in Z \tag{46}$$

We can find a set of basis functions using the mother wavelet $\psi[k]$, which forms an orthonormal basis and gives a multiresolution analysis framework. Therefore, the subspace of $L^{2}(R)$ covered by these functions is given by:

$$W_{j} = \overline{span\left\{\psi_{j,k}[n], k \in Z\right\}} \tag{47}$$

Where $[\psi_{j,k}]_{k \in Z}$ = the orthonormal base of W_j

For multiresolution analysis; sequence of consecutive approximation space V_j meets the condition

$$\subset V_{j+1} \subset V_{j} \subset V_{j-1} \cdots \tag{48}$$

$$\text{With } \overline{\bigcup_{j \in Z} V_{j}} = L^{2}(R) \tag{49}$$

$$\bigcap_{j \in Z} V_{j} = \{0\} \tag{50}$$

For all j ∈ z, we express W_j to be the orthogonal complement of V_j, hence V_{j-1} is given as:

$$V_{j-1} = V_{j} \oplus W_{j}, j \in Z \tag{51}$$

$V_j \perp W_j$, applying this function, we can easily loop the decomposition of

$$V_{j} = V_{j+1} \oplus W_{j+1} = V_{j+2} \oplus W_{j+2} \oplus W_{j+1} \tag{52}$$

To any level we desire.

Where V = the low frequency of signal referred to as approximation space

W = the high frequency signal known as detail space (detail space is the multiresolution analysis)

[114] Contains more information on the analytical theories of the implementation of wavelet analysis. The wavelet based fingerprint enhancement algorithm proposed in [112], to clarify ridge structure in order to improve minutiae detection and extraction consists of four main steps, viz:

i. **Normalization:** The essence of this is to tune the gray-level values of the original fingerprint image to obtain particular mean and variance values.

ii. **Decomposition:** The goal here is to obtain sub-images from decomposition of the input fingerprint through a specified filter in hierarchical relationship.

iii. **Gabor filtering:** Gabor filter is used to adjust the approximation image as well as its details to yield an enhanced image.

iv. **Reconstruction:** The final enhanced image is generated through the reconstruction of the enhanced approximation image and the detail images that where the output of the decomposition process.

From the above literatures, we see that a lot of techniques have been proposed towards the enhancement of fingerprint images. However, there is still need for fast, effective improvement as the optimal performance and reliability of any fingerprint based biometric system is greatly dependent on high quality fingerprint images.

Unfortunately, most fingerprints are corrupted by noise, creases, smudges, cuts, and uneven depression pressure during their acquisition. There is need to enhance such fingerprint images prior to extraction of minutiae, in order to ensure the acquisition of accurate minutiae while discarding irrelevant and false minutiae. This improves greatly the recognition rate of such biometric systems and consequently decrease rejection rate of authentic, enrolled users. This thesis will produce an algorithm that enhances fingerprint images using wavelet processing as an effort towards improving performance of fingerprint based biometric systems.

CHAPTER THREE

RESEARCH METHODOLOGY

3.0 In this chapter we will discuss the research methodology, develop fingerprint image enhancement using wavelet processing and present background information on wavelets. The research methodology consists of a series of steps that include:

i) **Input Fingerprint Image:** The FVC 2002 set B database was chosen as the source of the input fingerprint image as they are internationally available and recognized for fingerprint algorithm verification.

ii) **Normalization:** The input fingerprint image is normalized here in order to obtain an evenly illuminated fingerprint image. This is achieved by adjusting the mean and variance values of the matrix generated from the fingerprint image.

iii) **Decomposition:** Daubechiesô wavelet is employed to compute the wavelet transform of the normalized fingerprint image. This mathematical operation decomposes the normalized image into four sub-images namely: approximation detail, vertical detail, horizontal detail and diagonal detail.

iv) **Reconstruction:** The inverse wavelet transform of the image is computed using Daubechiesôwavelet too. This step generates an enhanced image for smoothing.

v) **Wavelet Filtering:** Here, the reconstructed fingerprint image is filtered by wavelet means to smooth the enhanced image.

vi) **Display enhanced image:** This last phase displays the enhanced fingerprint image which can be used for identification or verification purposes.

The block diagram of figure 11a shows an overview of the methodology.

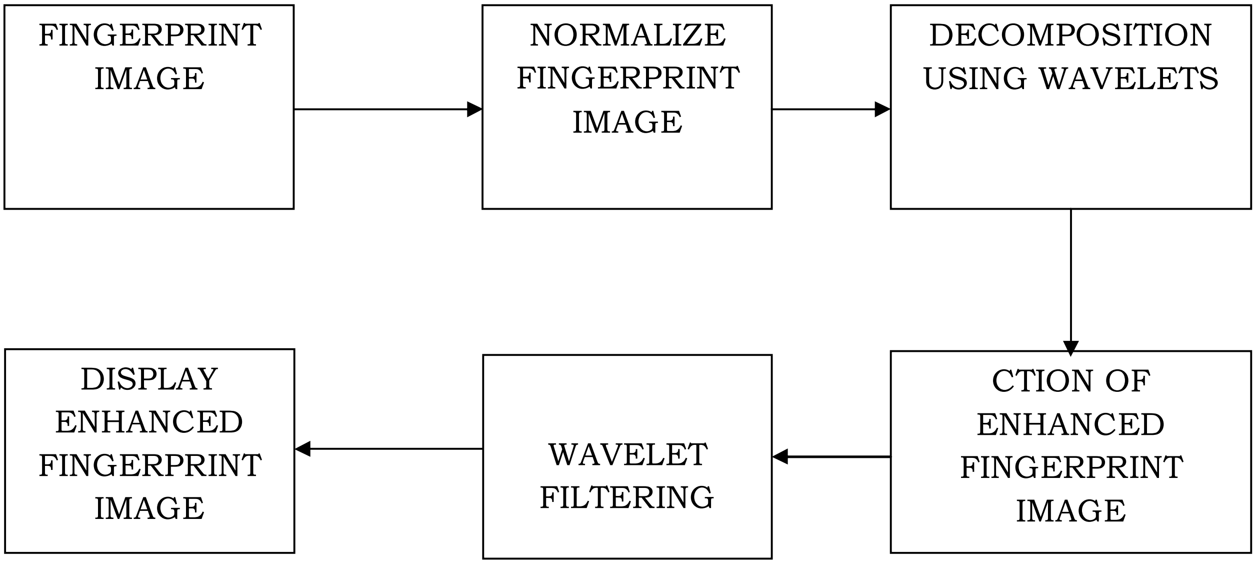

Figure 11a: Block diagram of fingerprint image enhancement algorithm using wavelet processing.

