EFFICACY OF STUDY QUESTIONS AND CONCEPT MAPS AS ADVANCE ORGANIZERS ON STUDENTS' ACHIEVEMENT AND INTEREST IN DIFFERENTIAL CALCULUS IN MATHEMATICS

BY

UZOIGWE, JOSEPH SYLVESTER
PG/MED/16/80537

DEPARTMENT OF SCIENCE EDUCATION,
UNIVERSITY OF NIGERIA, NSUKKA

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Title Page

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SUPERVISOR: DR. CHRISTIANA N. OBI

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APPROVAL PAGE

This thesis has been approved for the Department of Science Education, Faculty of Education, University of Nigeria, Nsukka.

BY

____

Dr. C. N. Obi
Supervisor

____

Dr. C. C. Ugwuanyi
Internal Examiner

____

Prof. Y.K. Kajuru
External Examiner

____

Prof. Nkadi Onyegegbu
Head of Department

____

Professor J.E. Umeifekwem
Dean, Faculty of Education

CERTIFICATION

Uzoigwe, Joseph Sylvester, a postgraduate student in the Department of Science Education with Registration Number PG/M.ED/16/50837, has satisfactorily completed the requirement for research work for the Degree of Master of Education, in Mathematics. The work embodied in the thesis is original and has not been submitted in part or full for any other diploma or degree of this or any other university.

Uzoigwe, Joseph Sylvester	**Dr. C. N. Obi**
Student	Supervisor

DEDICATION

This work is dedicated to Almighty God and my parents Mr and Mrs. Joseph Ajah.

ACKNOWLEDGEMENTS

The researcher wishes to express his sincere gratitude to the people who made the completion of this study possible. First, the researcher wants to thank the God Almighty for giving him the grace and strength to undertake this study. The researcher's gratitude also goes to his Supervisor, Dr. C.N Obi who, despite the throng of work on her, showed commitment, co-operation, patience and understanding at the various stages of this work. Dr. Obi's persistent wise counsel, guidance, and encouragement helped the researcher to successfully complete this study.

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Uzoigwe, Joseph Sylvester.

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ABSTRACT

This study was carried out to determine the efficacy of study questions and concept maps as advance organizers on students' achievement and interest in differential calculus. Six research questions and six null hypotheses guided the study. The study employed the 2×2 factorial design. The population of the study is one thousand, eight hundred (1800) SSIII mathematics students in the co-educational secondary schools in Nsukka Local Government Area consisting of eight hundred and twenty-eight (828) males and nine hundred and seventy-two (972) females. A sample of one hundred and eighty-four (184) SS III students consisting of ninety (90) male and ninety-four (94) female from 4 co-educational schools in Nsukka Local Government Area of Enugu State were selected through simple random sampling technique. Two instruments were use for the data collection: Differential calculus achievement test (DCAT) and Differential calculus interest scale (DCIS). A test blue print was developed to ensure the content validity of the DCAT while the DCIS was subjected to factor analysis to ensure its construct validity. The reliability index of DCAT was obtained using Kudder Richardson formula as 0.81 while the reliability index of DCIS was obtained as 0.92 using Cronbach Alpha. Mean and standard deviations were used to answer the research questions while analysis of covariance (ANCOVA) was used to test the null hypotheses at 0.05 level of significance. The results of the analysis showed that study questions as advance organizers is more effective than concept maps as advance organizers in enhancing students' achievement and interest in differential calculus. The results also showed that there was statistically significant difference in the mean achievement of male and female students in differential calculus. Furthermore, there was no statistically significant difference in the interest ratings of the male and female students in differential calculus. Based on these findings, it was recommended among others that mathematics teachers should incorporate the study questions as advance organizers strategy in the teaching and learning of mathematics.

CHAPTER ONE

INTRODUCTION

Background of the Study

The origin of mathematics can be traced to the existence of man in the world. Mathematics has been a human activity from as far back as written records exist (General Mathematics, 2018). The primitive men began counting, adding, subtracting, multiplying and dividing (the type of calculations called arithmetic) using their fingers, toes, stones, sticks and other objects alike. From then on, more elaborate ways of computations and problem solving evolved into various categories of quantitative fields such as algebra, geometry, statistics, mathematical modeling and calculus as man's quest for knowledge and search for solutions to emerging problems increased. The conglomeration of these fields gave birth to the discipline called mathematics.

There is a wide range of views about the meaning of mathematics. Petti (2015) defined mathematics as the science of patterns; such as patterns of reasoning and communicating, patterns of motion and change, patterns of shape, patterns of symmetry and regularity and patterns of position. This definition is line with Agwagah (2008) who defined mathematics as the study of quantity, structure, space and change. Mathematics is very important and permeates all human endeavours. Everyday activities of man revolve around mathematics. Everybody, in one way or the other thinks and applies mathematics in solving problems. Mathematics equips the individual with the capacity to, enumerate, calculate, measure, collate, group, analyze and relate quantities and ideas, among others (Agwagah, 2017).

Mathematics goes beyond sciences. Smith (2007) asserted that Mathematics has its application in natural sciences, social sciences, arts and humanities. In support of this view, Obi (2014) opined that in Arts and Humanities, mathematical concepts such as measurement, enlargement, symmetry, sequence, proportion, angle of elevation and depression, provide the baseline for the better understanding of some universal concepts. In this digital era, mathematics is an indispensable tool for technological advancement. It is an essential science which serves as the underlying knowledge of science and technology. It is in the recognition of this importance of mathematics that prompted the Federal Republic of Nigeria (FRN, 2013) in National Policy on Education to make mathematics compulsory requirement for students' entrance into the nation's higher institutions irrespective of the course they want to study and welcomes the participation of voluntary agencies, individuals and other stakeholders in education in ensuring that students achieve well in mathematics in order to meet this requirement.

Efforts have been made by stakeholders such as the National Mathematical Centre, (NMC), Mathematical Association of Nigeria (MAN), Science Teachers Association of Nigeria (STAN) and mathematics book publishers to improve the general achievement in mathematics. Each of these groups produces mathematics textbooks, teaching and learning materials that are cultural and environmental friendly with the Nigerian society, organizes seminars and workshops for teachers and learners on the best innovative practices that will help to improve the achievement of students in mathematics. It is expected that students' achievement should be commensurate with these efforts. But despite all these efforts, reports of students' achievement from the examinations conducted by the West African Examination Council (WAEC) over the years are not encouraging. It has been observed from the report of WAEC (2011-2016) (see appendix G, page 243) that the percentage of candidates that pass mathematics at credit level is yet to reach 50%. The alarming rate of poor achievement in mathematics among the students in secondary schools is often being discussed by concerned individuals. The poor achievement recorded in mathematics over the years can be attributed to several factors ranging from attitude shown towards the subject by the students, parents and teachers to issues relating to pedagogy.

Specifically, these factors include: poor understanding of mathematics language (Agwagah, 2017), mathematics phobia and lack of interest (Obi, 2014), teacher's traditional patterns of teaching that are over reliant in textbooks, diagrams and charts (Anyaemene, Nwokolo, Anyaechebelu & Anemelu, 2012), and most importantly poor teaching strategies adopted by teachers (Gimba, 2014). Most teachers adopt teaching strategies that involve mostly the talk-chalk approach that is teacher-centered. In other words, the teacher does most of the activities while the students are just passive listeners. The implication of these strategies is that they lead to rote learning- students memorize information just to pass examination without understanding the meaning of the concept and how it affects life. Mathematics teaching and learning has undergone a paradigm shift from passive process to an active construction and interpretation of experiences (Agommuoh & Ifeanacho, 2013). The authors added that the teaching and learning of Mathematics should aim at exposing the mind of the young to critical thinking, analysis and problem-solving strategies in fast-changing world like ours. The adoption of student-centered instructional strategies has been shown to enhance the active participation of students in the teaching and learning of Mathematics. These student-centered instructional strategies include among many the advance organizers strategy

An advance organizer strategy is a cognitive instructional strategy used to promote the learning and retention of new information. According to Keraro and Shihusa (2009), advance organizers are frameworks that enable students learn new ideas or information and meaningfully link these ideas to the existing cognitive structure. It is a term used to refer to all visual learning strategies such as concept mapping, mind mapping or webbing. The advance organizer strategy is credited to David Paul Ausubel's theory of learning. Ausubel (1963) submitted that advance organizers are introduced in advance of learning itself, and are also presented at a higher level of abstraction, generality, and inclusiveness. Since the substantive content of a given organizer or series of organizers is selected on the basis of its suitability for explaining, integrating, and interrelating the material they precede, this strategy simultaneously satisfies the substantive as well as the programming criteria for enhancing the organization strength of cognitive structure. Therefore, an advance organizer is a tool or device given prior to teaching so that the learners will be able to connect what they have learned with what they are about to learn. According to Pappas (2014), the advanced organizers used prior to instruction can be divided into the following four types: expository organizers that provide a description of new knowledge; narrative organizers that present the new information in a story format; skimming organizers that flick through the information; graphic organizers that include pictographs, descriptive, conceptual or study questions patterns and concept maps. The researcher's interest is on the use of study questions and concept maps as advance organizers because they do not only centre on concepts but pay much attention to exercises and problems; and show aspects of procedure for problem solving.