3.1 DEVELOPMENT OF FINGERPRINT IMAGE ENHANCEMENT ALGORITHM USING WAVELET PROCESSING

It is learnt from the previous chapter on literature review that wavelets have been around for a long time. In various literatures, wavelets and wavelet transforms have been adopted and proposed for image processing and compression applications, particularly in fingerprint image enhancement such as those of [115], [116], [117], [118], [119], [120] and [121].

The need for fingerprint image enhancement was discussed in chapter two; in this chapter, the primary objective is to develop an algorithm that will enhance fingerprint images by wavelet means. Most of the work on the development of wavelets was performed in the 1930s, from frequency analysis of Fourier to scale analysis of wavelets.

3.2 BACKGROUND INFORMATION ON WAVELET THEORY

It has been established in literature, that wavelets are little waves without dc components. They came to lime light as a result of the limitations of Fourier analysis to handle aperiodic signals, provide both frequency and time information of a signal concurrently. The search for a solution to these shortcomings of Fourier analysis led to a windowed version called short time Fourier transform (STFT) but this was also limited by the manner in which the window size is to be selected; as either too small or too wide a window size posed grave constraints to the computation requirements. A need for a solution to this was paramount; this was found in wavelets! The primary concept behind wavelets is analysis based on *scale*.

Wavelets are mathematical tools that enable us cut signals into pieces and analyse them simultaneously as desired. These wavelets contain both time and frequency information; their properties endeared them to signal and image processing applications.

Heisenberg principle states that a limitation exists in the measurement for time and frequency concurrently of a signal. The implication is that if an accurate and precise infinite measurement of time and frequency is done, the product of frequency and time is constrained according to the Heisenberg principle. Practically, Heisenberg stated that it is possible to measure either time or frequency separately with infinite precision. ôThe product of time interval, æt, and frequency interval, æf, is constantô [122].

Wavelet analysis is a multiresolution analysis. This implies that at low frequencies, the window size rectangle is horizontally elongated yielding better frequency resolution while at high frequencies; it is vertically elongated producing better time resolution [122].

The process of wavelet analysis involves choosing a wavelet prototype function referred to as *kernel or analysing wavelet or mother wavelet*. Wavelet analysis based on scale analysis implies that, analysis of a signal denoted by f(x) is done by generating mathematical structures that differ in scale. This is achieved by the construction of a function, delaying or shifting it by a certain factor, alter its scale [123] and adopt this structure in approximating signal f(x). Iterating this process keeps generating a new approximation of f(x). This scale analysis is immune to noise as it determines the average alternations of f(x) at varying scales. Wavelet functions are individually localized in space.

If we have the information of the mother wavelet, its basis functions can be determined. Waveletsô basis functions possess localization feature making them suitable for representing signals in time-frequency domain locally. These wavelet coefficients or basis functions are derived from dilation (or scaling) and translation of its mother wavelet.

We present a review of wavelets beginning from the traditional Fourier transform [i.e. to proceed from the more familiar domain to the unknown]. Let's analyze a continuous one dimensional instance for illustration purposes [124].

Given a function f(x), its continuous Fourier transform can be written as:

$$F(s) = \int_{-\infty}^{\infty} f(x) e^{-i(2\pi sx)} dx \text{ ------------------------- } (53)$$

Recall that the kernel, $e^{-i(2\pi sx)} = \cos(2\pi sx) - i\sin(2\pi sx)$ denotes a complex oscillatory function that is non-zero at all values of x with its modulus evenly unity for all x. At any point in x, the modulation frequency of f(x) is represented as s, the resultant F(s) is substantial; hence:

$$P(s) = |F(s)|^2 \text{ ------------------------------ } (54)$$

Where P(s) is the power spectrum being a measure of the energy content of each frequency component. Consequently:

$$E_f = \int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |F(s)|^2 ds \text{ -------------------- } (55)$$

With E_f being a measure of the entire 'power' and must be finite. Fourier transform is limited by its non-locality, hence all contents of f(x) irrespective of their position in x, constitute equal power to the total power P(s). Locality in x is denoted only in the phase framework of F(s). Another challenge is that in practice, most functions are unknown for the range -∞ to ∞. In order to address this, we may decide to adopt a window function w(x) at some location x_0 i.e. short time Fourier transform. This leads to:

$$F_w(s, x_0) = \int_{-\infty}^{\infty} f(x)\, w(x - x_0)\, e^{-i(2\pi sx)} dx = e^{-i2\pi s x_0} \left[F(s) * W(s) \right] \text{ ------ } (56)$$

Where W(s) = Fourier transform of w(x)

* denotes convolution.

NOTE:

i. There are two variables for the STFT

ii. The variables are frequency s and shift x_0 in the variable conjugate to s.

Fourier transform yields either x or s function but not both, but the insertion window of STFT provides both showing the desired localization characteristic. Various efforts have been documented to enable the choice of a good window function that can provide adequate localization in both spatial (x) and frequency (s) domains. These include: Gaussian window proposed by Gabor and Zak transform of Bastiaan.

Wavelets provide another approach to obtain localization i.e. representation of signals in both x and s domains. This is why wavelet transform is called a *multiresolution* technique. The wavelet transform functions at specific quality factor QF unlike STFT and Wigner distributions [124].

$$QF = \frac{|s|}{|\Delta s|} \text{-------------------------------} (57)$$

This relationship shows that the transform gives a high resolution at low frequency and vice versa. Wavelet transforms are invertible. An integration of these features makes wavelet transform the king of STFT and Wigner distribution.

Given a function $\psi(x)$, to find its mother wavelet, this condition must be satisfied:

$$C_{\psi} = \int_{0}^{\infty} \frac{|\psi(s)|^{2}}{s} \, ds < \infty \text{---------------------------} (58)$$

Where $\psi(s) =$ Fourier transform of $\psi(x)$

Therefore, any $\psi(x)$ that meets this "admissibility" condition of equation (58) may act as a mother wavelet.

We can find the coefficients or basis functions or daughter wavelets from the mother or kernel wavelet using:

$$\psi_{a,b}(x) = \frac{1}{\sqrt{a}} \psi\left(\frac{x - b}{a}\right) \text{---------------------------} (59)$$

Where a = the scale factor or magnification

b = the shift factor or translation

With these, given a function f(x) we can write the wavelet transform as:

$$W_f(a,b) = \langle f, \psi_{a,b} \rangle = \int_{-\infty}^{\infty} f(t)\,\psi^*_{a,b}(t)\,dt$$

$$= \sqrt{a}\int_{-\infty}^{\infty} F(\omega)\,\Psi^*(a\omega)\,e^{(j\omega b)}\,d\omega \tag{60}$$

With the inverse wavelet transform expressed as:

$$W^{-1}[W(a,b)] = f(t)$$

$$= \int_{-\infty}^{\infty} F(\omega')\left\{\frac{1}{C_\psi}\int_{0}^{\infty} \Psi^*(a\omega)\,\Psi(a\omega') \times \int_{-\infty}^{\infty} e^{[j(\omega-\omega')b]}\,db\,\frac{da}{a}\right\} \times e^{(j\omega t)}\,d\omega \tag{61}$$

Observe from equation (61) that:

$$\int_{-\infty}^{\infty} e^{[j(\omega-\omega')b]}\,db = 2\pi\,\delta(\omega - \omega') \tag{62}$$

To make the contents of {} in equation (61) equal 1, in order for equation (61) to denote the inverse Fourier transform, we must obtain:

$$C_\psi = \int_{0}^{\infty} \frac{|\Psi(\omega)|^2}{\omega}\,d\omega < \infty \tag{63}$$

Equation (63) is equivalent to equation (58) i.e. the wavelet admissibility condition. From the foregoing, it is evident that the inverse wavelet transform in a continuous state exists. If we analyse the admissibility condition, we notice that $\Psi(0) = 0$ for all admissible $\psi(t)$ implying that $\psi(t)$ cannot possess a "dc" component hence the name *'little wave' or 'wavelet'* [124].

In digital computation, "*dyadic*" (order of 2 integer) a special wavelet type is used; equation (64) shows their representation.

$$\psi_{j,k}(t) = 2^{\frac{-j}{2}}\,\psi(2^{-j}t - k) \tag{64}$$

Dyadic wavelets are fast wavelet transform with unequalled simplicity. With its computational difficulty being only of N order i.e. $0(N\log_2 L)$;

Where N = length of the data vector

L = length of the digital filter

In contrast with the ultra efficient fast Fourier transform (FFT) whose expression is 0(Nlog$_2$N) [125].

3.3 FINGERPRINT ENHANCEMENT ALGORITHM BASED ON WAVELETS

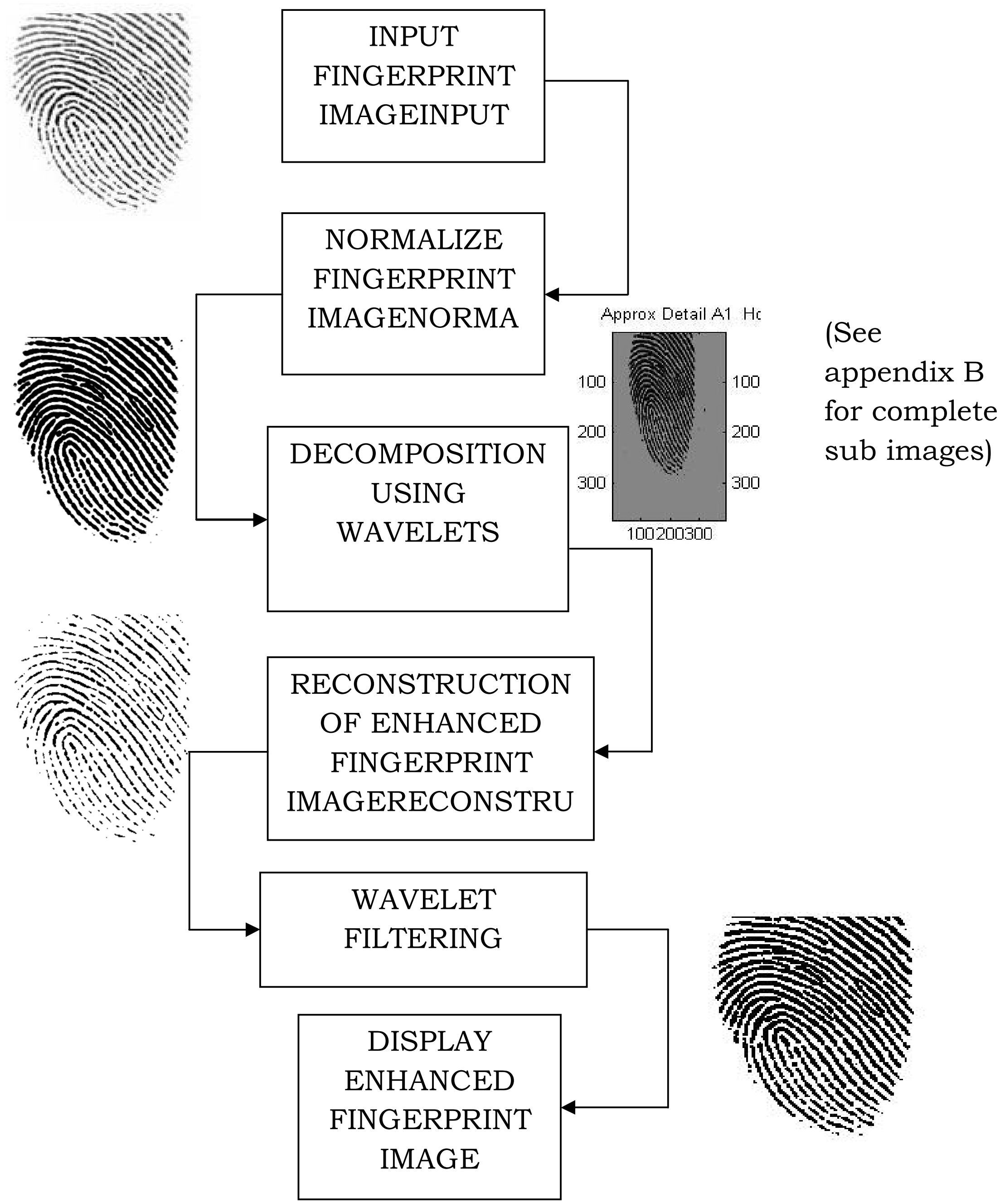

Figure 11b: Block diagram of fingerprint image enhancement algorithm using wavelet processing showing fingerprint image undergoing processing.