Study questions are questions given to students ahead of a lesson with the expectation that the answers will enable the learners to orient themselves with the topic they are about to learn. In other words, study questions direct students' attention to what is important in the upcoming lesson. The study questions as an advance organizer strategy helps to provide hints and clues about what will be taught as well as activate background knowledge and aid students in the process of filling in missing information (Helterbrand, 2012). They also help to highlight relationships among ideas or mathematical concepts, such as calculus, that will be presented and also remind students of the relevant information they already have. Hence, study questions as advance organizer strategy can be used to develop students' ability to learn independently. Researchers such as (Ogbeba & Agernor, 2016) reported that study questions as an advance organizers strategy is very effective in teaching Basic Science. Therefore, the researcher wanted to complement these works by investigating the efficacy of study questions and concept mapping strategy in teaching of mathematics concept-differential calculus.

Concept mapping is a strategy used to represent the relationships among a set of concept embedded in a framework of propositions. According to Roohi (2015), concept mapping is the technique for visualizing the hierarchical arrangement and the relationships among different concepts. It is a learning strategy that helps in understanding complex ideas and clarifying ambiguous relationships between key ideas in a topic which the students are about to learn (Nwoke, Iwu & Uzoma, 2015). According to Bot and Eze (2016), concept mapping as an instructional strategy refers to the process of teaching and learning whereby concepts to be learned including their meanings are planned and broken down using diagrams or maps.

The concept mapping strategy in mathematics enables mathematics teachers to select, organize and represent contents concisely. It helps mathematics teachers and students to observe, infer, classify and construct meanings from concepts in a hierarchy, easily and meaningfully. Furthermore, advance organizer helps students to visualize the relationships between the various mathematical concepts and their meanings and how the concepts are exemplified in theory and practice. The concept mapping strategy involves construction of concept maps. A concept map is a graph structure containing node that are interlinked by labeled directed areas. The nodes which are linked together into proposition show how students connect or link concepts (Nwoke, Iwu & Uzoma, 2015). The directionality of the link is indicated by arrows. The conceptualization of the materials by the students is indicated by the directionality and the connecting proposition. The proposition, thus illustrates the contextual relationship of the concepts to each other (Adeneye & Adeleye, 2011). Concept map is a valuable meta-cognitive tool which can be used to scaffold students' thinking and reasoning, to illustrate students' developmental and conceptual understanding, and to enhance efficiency in communicating mathematically as they learn new mathematics topics or solved mathematics problems (Afamasaga-Fuata'i, 2006). Studies have shown that the concept mapping strategy is very effective in teaching and learning. Researchers such as Nekang (2004) and Imoko (2005) reported that concept mapping enhances students achievement and interest in trigonometry while Nwoke, Iwu and Uzoma (2015) reported that concept maps enhances students achievement and interest in areas and perimeters of quadrilaterals. There is need to replicate these studies in other mathematics field such as differential calculus. Hence, the researcher investigated the efficacy of study questions and concept maps as advance organizers on students' achievement and interest in differential calculus.

Differential calculus is the study of derivatives. Derivative is a certain type of limit used to compute rates of change and slopes of tangents to curves. The process of finding a derivative is called differentiation (Douglas, 2009). Differentiation has applications to nearly all quantitative disciplines such as Physics, Chemistry, Operation Research and Economics. For example, in Physics, the derivative of the displacement of a moving body with respect to time is the velocity of the body and the derivative of velocity with respect to time is acceleration, etc. In Chemistry, the reaction rate of a chemical reaction is a derivative (Parong, 2013). In operations research, derivatives determine the most efficient ways to transport materials and design factories. Formal economic modeling began in the 19th century with the help of differential calculus, to represent and explain economic behaviour, such as utility maximization, an early economic application of mathematical modeling (World Heritage Encyclopedia, 2016). It is against the backdrop of this importance of differential calculus that informed the need for this study.

Differential calculus is one of the topics that was initially found only in senior secondary school Further Mathematics syllabus and specifically designed for students who would later read mathematics and other mathematics-oriented courses in the tertiary level. The implication of not including differential calculus in the general Mathematics curriculum is that those who initially read the general Mathematics instead of Further Mathematics and who would later want to read mathematics or, those courses where differential calculus is applied find themselves in disadvantaged position. It is because of this problem and the quest to achieve the objectives of Mathematics education at secondary school level, that there arose the need to include differential calculus in the senior secondary school General Mathematics curriculum. However, it is disheartening that despite the importance of calculus, students' achievement in calculus continues to be poor. According to WAEC chief examiner's reports (2017), students generally, achieved poorly in both integral and differential calculus. The WAEC chief examiner recommended that teachers and learners should put more effort in teaching and learning calculus. This recommendation suggests that teachers should try out more innovative strategies in teaching and learning of mathematics that can help to ameliorate this poor achievement. This is why the researcher investigated the efficacy of study questions and concept maps as an advance organizers strategy on students' interest and achievement with focus on differential calculus.

Interest is one variable that affects the way learners learn. Interest is feeling of like or dislike towards an activity (Ojo, 2015). This means that when someone is interested in an activity, one is likely to be more deeply involved in that activity (Ezeugwu, Onuorah, Asogwa & Ukoha, 2016). According to Areelu and Ladele (2018), interest can also be seen as the desire to re-engage in content over time, to seek answers to questions, to acquire more knowledge, and to promote achievement and excellence in education as well as professional careers, hence interest is an important force determining the quality of learning, educational and occupational choices. Therefore, students' interest in mathematics can be noticed from their level of zeal in learning of mathematics which is demonstrated by their attentiveness in class, participation in class activities, promptness in carrying out projects and assignments; and eagerness to engage in extra murals just to ensure that the overall learning objectives are achieved. This implies that students' interest in mathematics determines their attitude towards mathematics. In support of this view, Anigbo (2016) listed factors that militate against students' interest to include negative attitude of teachers and students and influence of other people's attitude. Another factor which can thwart students' interest is teachers' poor instructional strategy which in turn affects the students' achievement in mathematics. Anigbo recommended that government should organize refresher courses for mathematics teachers frequently from which teachers can be equipped with various instructional strategies with which they can use to teach students effectively to enhance their achievement in mathematics.

Achievement has been viewed from different opinions. Kulbir (2005) defined achievement as the measurement of the effect of specific programme of instruction or training. Relating this to education, Gimba (2014) defined academic achievement as the students' success in meeting short- or long- term goals in education. Therefore, academic achievement can be viewed as the individual's performance up to the desired level in a particular field. Hence, it is the measure of the behavioural changes that take place in the individual after passing through learning experiences. Furthermore, academic achievement is the knowledge attained or skills developed in the school subject usually designated by test scores or marks assigned by the teacher. Vences (2014) submitted that achievement of a learner in any field is evident in scores or type of grades the learner obtained at the end of the course as reported by the teacher. The scores or grades form the basis for promotion to the next class, admission into the university, award of degrees, certificates, prizes or scholarship. There are quite a large number of factors that influence achievement in Mathematics. These factors include, but not limited to gender.

Gender refers to the social attributes and opportunities associated with being male and female, the relationships between women and men and girls and boys, and the relations between women and between men. According to Ocha Tool Kit (2012), these attributes, opportunities and relationships are socially constructed and learned through the socialization processes. They are context-/time-specific and changeable. Gender determines what is expected, allowed and valued in a woman or a man in a given context. In most societies there are differences and inequalities between women and men in decision making opportunities, responsibilities assigned, activities undertaken, and access to and control over resources. All over the world there has been awareness about the need to bridge gender differences in education attainment. Hence, students' educational attainment or achievement is often analyzed in terms of gender difference.

Gender differences in terms of students' achievement in mathematics have been a topic of discourse in Nigeria. Research findings have majorly favoured the male gender more than the female in this regard, although there are still contrasting views as to who achieves better in mathematics. Ezeugo and Agwagah in Okeke (2014), revealed that male achieve significantly better than their female counterparts in mathematics. Also, Adekanye (2008); Nzewi (2009); Yang and Cheng (2010); Abonyi and Umeh (2014) and Agashi (2014) reported that male achieved better than the female counterparts in their different studies while Hydea and Mertzb (2009), Umeh (2011) and Malik, Farooq & Rabia (2016) reported otherwise. Furthermore, Achor, Imoko and Ajai (2010) reported no significant difference in the achievement of male and female students' achievement and interest in mathematics.