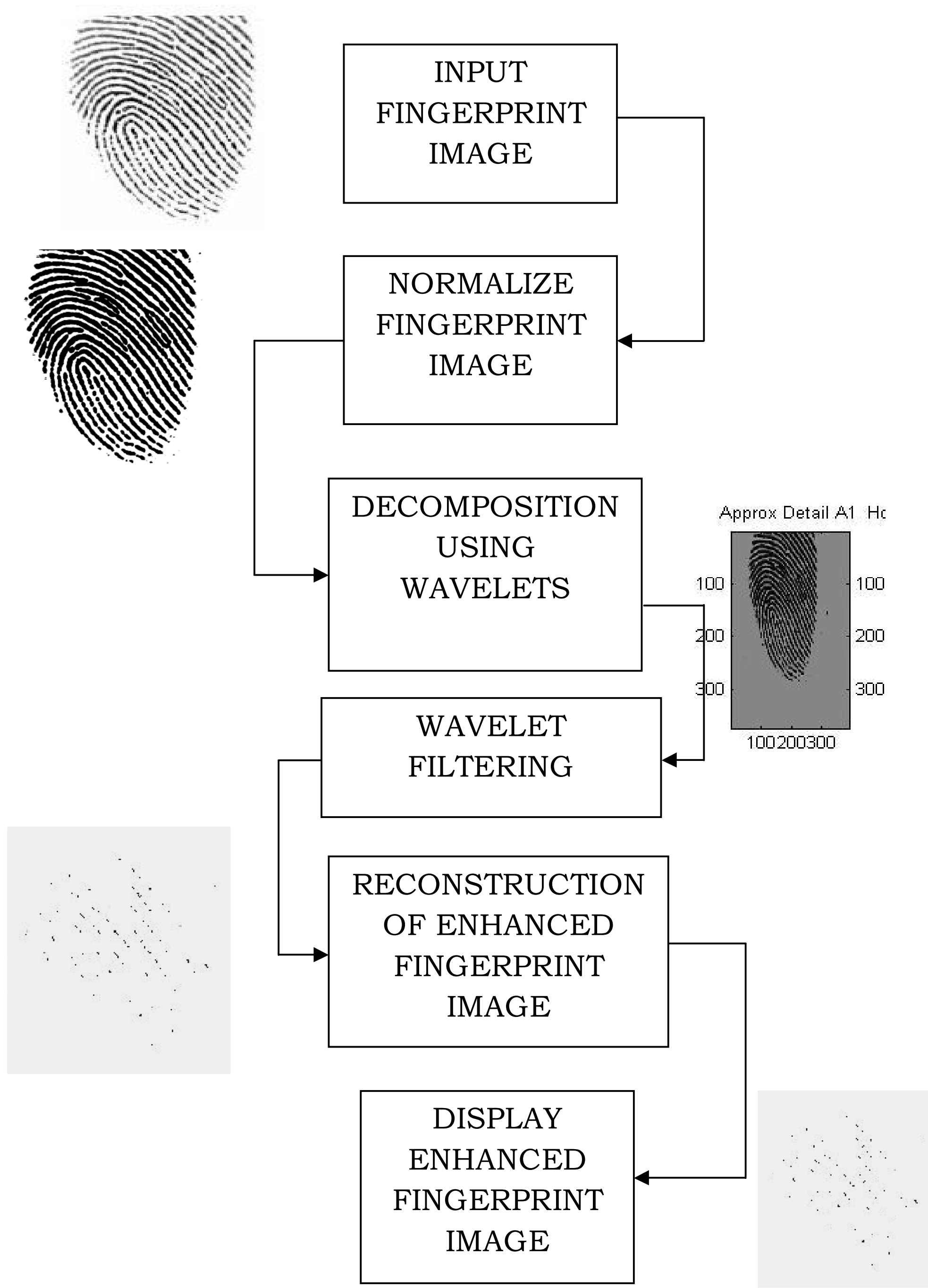

Figure 11c: Block diagram of fingerprint image enhancement algorithm using wavelet processing with wavelet filtering block interchanged with reconstruction block.

Figure 11b shows the block diagram of the developed algorithm for fingerprint image enhancement using wavelet processing. The input fingerprint image is taken from FVC 2002 set B database, which was then normalized.

The normalized fingerprint image is then decomposed at level 2 using Daubechiesô wavelets. This generates four sub details which are a quarter of the normalized fingerprint image. The complete details of the sub details are shown in appendix B. After decomposition, the enhanced fingerprint image is reconstructed by computing the Daubechiesô inverse wavelet transform; the enhanced image is then filtered by wavelet process to produce a smooth version of the enhanced fingerprint image. The enhanced fingerprint image is then suitable for identification or verification applications.

The wavelet filtering block was interchanged with the reconstruction block in figure 11c; the same fingerprint of figure 11b was also processed here. The purpose is to show that wavelet filtering prior to reconstruction of the enhanced image severely vanish the ridgelines as well as the valley structure of the fingerprint image. This implies that wavelet filtering though suitable for smoothing the enhanced fingerprint image is only effective after reconstruction has been performed.

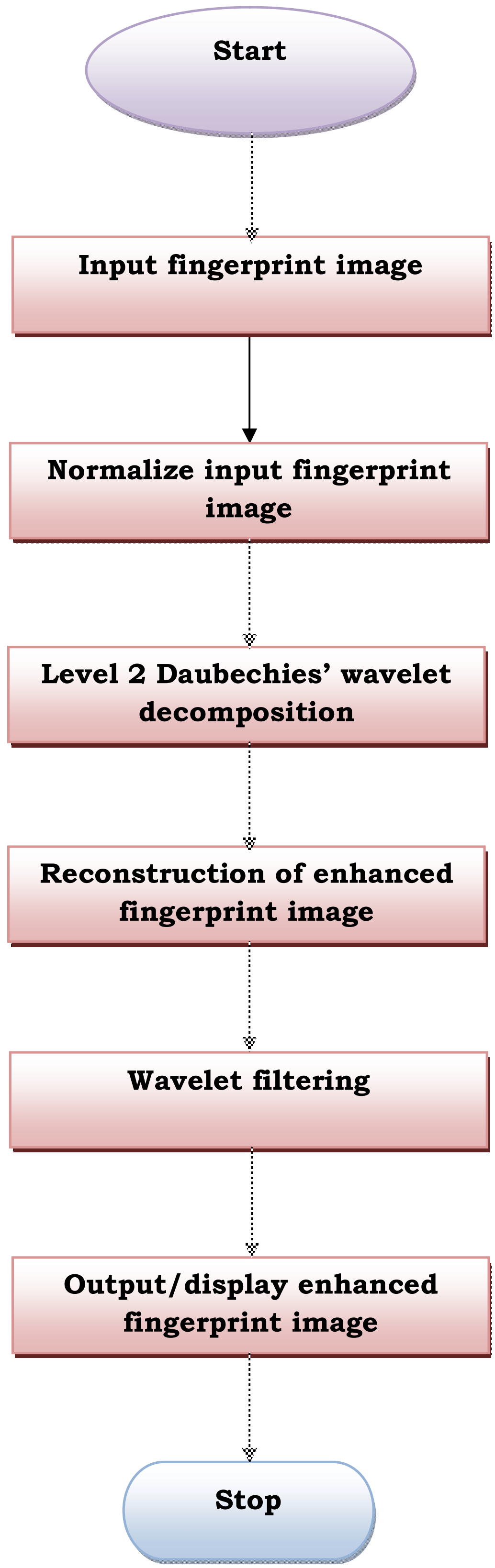

Figure 12: Program Flowchart for Algorithm implementation

3.3.1 ACQUISITION OF FINGERPRINT IMAGE

In this phase, the fingerprint image to be enhanced is acquired either directly from a fingerprint scanner or a database such as FVC 2002. Set B database of FVC 2002 was used to experiment the algorithm as it is freely available and contains the required fingerprint images for enhancement.

3.3.2 NORMALIZATION

Here, the primary purpose is to ensure that there is even illumination of the fingerprint image, through the generation of specific mean and variance values. This is done by adjusting the gray-level values of the input fingerprint image [126]. This phase modifies the gray-level values between ridges and gorges (furrows or valleys). The process of normalization eliminates the effects of sensor noise as well as gray-level corruption.

Let: F(i, j) represent the gray-level value at pixel location (i, j) of the input fingerprint image;

M_o denotes the estimated mean;

V_o represents the estimated variance of input fingerprint image F;

F_N (i, j) be the normalized output gray-level value at (i, j).

Then the normalized image function is given as [126]:

$$F_N(i,j) = M + \sqrt{\frac{V}{V_o}\left(F(i,j) - M_o\right)^2} \quad \text{-------------------------} (65)$$

Where M = predefined value of mean;

V = predefined variance value

Alternatively; if F(i, j) denotes the gray-level value of pixel (i,j);

M_o = estimated mean of the input fingerprint image;

V_o = estimated variance of the input fingerprint image;

F_N = normalized gray-level value at pixel (i, j);

Then F_N(i, j) is expressed as [126]:

$$F_N(i,j) = \begin{cases} M + \sqrt{\frac{V}{V_o}\left(F(i,j) - M_o\right)^2} \xrightarrow{\text{if}} F(i,j) > M_o \\ M - \sqrt{\frac{V}{V_o}\left(F(i,j) - M_o\right)^2} \quad \text{----------------------} (66) \end{cases}$$

Where M = the expected mean value;

V = the expected variance value;

Since normalization is a pixel-wise operation, it does not affect nor alter the ridge and furrow patterns (or structure). But it cannot modify the visibility of the ridge and valley patterns further.

3.3.3 WAVELET DECOMPOSITION

The purpose of this phase is to decompose or segment the normalized fingerprint image into approximation image and three sub-details which is a quarter of the original input fingerprint image. It is theoretically supported to decompose an image into sub-images at any level but it is limited to two decomposition levels in this research work. This is to prevent the disappearance of the orientation characteristics of the ridge structure due to over down sampling which occurs at too low a resolution level [119] [120].

The wavelet transform is applied to the normalized image so as to decompose it into a multiresolution representation. Daubechies wavelet was adopted because it provides sufficient information in the sub-images approximation [119] [120]. The choice of the level two decomposition in addition to the preservation of the ridge pattern to match filtering, also improves computational speed.

3.3.4 RECONSTRUCTION OF ENHANCED FINGERPRINT IMAGE

This last phase of the algorithm generates the final enhanced fingerprint image from reconstructing the enhanced approximation image together with the detail sub images of the decomposition phase. The same wavelet used during the decomposition process is also applied during reconstruction.

3.3.5 WAVELET FILTERING

This segment of the algorithm smoothes/denoise the fingerprint image using wavelet processing and soft thresholding techniques. The objective is to optimize the fingerprint image for feature extraction as well as compression for transmission purposes.

CHAPTER FOUR

EXPERIMENTS

4.1 DISCUSSION OF EXPERIMENTS AND PROCEDURE

STEP 1: Downloaded DB1_B of FVC- 2002 as the source for the input fingerprint images for testing the algorithm.

STEP 2: This database comprises of 80 images out of which 8 images where taken at random for experimentation.

STEP 3: The fingerprint images are generally of size 388 x 374 pixels with about 96dpi resolution.

STEP 4: The experimental results produced from the algorithm are shown in tables 1 and 2. The application of wavelet transform to fingerprint image enhancement, offers a robust technique that allows automatic fingerprint verification and identification systems improve their recognition rate. Daubechiesô wavelets where employed, as the sub details resulting from the decomposition contain enough information of the fingerprint image that lead to satisfactory reconstruction of the enhanced fingerprint image.

Although it is theoretically possible to decompose the fingerprint image at any level, it was restricted to level 2 in this work as it was observed that the ridge patterns and valley structures vanished at levels greater than 2 due to excessive down-sampling of the fingerprint image.

Wavelets also offer a very suitable image denoising feature that removes image noise without altering the image quality (i.e. does not compromise the integrity of the fingerprint ridge and valley structure); making it suitable for compression and consequently for transmission applications.

Another great feature of wavelets is their capability for multiresolution processing of images. The scale of the wavelet is proportional to the resolution; hence permitting us to choose any adequate scale that suits our resolution requirements. Wavelets are robust indeed as they provide information of the image signal in both spatial and frequency domains.

From the images of appendix B, it is apparent that in comparison with the input fingerprint images, the output images have been significantly improved.

The consequence of this outcome is improved recognition rate of fingerprint based biometric systems, reduced false rejection rate (FRR), rapid and accurate extraction of minutiae information, effective detection of core points and reduction of alias point detection.

4.2 QUANTIFICATION OF THE DEGREE OF ENHANCEMENT

In this section, we perform mathematical analysis with the aim of providing an estimate of the degree of enhancement generated by the algorithm. The purpose is to showcase the efficiency and importance of the application of wavelets to fingerprint image enhancement. This analysis will apply mathematical and statistical methods to investigate the performance of the algorithm.

It has been established that wavelet transform is an indispensable technique for image compression and denoising applications as they provide multiresolution capability which depends on scaling.

Wavelets provide a computationally robust and fast means of denoising images in comparison to other conventional methods.

We approach the task of quantifying the degree of enhancement achieved by this algorithm using performance evaluation technique:

Application of the performance evaluation (PE) formula to compute the performance of the algorithm as stated in [115] yields:

$$PE = \frac{N_M - N_S - N_L}{Total_H} \quad (67)$$

Where: N_M = the number of minutiae that are similar both in algorithm as well as with human expert.

N_S = number of super abundant minutiae.

N_L = number of lost minutiae.

$Total_H$ = the number of minutiae extracted by human expert.

From equation (67), we observe that in ideal situation, the performance evaluation (PE) attains its maximum value which is 1 if the number of lost minutiae equals zero and if there are no super abundant minutiae; this implies that the number of minutiae that are similar both in algorithm as well as with human expert are equal.