Statement of the Problem

One of the branches of mathematics that provide a sound background for fields such as engineering, sciences and economics is differential calculus. Evidence from the West African Examination Council (WAEC) chief examiner's report and various research reports shows that, students' achievement in mathematics, particularly, differential calculus is poor. The poor achievement of students in differential calculus has been attributed to some factors such as negative attitude of teachers and students, mathematics phobia, lack of interest and most importantly instructional strategies adopted by teachers. Most teachers adopt teacher-centered strategies that involve mostly the talk-chalk approach. These strategies have not proved to be effective in improving achievement in mathematics. This has also affected students' interest in mathematics. This has become a source of worry to the researcher. The practical nature of mathematics demands that mathematics teachers should adopt innovative teaching strategies that promote meaningful learning and support deeper mathematical thinking and reasoning such as the use of advance organizers suggested by Paul David Ausubel's subsumption theory. Some advance organizer strategies such as the use of behavioural objectives, instructional games and analogy, concept maps and study questions as advance organizers have been reported to be effective in teaching and learning of some topics in mathematics such as trigonometry, commercial arithmetic and probability. However, no such reports on differential calculus are available in Nigeria. Moreover, these reports seem to pay limited or no attention to students' interest in mathematics and seem to have not ended the debate on gender disparity.

Gender disparity in mathematics achievement has been a subject of debate over the years. Research reports on gender and mathematics achievement are still inconclusive as there are more reports in favour of the male than the female. This informed the need to conduct a further research on gender and mathematics achievement. It is on the basis of the students' poor achievement and interest in differential calculus and inconclusive reports on gender and mathematics achievement and interest that this study investigated the efficacy of study questions and concept maps as advance organizers on students' achievement and interest in differential calculus.

Purpose of the Study

The purpose of this study was to investigate the efficacy of study questions and concept maps as advance organizer on students' achievement and interest in differential calculus.

Specifically, this study sought the:

i. effect of study questions and concept maps as advance organizers on students' achievement in differential calculus.

ii. influence of gender on students' mean achievement score in differential calculus.

iii. interaction effect of advance organizer strategies and gender on students' achievement in differential calculus.

iv. effect of study questions and concept maps as advance organizers on students' interest of in differential calculus.

v. influence of gender on students' mean interest ratings in differential calculus.

vi. interaction effect of advance organizer strategies and gender on students' interest in differential calculus.

Significance of the Study

The study has both theoretical and practical significance. Theoretically, the result of this study provides insight into the Paul David Ausubel's subsumption theory on advance organizer (1962-1963) which is concerned with how individuals learn large amounts of meaningful material from verbal/textual presentations in school setting. This theory believes that an individual's existing cognitive structure (organization, stability and clarity of knowledge in a particular subject) is the principal and basic factor influencing the learning and retention of meaningful new material. It describes the importance of relating new ideas to a student's existing knowledge base before the new material is presented through the use of advance organizers. The implication is that advance organizers can be used to assist learners in assimilating new information thereby enhancing their interest and improving their achievement in mathematics in general and differential calculus in particular. Therefore, the result of this study lends credence to Ausubel's theory because it explored study questions and concept maps as advance organizers.

Practically, the findings of this study, is of benefit to mathematics teachers, mathematics students, curriculum planners and members of professional associations such as Mathematical Association of Nigeria (MAN) and Science Teachers Association of Nigeria (STAN). One reason why mathematics teachers glue to some conventional strategies of teaching is lack of awareness of innovative teaching strategies that abound. Therefore, if this study is published, it will create awareness to mathematics teachers about the impact of the study questions and concept maps as advance organizer strategy on the teaching and learning of mathematics. This will enable the mathematics teachers begin to discard the conventional instructional strategies that does not produce the much needed result. Also, the awareness, through seminars and workshops, on how best to use these advance organizer strategies in order to produce optimum result will add to the teachers' wealth of pedagogical skills.

The finding of this study if implemented in the classroom will be of great benefit to mathematics students. It will help to enhance students' interest in differential calculus thereby improving their overall achievement in it. In other words, this study will help to bridge the purported gap existing in the male and female students' interest in mathematics.

The result of this study if published will motivate the mathematics curriculum planners to include this strategy into the teacher's activities and students' activities components of the senior secondary mathematics syllabus, and also include the use of study questions as advance organizer in the pre-service training of mathematics teachers. It will also motivate the mathematics curriculum planners, to sponsor further researches that will extend this study to other branches of mathematics such as the geometry, trigonometry, algebra, etc.

Finally, members of professional associations such as Mathematical Association of Nigeria (MAN) and Science Teachers Association of Nigeria (STAN) will in turn benefit from the result of this study if it is made available to them as it will serve as a source of theme or subthemes for conferences, seminars and workshops.

Scope of the Study

This study was conducted using the SS III mathematics students in Nsukka Local Government Area of Enugu and focused on differential calculus. The reason for the choice of Nsukka Local Government Area was because of students' poor achievement in external examinations in mathematics and differential calculus in this area. SS III was chosen because differential calculus is found only in SS III scheme of work and it is newly introduced in the senior secondary school III mathematics curriculum. The researcher focused on the following contents in differential calculus: differentiation from the first principle, differentiation of polynomials, chain rule of differentiation, product and quotient rule of differentiation and application of differentiation.

Research Questions

The following research questions guided the study:

1. What are the mean achievement scores of students taught differential calculus with study questions as advance organizers and those students taught with the concept mapping strategy?
2. What are the mean achievement scores of male and female students in differential calculus?
3. Is there any interaction effect of advance organizers strategies and gender on students' achievement in differential calculus?

4. What are the mean interest ratings of students taught differential calculus with study questions as advance organizers and those taught using the concept mapping strategy?
5. What are the mean interest ratings of male and female students in differential calculus?
6. Is there any interaction effect of advance organizers strategies and gender on students' interest in differential calculus?

Hypotheses

The following null hypotheses were formulated to guide the study and were tested at 0.05 level of significance:

HO_1: There is no significant difference in the mean achievement scores of students taught differential calculus with study questions as advance organizers strategy and those students taught with the concept mapping strategy.

HO_2: There is no significant difference in the mean achievement scores of male and female students in differential calculus.

HO_3: There is no significant interaction effect of advance organizer strategies and gender on students' achievement in differential calculus.

HO_4: There is no significant difference in the mean interest ratings of students taught differential calculus with study questions as advance organizers strategy and those students taught with the concept mapping strategy.

HO_5: There is no significant difference in the mean interest ratings of male and female students in differential calculus.

HO_6: There is no significant interaction effect of advance organizer strategies and gender on students' interest in differential calculus.

CHAPTER TWO

LITERATURE REVIEW

In this chapter, works related to the present study were reviewed under the following subheadings:

Conceptual Framework

Theoretical Framework

- Ausubel's learning theory

Related Subthemes

- Calculus
- Advance organizers
- Students' achievement in mathematics
- Students' interest in mathematics
- Gender and mathematics learning

Review of Empirical Studies

- Studies on advance organizers
- Studies on students' achievement in mathematics
- Studies on students' interest in mathematics
- Studies on gender and achievements in mathematics
- Studies on gender and interest in mathematics