Mathematically speaking, this occurs when $N_L = 0$, $N_S = 0$, hence $N_M = Total_H$.

The algorithm was experimented on 8 fingerprint images which were obtained from DB1-set B of FVC 2002 database; the images were generally of size 388 x 374 pixel.

The test fingerprint image was normalized, decomposed using Daubechiesô wavelet; its inverse wavelet transform was computed and then denoised.

Arithmetic mean (μ) was computed using the formula:

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i - (68)$$

Where μ = Mean

x_i = PE value at i

N = 8

And finally the standard deviation (σ) from the mean was calculated using the expression:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2} - (69)$$

Where σ = Standard deviation

xi = PE value at i

μ = Mean

N = 8

Table 1: Performance evaluation (quantification of the degree of enhancement) of the developed algorithm

Serial number	Performance Evaluation (PE)									
	Pre-enhancement parameters					Post-enhancement parameters				
	N_m	N_l	N_s	$Total_H$	PE	N_m	N_l	N_s	$Total_H$	PE
1	19	7	0	19	0.6316	26	0	3	26	0.8846
2	14	5	0	14	0.6429	19	2	1	19	0.8421
3	55	18	0	55	0.6727	73	2	3	73	0.9315
4	39	8	0	39	0.7949	47	1	3	47	0.9149
5	39	1	0	39	0.9744	40	0	3	40	0.9250
6	31	5	0	31	0.8387	36	1	2	36	0.9167
7	30	1	0	30	0.9667	31	1	2	31	0.9032
8	14	30	2	14	-1.2857	50	2	5	50	0.8600
Mean					0.4302					0.8972
Standard deviation					0.7047					0.0301

Table 2 [117]: Comparison of this algorithm with other works.

Performance	ALGORITHMS				
	Sherlock [115]	Wahab [117]	Hsieh [115]	Hatami [117]	Thesis
PE	0.49	0.40	0.64	0.79	0.89

4.3 ANALYSIS OF RESULTS OF FINGERPRINT IMAGE ENHANCEMENT ALGORITHM BASED ON WAVELET PROCESSING

The FVC databases are internationally available databases developed for International Competition for Fingerprint Verification Algorithms. FVC is an acronym for *fingerprint verification competition*. The databases are updated periodically to ensure continuity and availability of useful data for researchers in the biometric industry. The purpose of this competition is to set a benchmark that will serve as a reference for practitioners in the industry as well as the academia.

The FVC 2002 [set B] database which is available online at [127] is one of the FVC databases that are generally adopted for evaluating and categorizing fingerprint based biometric system. These databases contain various types of fingerprint images that were captured at varying degrees of quality.

The developed algorithm was tested on images from FVC 2002-DB1 set B database as they are freely available. Set B database contains 80 fingerprint images out of which 8 fingerprint images were randomly selected for experimentation.

The images contained in FVC 2002 databases were captured from four different sensors in order to accommodate recent progresses that were made in fingerprint capturing techniques.

Optical and capacitive sensors that are generally small in size and cheap were applied in generating the fingerprint images of databases 1 and 2; while a higher quality optical sensor was deployed for the images of database 3 [116]. The fourth database images were artificially produced by applying the technique presented in [128].

4.4 COMMENTS ON IMPROVEMENTS BY WAVELET PROCESSING

It has been established that wavelets are very robust and suitable for fingerprint image enhancement. It is worthy of note that the enhanced images still have room for improvement as they are not perfect reconstructions from the sub-detail images. This arises due to loss of some signals due to noise in the sub-details; hence they are attenuated together with the useful image information signals during filtering.

This can be improved by incorporating a suitable band pass filter such as Gabor filters after decomposition prior to reconstruction using the inverse wavelet transform. Some other techniques proposed and employed in literature include:

i) Application of second derivative Gaussian filter

ii) Use of Directional wavelet transform

iii) Stationary wavelet transform based on composite method

iv) Contourlet transform methods (ñContourlet forms a multiresolution directional tight frame to efficiently approximate images and make smooth regions separated by boundariesò) [109].

4.4.1 SPECIFIC APPLICATION AREAS FOR DEPLOYMENT OF THE ALGORITHM

This algorithm is designed specifically for integration into any fingerprint based biometric systems, to act as a preprocessing step for fingerprint image processing. The human purpose of this is to contribute a reliable fingerprint image enhancement algorithm. That will aid the production of fingerprint based biometric systems which provide high recognition rate with significantly negligible false rejection rate (FRR).

Biometric systems that can incorporate this algorithm include:

i) Fingerprint based point of sale terminal machines.

ii) Fingerprint based identification systems.

iii) Fingerprint based employee attendance machines.

iv) Fingerprint based passport control systems.

v) Biometric access control doors.
vi) Computer and network resources control systems.
vii) Fingerprint based voting machines.

There are numerous fields in which biometric systems are deployed, of which fingerprint based systems are in the majority. Ranging from government installations to security, the application of fingerprint based biometric systems are inevitable, indispensable and essential.

As the future of our civilization continues to evolve and revolve around computers and mobile communication networks. Fingerprint based biometric systems are guaranteed a stable, relevant and indispensable application.

The need for positive and reliable identification of humans are here to stay as crimes, security, payroll, and bank transactions continue to be at the core of our modern and future civilizations.

4.5 LIMITATIONS OF BIOMETRIC SYSTEMS

Despite the numerous advantages of biometrics systems, it is like every other system restricted by some disadvantages. Although there are numerous beneficial applications of biometric systems, there are inherent risks associated with their use. These include: theft of biometric information, brute force attack, rejection by end users, and false rejection of authentic enrollees.

In this section, an overview of the demerits of biometric systems is presented:

i) **Stolen biometrics:** A criminal seeking access to a biometrically secured resource, facility or location can monitor authorized personnel; in order to steal his/her biometric information so as to breach the security of such biometric system. For instance, the criminal may lift a latent fingerprint which can be transferred to an artificial three dimensional model. He can then use such model to circumvent the biometric system. Despite the risk posed by such theft, it is still a herculean task to produce such a model. This difficulty is not simplified with increase in the number of latent fingerprints to be modeled hence biometrically secured resources like computer networks are safer than those secured by traditional methods.

The reason being that a hacker does not require knowledge of the network users personally, prior to launching a simple password guess with brute force attack which is very likely to breach the networkÕs security.

ii) **Irreplaceable:** It is practically impossible to retrieve a lost biometric as it cannot be easily cancelled or blocked, unlike a bankÕs debit card that can be easily canceled and a new one reissued on short notice. A single stolen biometric may provide access to different applications if those applicationsÕ securities rely on the biometric feature that was compromised. An effective approach to tackle this situation is to combine cryptographic methods with biometric matches [9].

iii) **Privacy:** Another limitation of biometrics is in the area of privacy violations. Biometrics generates three major privacy concerns categorized into: unintended functional scope, unintended application scope and covert recognition as discussed in [9].

iv) **Performance limitation** [129]**:** Unlike traditional identification and verification systems that are based on keys, tokens and passwords, biometric recognition systems suffer from errors due to verification of raw data. This is generally indicated by a match percentage, which is always less than 100%. These errors due to verification are caused by factors like: uniqueness of human features, environmental conditions during capture as well as cross device matching.

v) **Acceptance:** The applications of biometric systems are limited by usersÕ satisfaction and acceptance.

vi) **Architectural drawbacks:** Loopholes that are inherent in the architectural designs of biometric systems present a severe constraint leading to circumvention, repudiation, contamination, collusion and coercion threats of biometric systems as discussed in [8].

CHAPTER FIVE

5.1 RECOMMENDATIONS

From the results of the experiments with the algorithm, it is recommended that fingerprint images be enhanced prior to minutiae extraction in order to increase the recognition rate and consequently reduce false rejection rate (FRR).

For future works in this area, it is recommended that Gabor filtering be included as a step in the enhancement process. This should be inserted after the wavelet decomposition phase before the computation of the inverse wavelet transform. The reason for this is that Gabor filters possess both frequency and orientation selective properties that will preserve the ridge and valley patterns of the sub-detail images aiding a better reconstruction of the final enhanced image. Wavelet denoising approach was found to diminish the ridge and valley structures after decomposition. Hence, the inverse wavelet transform of the enhanced image was computed prior to wavelet denoising.

As a prerequisite for further research in biometrics, the scholar should possess a working knowledge of the MATLAB programming language, as this is the primary tool for implementing most fingerprint image enhancement algorithms.

Set A of FVC 2002 or FVC 2004 or FVC 2006 database should be employed in future works for input fingerprint images to be enhanced as they contain updated fingerprint images from recent advancements in fingerprint image capturing technologies.

For biometric systems to mitigate the various limitations mentioned previously, the following steps can be taken:

i) For most commercial deployment of biometric systems, passwords or tokens can be integrated with biometrics to address theft of fingerprints of authentic users.

ii) In high risk and sensitive areas such as military and power plant installations, each component of the biometric system should implement a standalone security measure. When coupled, the components should offer additional security.

iii) Design and implementation of vitality detection mechanism (to check for signs of life such as pulse and/papillary hippus) [9] which can be integrated into the biometric systemǿs software and hardware.

iv) Perhaps the most formidable approach to biometric security concerns is to design a biometric system that applies two or more biometric features for recognition such as fingerprints, gait, face and voiceprints. These biometric systems are referred to as multimodal biometric systems.

v) A unimodal biometric system based on fingerprint can improve its security by requiring the enrollment of several fingerprints and request that they are provided in a specific order during recognition.

5.2 CONCLUSION

In this thesis, an effective and efficient fingerprint image enhancement algorithm that is based on wavelet processing has been developed. The algorithm reads an input fingerprint image of uneven illumination, normalizes the illumination, decomposes it into approximation, horizontal, vertical and diagonal sub details using Daubechiesô wavelets. Its inverse wavelet transform was computed and finally denoised the enhanced fingerprint image using wavelet techniques.

The algorithm provides high performance results as can be seen from section 4.4.

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Appendix A

MATLAB code for thesis implementation

Appendix B

Experimental images

Appendix C [130]

Greek alphabets

A MATLAB CODE FOR IMPLEMENTATION

INPUT PHASE:

f = imread (‘filename’);

Where filename is the complete filename of the fingerprint image to be enhanced together with its extension and root directory path if the image is not in MATLAB’s current working directory or search path.

NORMALIZATION PHASE:

```
function [fn] = normalization_phase_new(M,V,Mo,Vo, f)
%NORMALIZATION normalizes the illumination of input fingerprint image using
%the expression: fn= M + sqrt(V/Vo)*(f-Mo), using M,V,Mo,Vo and f as %input
%parameters.
% Where: M = predefined value of mean,
% V = predefined value of variance,
% Mo = the estimated or assumed mean,
% Vo = the estimated variance of input fingerprint image,
% f = input fingerprint image, and
% fn is the normalized output fingerprint image.
% Written by Nwabunwanne S. F. C. PG/MENGR/11/59401
z= V/Vo;
p= f-Mo;
fn= M + (sqrt(z)*p);
q=mat2gray(fn);
imwrite(q,'normalized.tif','compression','none','resolution',[80,80]);
figure;imshow(q);
```

DECOMPOSITION PHASE:

```
function [A1, H1, V1, D1, A2, H2, V2, D2] = decomposition_adopted(q)
%DECOMPOSITION decomposes a give fingerprint image into its approximation
% subimage and detail subimages using 2nd level Daubechies' discrete
% wavelet transform
[c, s]= wavedec2(q, 2, 'db2');
%Extract the approximation and detail coefficients of the discrete wavelet
%transform computation.
cA1 = appcoef2(c, s, 'db2', 1);
cA2 = appcoef2(c, s, 'db2', 2);
[cH1, cV1, cD1] = detcoef2('all', c, s, 1);
[cH2, cV2, cD2] = detcoef2('all', c, s, 2);
% Reconstruct levels 1 and 2 approximation as well as detail subimages % or
coefficients.
A1 = wrcoef2('a', c, s, 'db2', 1);
A2 = wrcoef2('a', c, s, 'db2', 2);
H1 = wrcoef2('h', c, s, 'db2', 1);
V1 = wrcoef2('v', c, s, 'db2', 1);
D1 = wrcoef2('d', c, s, 'db2', 1);
H2 = wrcoef2('h', c, s, 'db2', 2);
V2 = wrcoef2('v', c, s, 'db2', 2);
D2 = wrcoef2('v', c, s, 'db2', 2);
% Display the results of the level 2 Daubechies' wavelet decomposition
colormap (gray);
subplot (2,4,1); image(wcodemat (A1, 192));
title ('Approx Detail A1')
subplot (2,4,2); image (wcodemat (H1, 192));
title ('Horizontal Detail H1')
subplot (2,4,3); image (wcodemat(V1, 192));
title ('Vertical Detail V1')
```

```
subplot (2,4,4); image (wcodemat(D1, 192));
title ('Diagonal Detail D1')
subplot(2,4,5); image (wcodemat(A2, 192));
title('Approx Detail A2')
subplot(2,4,6); image(wcodemat(H2, 192));
title('Horizontal Detail H2')
subplot(2,4,7); image(wcodemat(V2, 192));
title('Vertical Detail V2')
subplot(2,4,8); image(wcodemat(D2, 192));
title('Diagonal Detail D2')
```