Summary of Literature Review

Conceptual Framework

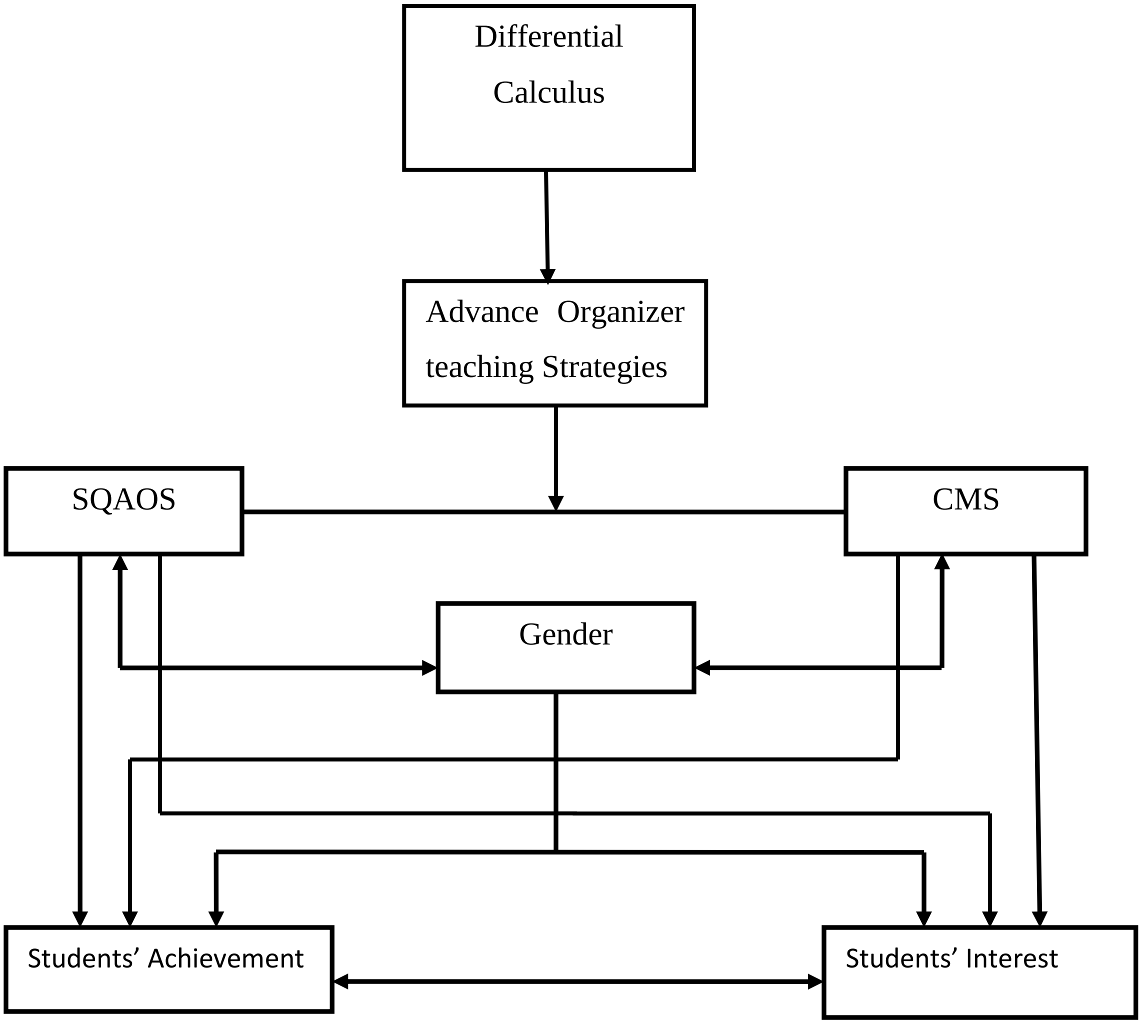

Figure 1: Schematic diagram showing the relationship among the key variables in the study.

The diagram above shows the interconnections among the variables that hold this study. The independent variables are the two teaching strategies (study questions as advance organizers strategy (SQAOS) and concept mapping strategy (CMS)) that were used in teaching differential calculus in mathematics. Students' achievement and interest in differential calculus are the dependent variables while gender (which has two levels: male and female) were the moderating variable. The diagram shows that each of the gender may affect students' achievement and interest in differential calculus. It also shows that both study questions as advance organizers

(SQAOS) and concept mapping strategies (CMS) interact with gender and this may significantly affect students' achievement and interest in differential calculus.

Theoretical Framework

Ausubel's Subsumption Theory

Ausubel subsumption theory states that learning is based upon kinds of super ordinate, representational, and combinational processes that occur during the reception of information. The proponent of the Ausubel subsumption theory was David Paul Ausubel, an American educator, psychologist and psychiatrist. Ausubel's theory is concerned with how individuals learn large amounts of meaningful material from verbal/textual presentations in a school setting (in contrast to theories developed in the context of laboratory experiments). Subsumption is the central idea running through the whole of Ausubel's learning theory. The key tenets of the Ausubel's learning theory are as follows:

1. Learners should be presented with the most general concepts first, and then their analysis.
2. The instructional materials should include new, as well as previously acquired information.
3. Comparisons between new and old concepts are crucial. Existing cognitive structures should not be developed, but merely reorganized within the learners' memory.
4. The role of the instructor is to bridge the gap between what's already known and what is about to be learned.

A primary process in learning is subsumption in which new material is related to relevant ideas in the exiting cognitive structure on substantive, non verbatim basis. Cognitive structures represent the residue of all learning experiences; forgetting occurs because certain details get integrated and lose their individual identity. According to Ausubel (1963), present experience is always fitted into what the learner already knows. A cognitive structure that is clear and well organized facilitates the learning and retention of new information. A cognitive structure that is confused and disorderly on the other hand, inhibits learning and retention. Learning can be enhanced by strengthening relevant aspects of cognitive structure. The major concepts (subsumers) in the cognitive structure act as anchoring posts for new information. The availability of anchoring ideas facilitates meaningful learning. The cognitive stability provided by anchoring ideas helps to explain why meaningful learning is retained longer than rote learning. Meaningful learning is anchored, rote learning is not.

A major instructional mechanism proposed by Ausubel is the use of advance organizers. "These organizers are introduced in advance of learning itself, and are also presented at a higher level of abstraction, generality, and inclusiveness; and since the substantive content of a given organizer or series of organizers is selected on the basis of its suitability for explaining, integrating, and interrelating the material they precede, this strategy simultaneously satisfies the substantive as well as the programming criteria for enhancing the organization strength of cognitive structure" (Ausubel, 1963, p. 81).

Ausubel emphasizes that advance organizers are different from overviews and summaries which simply emphasize key ideas and are presented at the same level of abstraction and generality as the rest of the material. Organizers act as subsuming bridge between new learning material and existing related ideas. This theory is relevant to the present study because its constructs elaborated how new meaningful materials can be incorporated in the cognitive structure; that is, how the present experience is always fitted into what the learner already knows for retention of learning through the use of advance organizers which act as subsuming bridge between new learning material and existing related ideas. The use of advance organizers in innovative ways helps to support students' mathematical thinking and reasoning in deeper and more conceptually based ways. This study involves the use of advance organizer that will help students to connect what they already know in mathematics calculus with what they are expected to learn through the use of study questions and concept maps.

Related Subthemes

Calculus

Calculus is arguably one of the most important topics in the senior secondary school mathematics in Nigeria. It is a peak of modern mathematics that lends itself to many fields such as engineering, medicine, economics and so on. According to Zakaria & Salle, (2013) calculus is a doorway to engineering careers. Calculus can be defined as mathematics of motion and change. According to Douglas (2009), it is the study of rates of change and much more. Calculus provides answers to some questions such as: how much does one value change when another value changes? How fast does an object move? How steep is a slope? These problems can be solved by calculating the derivative, which also allows you to answer the question: what is the highest or lowest value? Calculus is divided into branches: integral calculus and differential calculus.

Differential calculus is the study of derivatives. Derivative is a certain type of limit used to compute rates of change and slopes of tangents to curves (Douglas, 2009). The process of finding a derivative is called differentiation. Differentiation has applications to nearly all quantitative disciplines. For example, in Physics, the derivative of the displacement of a moving body with respect to time is the velocity of the body and the derivative of velocity with respect to time is acceleration, etc. The reaction rate of a chemical reaction is a derivative (Parong, 2013). In operations research, derivatives determine the most efficient ways to transport materials and design factories. Formal economic modeling began in the 19^{th} century with the help of differential calculus, to represent and explain economic behaviour, such as utility maximization, an early economic application of mathematical modeling (World Heritage Encyclopedia, 2016).

Since the inclusion of Calculus in the general senior secondary school mathematics curriculum, students' achievement has been very poor. The poor achievement in calculus is also recorded for Further Mathematics. This poor achievement could be as a result of poor teaching strategies employed by the mathematics teachers. The use of advance organizer strategy has been reported to be efficacious in teaching of some science subjects such as Physics, Biology and Geometry (Eze, 1992); (Sidhu & Singh, 2005); (Kararo & Shihusa, 2009); (Onwioduikit & Akinbobola, 2005); (Githua & Nyabwa, 2008); and (Agashi, 2014). However, no work has been reported in Nigeria on the effect of advance organizers on students' achievement and interests in differential calculus. This informed the need to explore the effect of study questions and concept maps as advance organizers on students' achievement and interest in differential calculus.

Advance Organizer

Advance organizers are statements, activities, or graphic organizers that help the learner anticipate and organize new information. According to Meyer (2003), it is information that is presented prior to learning that can be used by the learner to organize and interpret new incoming information. They are used at the beginning of lessons in which new information is to be learned. They indicate to the learner what information from a lesson will be important. Advance organizers are a visual organization practice which can be used at the beginning of a class or a new unit of study to provide a framework for how new information will be presented during the lesson.