RECONSTRUCTION PHASE:

```
qr = waverec2(c, s, 'db2');
figure, imshow (qr)
imwrite(qr, 'reconstructed.tif', 'compression', 'none', 'resolution', [80,80]);
```

FILTRATION/DENOISING/SMOOTHING PHASE:

```
%DENOISE filters and compresses the enhanced fingerprint image. It %also applies
softthresholding and wavelet deonising technique to %provide
% quality fingerprint image for improved feature extraction without
% increasing the file size. This optimizes the fingerprint image for
% transmisssion over a communication channel.
[thr, sorh, keepapp]= ddencmp('cmp', 'wv', f); % computes the default
% parameters of the original fingerprint image.
[Xcomp,CXC,LXC,PERFO,PERFL2] = ...
   wdencmp('gbl', c, s, 'db2', 2, thr, sorh, keepapp);
% Display the Denoised image( final enhanced image) along side the original
% fingerprint image.
colormap(gray)
figure; subplot (121); imshow (f); title ('Original Fingerprint Image');
axis square
subplot (122); imshow (Xcomp); title('Enhanced Fingerprint Image');
axis square
```

B

EXPERIMENTAL IMAGES

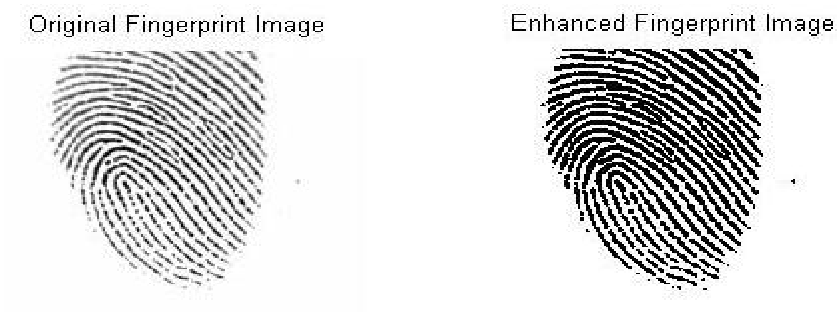

Figure 101_2: a) Original fingerprint image b) Enhanced fingerprint image

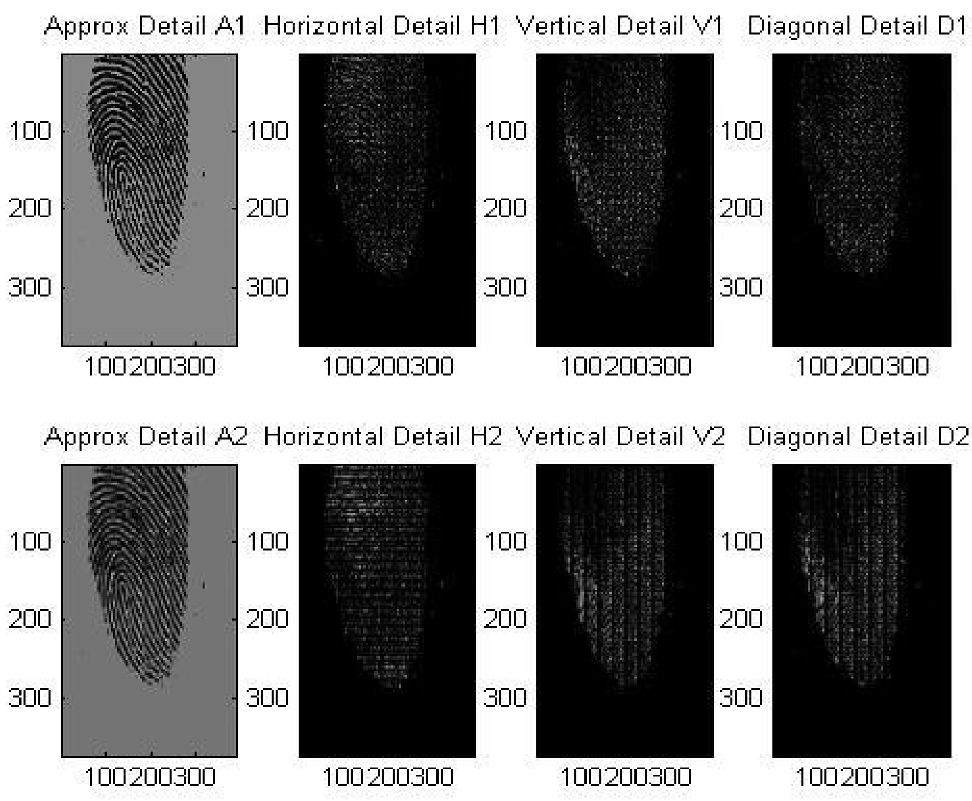

Figure 101_2c: Level 2 Daubechiesô wavelet decomposition showing approximation and sub-detail images.

101_1

101_1 enhanced

101_2

101_2 enhanced

102_3

102_3 enhanced

102_4

102_4 enhanced

103_5

103_5 enhanced

103_6

103_6 enhanced


104_7


104_7 enhanced


104_8


104_8 enhanced


101_2 Vanished ridge structure

C

GREEK ALPHABETS

Greek Symbol		Greek Letter Name	English Equivalent	Pronunciation
Upper Case	Lower Case			
ȷ	Ŭ	Alpha	a	al-fa
ȸ	ɓ	Beta	b	be-ta
ũ	ɔ	Gamma	g	ga-ma
ȹ	ŭ	Delta	d	del-ta
Ⱥ	Ů	Epsilon	e	ep-si-lon
Ȼ	ɕ	Zeta	z	ze-ta
ȼ	ɖ	Eta	h	eh-ta
Ū	ɗ	Theta	th	te-ta
Ƚ	ɘ	Iota	i	io-ta
Ⱦ	ə	Kappa	k	ka-pa
ȿ	ɚ	Lambda	l	lam-da
ɀ	ɛ	Mu	m	m-yoo
Ɂ	ɜ	Nu	n	noo
ɂ	ɝ	Xi	x	x-ee
Ƀ	ɞ	Omicron	o	o-mee-c-ron
Ʉ	´	Pi	p	pa-yee
Ʌ	ɟ	Rho	r	row
Ɇ	ů	Sigma	s	sig-ma
ɇ	Ű	Tau	t	ta-oo

Ɉ	ƍ	Upsilon	u	oo-psi-lon
ū	ű	Phi	ph	f-ee
ɉ	ɢ	Chi	ch	kh-ee
Ɋ	ɣ	Psi	ps	p-see
ɋ	ɤ	Omega	o	o-me-ga