Organizers offer a tool for educational scaffolding and can provide a variety of note taking techniques used to move students progressively toward stronger understanding and greater independence in the learning process. Scaffolding in teaching involves breaking down task into smaller steps to ease learning. It is also the process in which an individual builds off of prior knowledge in order to facilitate the learning of new complex material. The best use of scaffolding is when it allows the students to figure out the problems on their own by providing a temporary support (Ponchet, 2013).

Additionally, advance organizers can be used to facilitate active learning, something that increases student performance in test taking and concept application (Bohay, Blakely, Tamplin & Radvansky, 2011). Advance organizers provide an organizational framework that instructors develop prior to presenting information to students. This framework prepares students for what they are about to learn and can help provide links between new ideas and similar concepts. Advance organizers help instructors and students focus on information that is important and essential. Jafari and Hashim (2012) opined that the use of advance organizers aid in listening, comprehension, connecting information with prior knowledge, building confidence, motivation, and attentiveness. The format of advance organizers can be very basic or visually creative and will depend on the instructor’s style, the material to be learned, and the learner's characteristics. The methods for presenting information to students will also influence the design of the organizer. Advance organizers are especially effective when created using principles such as consistence, coherency, and creativity. It is on this basis that Agashi (2014) suggested that in using the advance organizer during lesson, the teacher should endeavour to: clarify the aims of the lesson, prompt awareness of relevant knowledge, and make links to / from the organizer and make organization and logical order of learning materials explicit. The teacher should strengthen the cognitive organization by allowing the students to make summaries, to point differences, to relate new example with the organizer.

There are two key types of advance organizer - expository and comparative organizers. According to James, Wirtz and Woodward (2017), an expository organizer can be used when material that will be presented is unfamiliar or new to students. For example, when an instructor presents a new lesson on a topic that has never been discussed and helps students make connections to what they may already know. It can be presented graphically or verbally and is highly effective when used at the start of a unit. The organizer is neatly displayed using pictures or text, and the organizer provides an example that shows a relationship between the organizer and the new material. A comparative organizer on the other hand, is presented when material is relatively familiar or when new ideas will be integrated with prior knowledge.

Advance organizer can take different forms. These forms include narrative organizer, graphic organizers or text organizers (Konrad, Joseph & Itoi, 2010). Narrative advance organizers use stories to activate background knowledge allowing students to make connections to things they already know. Graphic organizers capitalize on both linguistic and non-linguistic styles of information storage. There are many different kinds of graphic organizers. Some include thinking maps, concept maps **(**Venn diagrams, or; cluster maps) (Coffey, 2006).

Concept mapping is a strategy that involves the hierarchical arrangement and the relationships among different concepts. This is according to Roohi (2015) who asserted that concept mapping is an interlocking network of existing and fresh knowledge of the learner. It is a strategy that helps students in understanding complex ideas and clarifying ambiguous relationships (Ajaja, 2009). Concept mapping involves the construction of a concept map. A concept map is a graph structure containing node that are interlinked by labeled directed areas. The nodes which are linked together into proposition show how students connect or link concepts. The propositions are represented by arrows to connect individual concepts together. Concepts are usually depicted by rectangles or circles, forming the nodes of the new work by labeled links (Nwoke, Iwu & Uzoma, 2015). The directionality of the link is indicated by arrow. The conceptualization of the materials by the students is indicated by the directionality and the connecting proposition. The proposition, thus illustrates the contextual relationship of the concepts to each other (Adeneye & Adeleye, 2011).

However, it important to point out that the teaching and learning of mathematics not only centres on concepts, but it must also pay much attention to exercises and problems as well as the development of the abilities to resolve them. For this reason, concept mapping not only covers concepts, but it is also important to show aspects of the procedure for problem solving (Rafael, 2008). (See appendix C_4, page 190 for a sample of concept maps advance organizer used in this study).

Text organizers can consist of guided notes, verbal directions, or pre-questioning strategy. Students who use pre-questioning organizers benefit from the detailed information provided in an organized fashion when having to draw conclusions (Larson, 2009). Pre-questioning techniques or strategies are used to provide hints and clues about what will be taught as well as activate background knowledge and aid students in the process of filling in missing information (Helterbrand, 2012). It is the pre-questioning strategy in the context of this study that is referred to as study questions strategy. Study questions are questions given to students ahead of a lesson with the expectation that the answers will enable the learners to orient themselves with the topic they are about to learn. In other words, study questions direct students' attention to what is important in the upcoming differential calculus lesson. They also help to highlight relationships among ideas or mathematical concepts, such as differential calculus, that will be presented and also remind students of the relevant information they already have. It is also a strategy that can be used to develop students' capacity to learn independently (Nwoke, Iwu & Uzoma, 2015). The concept mapping strategy is closely related to the study questions strategy in that they are both given to students in advance of the lesson and they both help to highlight relationships among concepts that will be presented. However, while the former makes use of concept maps, the latter involves the use of questions. The questions are constructed in logical and sequential manner such that each preceding question serves as a hint or link to the succeeding questions. (See appendix C_3, page 127 for a sample of study questions as an advance organizer used in this study).

Therefore, there is need to conduct research into innovative ways of supporting mathematical thinking and reasoning in deeper and more conceptually based ways. This is why the researcher wanted to find out whether the use of study questions as advance organizer will be efficacious in enhancing students' achievement and interest in differential calculus.

Students' Achievement in Mathematics

Achievement can be seen generally as accomplishment or success. According to Abugu (2017), achievement is individual attainment in given task. This is in line with Gimba (2014) who opined that achievement is something that somebody has succeeded in doing after a lot of effort. In other words, it is an attainment of desired goals or objectives. Relating this education, academic achievement is the students' success in meeting short- or long- term goals in education. Gimba concluded that academic achievement can be viewed as the individual's performance up to the desired level in a particular field. Therefore, academic achievement is the knowledge attained or skills developed in the school subject usually designated by test scores or marks. This implies that achievement of learner in mathematics is evident in scores or type of grades the learner obtained at the end of the mathematics course as reported by the teacher. The scores or grades form the basis for promotion to the next class, award of degrees, certificates, prizes, scholarship or admission into the university.

Achievement of students in mathematics over the years has become overwhelmly poor. Apart from 2016 where WAEC chief examiner's report shows that 53.8% of candidates who registered for the May/June West African Secondary School Certificate Examination (WASSCE) had at least credit pass, reports for other years show that the performance level of students who had at least credit pass in mathematics is lower than 50% (Gogo & Nduka, 2017). The breakdown of the percentage of students that passed with at least credit over the last decade as reported by the Test Development Division, West Afican Examination Council (WAEC) were recorded as follows: 2007-15.56%, 2008-23.00%; 2009-31.00%; 2010-33.55%; 2011-38.98%; 2012-49.00%; 2013-36.00%; 2014-31.30%; 2015-34.18% (see appendix G, page 243**)**. These poor performances recorded in mathematics over the years has been attributed to several factors such as poor understanding of mathematics language (Agwagah, 2017), mathematics phobia and lack of interest (Obi, 2014), and most importantly poor teaching strategies adopted by teachers (Gimba, 2014).

The poor teaching strategies involve the use of the traditional approach which is over reliant in textbooks, diagrams and charts. These approach leads to rote learning and does not expose the mind of the learners to critical thinking, analysis and problem-solving. Several instructional strategies such as the problem solving strategy, cooperative learning strategy, constructivist learning and concept mapping strategy have been reported to be more efficacious in improving students' achievement in mathematics and other mathematics related fields. The researcher wanted to find out whether the use of study questions and concept maps as advance organizers will also be efficacious in improving students' achievement and interest in differential calculus.

Students' Interest in Mathematics

One variable that determine an individual's active participation in an activity is interest. When someone is interested in an activity, one is likely to be more deeply involved in that activity (Imoko and Agwagah, 2006). Interest can be defined as a subjective feeling of concentration or curiosity over something. It is a motivational variable that is linked with educational attainment in that students are more likely to engage in an academic activity, pay more attention, and generate higher performances if students are interested in the topic (Schunk, Pintrich and Meece, 2008). It can also be seen as the desire to re-engage in content over time, to seek answers to questions, to acquire more knowledge, and to promote achievement and excellence in education as well as professional careers, hence interest is an important force determining the quality of learning, educational and occupational choices (Areelu and Ladele, 2018).

Interest promotes learning because very little learning can take place without the learner becoming interested in the subject matter. According to Akogwu (2015), a child though intellectually and physically capable of learning, may never learn until the child's interest is stimulated. This is in line with the proverbial statement that "you can lead a horse to the water but you cannot force him to drink water''. Hence, once there is interest, the child's attention is drawn and this allows learning to take place. Interest produces efforts, efforts increase interest and a combination of the two usually results to success or achievement (Onyishi, 2009). Therefore, interest is an important variable in learning mathematics because when students become interested in mathematics, they are likely to be more deeply involved in it (Okigbo and Okeke, 2011). In order to sustain students' interest in mathematics, Abugu (2017) suggested that teachers should use the following strategies:

i. Clearly articulate learning goals;
ii. Show relevance to students' academic lives;
iii. Demonstrate relevance to students' professional lives;
iv. Highlight real-world applications of knowledge and skills
v. Connect to students' personal interest; and
vi. Show your own passion and enthusiasm.

When the students' interests are directly stimulated by following the above strategies, students will concentrate without duress. This implies that teachers are a key factor that influences students' interest in mathematics. In support of this view, Kessels and Hannover (2007) opined that lack of interest in mathematics is being worsened by factors arising from teachers, school, parents and government. On the teacher factor, (Anigbo 2016) submitted that an instructional strategy adopted by teachers is one of the key factors that affects students' interest in mathematics. Students' inactivity in the classroom as a result of low interest in mathematics is caused by the traditional method of teaching in senior secondary schools (Areelu & Dawodu, 2015). Thus, the strategies that will familiarize the students with the contents of instruction, increase their interest, empower them with sufficient level of mathematical proficiency and enhance their active participation in the subject and also efficacious in improving their interaction with the environment are highly needed (Areelu and Ladele, 2018).

The use advance organizer strategy in teaching has been reported by different researchers. Eze (1992) first reported that the use of study questions as advance organizers is efficacious in improving students' retention and interest in Integrated Science. Sidhu and Singh (2005) reported that advance organizer model is very effective in teaching of Physics. Kararo and Shihusa (2009) discovered that students taught Biology using advance organizer had a high level of motivation. Specifically, certain types of advance organizers as they affect students' achievement and interest in some branches of mathematics such as concept maps in trigonometry (Nekang, 2004 and Imoko, 2005); Commercial Arithmetic (Githua and Nyabwa, 2008); game and instructional analogy in numbers and numeration and algebraic process (Okigbo and Agu, 2010) advance organizer model in geometry (Agashi, 2014) and graphic organizers in word problems (Owolabi and Adaramati, 2015) have also been reported.

Although several studies have been done on the advance organizer strategy in teaching science fields and some branches of mathematics such as Commercial Arithmetic, Geometry and Trigonometry, no studies on advance organizers, particularly the use of study questions and concept maps as advance organizers have been done in differential calculus. Hence, the researcher wants to investigate the efficacy of study questions and concept maps as advance organizers on students' achievement and interest in differential calculus. However, Newen (2013) observed that gender as a variable is related to achievement.

Gender and Mathematics Learning

Gender inequality has over the years become a contentious issue as far as science, mathematics and technology are concerned. Several research reports showed that there were fewer female in mathematics than males. Historically, women were not found among the great ancient mathematicians who had made footprints in sand of time. Among the ancient mathematicians, the celebrated names such Euclid, Eratosthenes, Pythagoras, Pascal, Chike Obi and others were all men (Obi, 2014).

Culturally, males are believed to be superior to females. Females culturally, are believed to be significant for child-rearing and home-keeping; hence both their education and aspirations for means of livelihood have often been directed and tailored towards this job. Therefore, girls were made to do subjects like needle work, home management, nutrition, domestic science, housewifery, languages and literature so that they will be prepared for their roles as mothers and housewives which are societal expectation of them (Obi, 2014). This is why boys choose science courses in high schools and show greater interest in mathematics than the girls.

Socio-cultural factors play a major role in students' perceptions about the degree to which females and males are good at mathematics and the utility of studying the subject. Negative views held by influential individuals such as parents and teachers underscore these messages and are internalized by girls, negating their interest in mathematics. When considering the long-term impact, lack of interest in mathematics among girls is directly related to fewer women pursuing degrees in mathematics-related careers, including science, technology, and engineering (Watt, 2006). These pervasive social, cultural, and historical messages that mathematics is not useful to women, mathematics careers are masculine, and women are more interested in social fields are communicated explicitly and implicitly to girls from a very young age. For instance, parents tend to view mathematics as a more masculine field and buy more mathematics-related products for their sons than for their daughters (Amelink, 2012). This belief constrains girls from pursuing a career in mathematics-related disciplines. Even the girls who venture to try them do that with fear and anxiety. Thus, girls tend to show lack of interest to mathematics more than boys.

Consequently, the achievement of the male students in mathematics is usually higher than the female students' thereby creating imbalance in mathematics achievement. This is evident in several research reports in favour of the male as against the fewer reports in favour of the female students. Anyamene and Anyachebelu (2009); Nekang (2013); Agwagah (2013) and Areelu and Ladele (2018) all reported that males achieved significantly higher in mathematics than the females. Some other researchers have conflicting findings. Hydea and Mertzb (2009), Umeh (2011) and Malik, Farooq and Rabia (2016) reported high mathematics achievement in favour of the females.

In order to curb the imbalance in mathematics achievement, adequate motivation, good learning environment and the right teaching strategy should be employed. This means that when the right teaching strategy is employed, girls will achieve in mathematics as much as the boys. In other words, when the right teaching strategy is employed in teaching mathematics, the imbalance in mathematics achievement along gender will be mitigated. Hence this study explored how efficacious the use of advance organizer strategy in form of study questions and concept maps could be in solving the problem of gender disparity in mathematics achievement.

Review of Empirical Studies

Studies on Advance Organizers and Students' Achievement

Githua and Nyabwa (2008) carried out a study on the effects of analogies as advance organizer strategy during instruction on secondary school students' mathematics achievement in Kenya's Nakuru district. The purpose of this study was to develop and use advance organizers to augment the teaching of commercial arithmetic and then investigate its effects on students' achievement in the topic. Solomon four-group design was employed for the study. A simple random sample of four provincial mixed-sex secondary schools in Nakuru district was obtained. The experimental groups received the advance organizers as treatment and two control groups were taught in the conventional way. The sample size was 142 students. A mathematics achievement test (MAT) was used for data collection. The reliability of the MAT was obtained as 0.7564 using Kudder-Richardson (K-R 20) formula. The MAT was administered to two groups before the teaching of the topic and then to all four groups after learning the topic of commercial arithmetic. Descriptive statistics (mean, standard deviation, percentages) and inferential statistics ANCOVA was used for data analysis. The level of significance for acceptance or rejection of hypothesis was set at 0.05 α-level. The results indicated that students taught using advance organizers had significantly higher scores in MAT than those taught in the conventional way. The result also showed that there was no significant effect of gender on mathematics achievement. The above study and the present study are similar as they both sought to determine the effect of advance organizer on mathematics achievement. However, Githua and Ayieko's study differs from the present study in scope of the study, area of study, design of the study and some tools for data analysis. Moreover, while Githua and Ayieko's study focused on investigating the effect of analogies as advance organizers on only students' achievement on

Commercial Arithmetic, the present study focused on the use of study questions and concept maps as advance organizers on students' achievement and interest on differential calculus.

Okigbo and Agu (2010) did a study on the effects of mathematical game and instructional analogy as advance organizers on students' achievement in secondary school mathematics in Awka and Ogidi education zones of Anambra state of Nigeria. The purpose of the study was to investigate the effects of mathematical game and instructional analogy on students' achievement in junior secondary school mathematics. A 3×2 factorial design was adopted in the research. A total of 246 Junior Secondary Two (JS2) Mathematics students were involved in the study. The instrument for data collection was Mathematics Achievement Test (MAT) structured from Junior WAEC past question papers. The reliability of MAT was established to be 0.89 using Kuder Richardson formula 21 (kR-21). Data collected were analyzed using z-test (which is questionable) and analysis of covariance (ANCOVA). The findings, showed that: both game and analogy enhance achievement in mathematics; analogy was found to be more effective in facilitating students' achievement in mathematics than game; a non significant difference existed between the achievement of male and female mathematics students taught with either game or analogy; and there was no significant interaction between the use/non-use of advance organizers and gender on mathematics students' achievement. The study above has some similarities to the present study as both of them investigated the effect of advance organizers with consideration of students' gender. However, the study differs in the type of advance organizer studied, area of study, type of factorial design and method of data analysis.

Ifamunyinwa (2011) investigated the effect of behavioural objectives on students' achievement in senior secondary school mathematics instructions when used as advance organizers in Ibadan metropolis, Oyo State, Nigeria. The purpose of the study was to investigate the effect of behavioural objectives as advanced organizers on students' mathematics achievement. The study adopted a pretest, posttest control group quasi experimental design. Ninety SSS 2 students from two purposively selected senior secondary schools, who were the participants, responded to the researcher developed and validated mathematics achievement test (MAT), the research instrument. Reliability coefficient of 0.81, using the Kuder-Richardson K-R 21 formula was obtained for the MAT. The t-test of significance was the main statistical tool used for data analysis. Results of the study showed that the use of behavioural objectives as advance organizers is an effective strategy for teaching and learning mathematics as the experimental class which received the advance organizers obtained mean posttest score which was significantly higher than the mean posttest score of the control class. The results also revealed that the strategy is also capable of improving students' mastery of content at the comprehension level than at the knowledge level of cognition. Since retention is closely related to achievement one can say that this study is somewhat similar to the present study because they both investigated effect of advance organizers. However, the two studies differ in scope, area of study, the level of students studied, design of the study, topics taught and the type of advance organizers used for the study. Additionally, Ifamunyinwa investigated the effect of behavioural objectives as advance organizers on students' achievement without considering gender while the present study investigated the effect of study questions and concept maps as advance organizers on students' achievement and interest in differential calculus with much consideration on gender. Educational research that involves experimental studies should be guided by hypotheses. This is absence in Ifamuyinwa's study. Moreover, Ifamuyinwa used t-test for hypotheses analysis which is wrong since intact classes and quasi experimental design was used. The present study used analysis of covariance (ANCOVA).

Agashi (2014) studied the effect of advance organizer and concept attainment models on the achievement and retention of pre- NCE students in Geometry. The study was carried out in

Kogi and Benue states of Nigeria. The design of the study was pre-test post-test non equivalent control group design or quasi experimental design. The population of the study is 1100 pre-NCE students in the two states. The sample for the study composed of 830 pre-NCE students who offered geometry drawn from three out of the four public colleges of education. The instrument used for data collection was Pre-NCE Geometry Achievement Test (PNGAT). The reliability of the PNGAT was determined as 0.74 using Kudder-Richardson K-R20 formula. Scores from the PNGAT were analyzed using the means and analysis of covariance (ANCOVA). The findings showed that the use of advance organizer and concept attainment model were both more effective than the conventional method. Agashi's work and the present study have some similarities. Both studies investigated the use of advance organizer. The major difference however, is that Agashi did not include interest in his investigation which the present study is investigating. Also, while Agashi's study focused on Pre-NCE Geometry, the present study focused on senior secondary school differential calculus.

Ogbeba and Agernor (2016) investigated the effect of study questions as advance organizers on students' retention in Basic Science in Makurdi Local Government Area of Benue State, Nigeria. The study adopted a quasi-experimental, non-randomized, control-group, pre-test, posttest, research design. Purposive sampling was used to draw 167 students from a population of 1682 Upper Basic 2 students who were used for the study. Data were collected using the Basic Science Achievement Test (BSAT). The reliability coefficient of the BSAT using Kuder Richardson 20 (K-R20) formula was established as 0.86. Mean and Standard Deviation scores were used to answer the research questions while Analysis of Covariance (ANCOVA) was used to test the hypotheses at 0.05 level of significance. The researchers discovered that experimental subjects significantly outperformed the control subjects in mean retention while there was no significant difference between male and female students in retention. Since retention is closely related to achievement one can say that this study is somewhat similar to the present study because they both investigated the efficacy of study questions as an advance organizer. Moreover, both studies employed quasi-experimental design in their investigation and were much interested in gender factor. However, the two studies differ in area of study, subjects, fields and variables studied. While Ogbeba and Agernor investigated the effect of study questions as advance organizers on students' retention in Basic Science in Makurdi Local Government Area of Benue State, Nigeria, the present study investigated the effect of study questions as advance organizer on students' achievement and interest in mathematics differential calculus in Nsukka Local Government Area of Enugu State Nigeria.

Okeke, Okigbo and Mbakwe (2016) investigated the effect of the use of advance (instructional graphic organizer and analogy) organizers on academic achievement of introverted and extroverted students in mathematics in Senior Secondary Schools (SSS) in Awka South Local Government Area of Anambra State, Nigeria. Quasi-experimental design, specifically pretest–posttest nonrandomized control group was adopted for the study. The sample was made up of 71 SSS 2 students randomly selected from two schools using their intact classes. The mathematics achievement test was used for data collection. Kunder Richardson formula K-R20 was used to establish the coefficient of internal consistency for MAT and the value was 0.88. The mean and standard deviation were used to answer the research questions while the Analysis of Covariance (ANCOVA) was used to test the hypotheses at 0.05 levels of significance. Findings from the results revealed that students taught mathematics using advance organizer achieved higher than those taught without it. There was significant difference between the mean achievement scores of students in the two groups in favour of experimental group. Extroverts achieved slightly higher than introverts taught with advance organizers but the difference was not significant. The study above is somewhat similar to the present study as both of them investigated the effect of advance organizer on students' mathematics achievement. Additionally, instrument for data analysis were the same. Okeke,Okigbo and Mbakwe's study however, differs from the present study in scope, area of study, deisgn, the type and level of learners (introvert and extrovert SSS II students) sampled. Therefore, the researcher delimited the present study to the effect of study questions as advance organizers on students' achievement and interest in differential calculus.

Anyor and Amo (2017) studied the effect of mathematical games as advance organizers on upper basic two students' retention in mathematics. The purpose of the study was to find out the effect of advance organizers on upper basic two students' retention in mathematics. The quasi-experimental design of pre-test post-test control group type was adopted. A sample of 186 from a population of 672 upper basic two students in Gboko Local Government Area of Benue State was drawn for the study using simple random sampling. The instruments for data collection were Mathematics Achievement Test (MAT) and Mathematics Retention Test (MRT). using The reliability Kuder-Richardson formula 21 (K – R21), was obtained as 0.87 for the MAT items. Mean and standard deviation were used to answer research questions while hypotheses were tested at 0.05 level of significant using ANCOVA. Findings of the study revealed that students taught with advance organizers retained significantly higher than their counterpart taught without advance organizers. The study also revealed that there was no significant difference in the mean retention scores of male and female students taught mathematics with advance organizers. The study above is similar to the present study in the design, method of data analysis and the contribution of advance organizer to mathematics as it concerns gender. However, the area of study, subjects and dependent variables studied are different. While the Amo and Anyor studied the effect of advance organizers on upper basic two students' retention in mathematics in Gboko Local Government Area of Benue State, the present study delimited its study to the effect of study questions as advance organizers on Senior Secondary School (SSS III) students' achievement and interest on differential calculus in Nsukka Local Government Area of Nsukka Education Zone.

Among the studies reviewed, all of them revealed that advance organizers have positive effect on students' achievement on certain areas in mathematics except differential calculus which the research is investigating. Moreover, these studies on advance organizers seem to pay limited attention on their effects on students' interest in mathematics. Apart from the Basic Sciences, effect of study questions as advance organizers on students' achievement and interest on any branches of mathematics, have not been explored, hence this study.

Studies on Interest in Mathematics

Nekang (2004) investigated the effect of concept mapping on two groups of form four (final year secondary school) students' achievement and interest on elementary probability in Biu Division, in North West Province in Cameroun. The study also sought to find out the differential effect of concept mapping on the achievement of male and female subjects in elementary probability. A 17-item-essay type achievement test (ATEP) and a 13-item probability and statistics interest inventory (PSII) were administered on 154 subjects before and after teaching. The design of the study was the non randomized control-group pretest-posttest quasi-experimental design. Data were analyzed using the mean and standard deviations to answer the three research questions and a 2 way (2x2) analysis of covariance (ANCOVA) to test the two hypotheses at 0.05 level of significance. The results showed that concept mapping enhances students' interest in probability. Nekang's study and the present study are related as both studies explored the effect of concept mapping strategy on students' achievement and interest with much focus on gender. However, the two studies differ in the area of study, the mathematics topic studied, design and type of achievement test used to collect data. Nekang made use essay type test while the present study used objective type achievement test. Moreover, while Nekang used concept mapping strategy for treatment and conventional strategy as control, the present study used both the study questions as advance organizer and the concept mapping strategies for treatments.

Imoko (2005) investigated the efficacy of concept mapping in enhancing the achievement and interest in trigonometry in Benue state. The study was designed to examine the differential effects of concept mapping on the achievement and interest of male and female students as well as urban and rural students. Data were collected from 297 Senior Secondary Two (SS II) students using two instruments- Trigonometry Achievement Test (TAT) and Trigonometry Interest Inventory (TII). Concept maps and lesson plans on sine and cosine rule were used for the treatment. Research questions were answered using means and standard deviations, while the hypotheses were tested at 0.05 level of significant using the 3- way ANCOVA. The results showed that students exposed to the concept mapping strategy showed greater interest in the trigonometry concepts than those who were not. Also the mean interest scores of students in the urban areas are significantly higher than those in the rural areas.

Imoko’s study is similar to the present study to some extent because of relatedness of concept mapping to the advance organizer concept. Moreover, both work studied the same dependent variables- achievement and interest. Imoko’s study differs from the present study in terms of area of study, design and the content area of mathematics studied.

Akogwu (2015) conducted a study to determine the effect of Geometer’s sketchpad on students’ achievement and interest in Geometrical concepts in Junior Secondary Schools. The study sampled one hundred and fifty-seven (157) JS3 mathematics students randomly drawn from five schools in Nsukka Education Zone. The study employed a non-equivalent pretest-posttest quasi-experimental. The two groups were given pretest and posttest using the researcher's constructed instruments- the Geometry Achievement Test (GAT) and Geometry Interest Scale (GIS). The mean and standard deviation were used to provide answers to the research questions while the analysis of covariance (ANCOVA) was used to test the null hypotheses at 0.05 level of significance. The result of the study indicates there is significant difference between the mean interest scores of students taught geometry with geometer's sketchpad and those taught geometry without the geometer's sketchpad in favour of the experimental group. The result of the study also revealed that male students had higher interest score than their female counterparts. However, further analysis showed that gender has no significant influence on students' interest in geometry.

Akogwu's study is somewhat similar to the present study in terms of area of study and the subject. They both sought to determine how achievement and interest could be enhanced. However, they differ in the area of content coverage, and level of the subjects chosen for the study. Also, while Akogwu's study made use of software the present study used study questions and concept maps as advance organizers.

Ekon, Ekwueme and Meremikwu (2014) studied the effect of five phases of constructivist instructional model (CIM) on Junior Secondary School two (JSS 2) students' cognitive achievement and interest in basic science and mathematics in Calabar municipality of Nigeria. The main objective of the study is to investigate the effect of instructional models on students' achievement and interest in mathematics and basic science. A non-equivalent pre-test post-test quasi-experimental design was used. Two hundred and sixty-eight (268) participated in the study out of a total population of about 4,280 JSS 2 students in state government-owned schools using a simple random sampling technique. Two instruments: Cognitive Achievement Test (CAT) and Basic Science and Mathematics Interest Inventory (BSMII) were developed and administered to the students. Test-retest reliability method was used determine the reliability of the CAT and BSMII and Pearson moment correlation coefficients of 0.790 and 0.793 were obtained for the CAT and BSMII respectively . Mean Scores and standard deviations were used to answer the research questions. The result revealed significant difference in mean achievement and interest scores of the students in the experimental group indicating that use of the CIM enhanced students' academic achievement and interest in mathematics.

Ekon, Ekwueme and Meremikwu's study is similar to the present study to some extent as both of them explored the effect of some teaching strategies on students' achievement and interest in mathematics. Also, both studies used cognitive achievement test for data collection. However, their study differs from the present study in the type of the teaching strategies investigated, the level of learners, area of study, design of the study and the tools used for data analysis. The gender factor as it concerns students' achievement and interest in mathematics which is absence in their study was investigated in the present study.

Studies on Gender and Achievement in Mathematics

There are many contrasting research findings on gender and achievement in mathematics. Obi (2006) carried out a comparative study on the effects of Greeno problem solving models (GREPSOM) and Metes, Pilot, Rooznick and Krammers-Pa problem solving model (MEPSOM) on students' achievement and interest on word problems in algebra. The study was carried out in Nsukka Local Government Area of Enugu state. Obi employed 2×2 factorial type of quasi-experimental design. The study made use of a sample of one hundred and thirty (130) students of JSSIII drawn from three (3) out of fourteen (14) co- educational in Nsukka Educational Zone. Two instruments were used for data collection namely; mathematics achievement test (MAT) and variation interest scale (VIS). Data analysis was done using mean, standard deviation and ANCOVA. The result of the study revealed that male students achieved higher than their female counterparts. Obi's study is similar to the present study in area, design of study, and techniques used for data analysis. It differs however from the present study in terms of the levels of learners that were sampled and content area. While Obi's study investigated algebra JSS III, the present study focused on differential calculus in SSS III.

Onyishi (2009) conducted a study on the effect of mind maps on students' achievement in measures of central tendency in mathematics. The study was conducted in Nsukka Education Zone. The study adopted a non- equivalent control group design with the sample of 350 students drawn from four schools. The measure of central tendency achievement test (MCAT) was used for data collection from JSS II students. The data collected were analyzed using the mean, standard deviation and ANCOVA. The results showed that male students performed higher in measures of central tendency test than their female counterparts. The above study is similar with the present study in the area of study, method of data collection and data analyses. Also, they also investigated the influence of gender on mathematics achievement. However, the two studies differ in the teaching strategies, the content area of mathematics and the levels of learners studied and design.

Nekang (2013) investigated the effect of Rusbult's Problem Solving Strategy (RUPSS) on secondary school students' achievement in trigonometry in Fako Division in Cameroon. The purpose of this study was to investigate the effect of Rusbult's Problem Solving Strategy (RUPSS) on secondary school students' achievement in trigonometry in Fako Division. The design of the study was the non-equivalent control group design. A sample of 366 form four students consisting of 186 males and 180 females were drawn from three colleges in the division by a multi-stage sampling technique. The Trigonometry Achievement Test (TAT) was used for data collection. The test retest (stability) index of TAT was computed to be .91 using Pearson's product moment correlation coefficient. The data collected were analyzed using the mean, standard deviation and ANCOVA. The findings showed that male students obtained a higher mean score compared to their female counterparts supporting earlier findings by Orji (2002) who reported that there were significant different in mean achievement in favour of males. The above study is similar with the present study in the method of data collection and data analyses. Also, they also investigated the influence of gender on mathematics achievement. However, the two studies differ in the teaching strategies, the content area of mathematics and the levels of learners studied. While Nekang's study focused on the effect of Rusbult's problem solving strategy on form four students' trigonometry achievement, the present study investigated the effect of study questions as advance organizers on students' differential calculus achievement and interest.

However, there were contradictory reports on gender and achievement in mathematics. Gimba (2014) investigated the effect of computer assisted instructional package on achievement, retention and interest in set theory among senior secondary school students in Niger state. Gimba drew a sample of three hundred (300) students comprising of one hundred and eighty nine (189) males and one hundred and eleven (111) females selected from twenty-four thousand, four hundred and sixty-nine SS I students in thirty (30) senior secondary schools in two education zones. The design of the study was pretest posttest non equivalent control group quasi-experimental design. Data were collected using the achievement test on set theory (ATOST), interest inventory on set theory (INIOST) and retention test on set theory (RETOST). Data collected were analyzed using the mean, standard deviation and ANCOVA.

The result showed that female students achieved higher than the male students, though the difference was not statistically significant. This study is similar to the present study in terms of the design. Moreover, they both investigated gender differences in mathematics achievement. They differ however in the area of study and mathematics content area.

Abugu (2017) investigated the effect of mathematical modeling on senior secondary school students' achievement and interest in Geometry. The purpose of the study was to determine the effect of mathematical modeling on senior secondary school students' achievement and interest in Geometry. The study also examined the influence of gender on students' achievement and interest in geometry. Four research questions and six hypotheses were formed to guide the study. The study employed a non-equivalent pretest-posttest quasi-experimental design. A sample of 170 students was randomly selected from 2 out of 48 schools in Obollo Afor Education Zone, Nigeria. Data were collected by using the Geometry Achievement Test (GAT) and Geometry Interest Scale (GIS) and analysed using mean, standard deviation and Analysis of Covariance (ANCOVA). The result of the study showed that gender has no significant influence on the achievement of students' on Geometry. This result supported earlier findings by Yusuf (2009) who reported that gender has no effect on students' mathematics achievement scores.

Abugu's study is similar to the present study as they all explored the influence of gender on students' achievement using teaching strategies. Additionally, the design, method of data collection and data analyses are similar. However, both studies differ in the area, the mathematics content, level of learners and the teaching strategies explored.

Studies on Gender and Interest in Mathematics

Gender as it relates to students' interest in mathematics has been studied by different researchers. Ojo (2015) carried out a study on the effect of animated computer-based instructional packages on achievement and interest of junior secondary school students in algebra in Bwari Local Government Council of the Federal Capital Territory, Abuja, Nigeria. The purpose of the study was to determine the effect of animated computer-based instructional packages on students' achievement and interest in algebra. The design for the study was non-equivalent control group design. The sample size was 80, consisting of 40 males and 40 females. The instruments used for the study were the Algebra Achievement Test (AAT) comprising of objective and essay- type tests and Mathematics Interest Inventory (MII). The reliability coefficient of the objective AAT was obtained as 0.893 using kudder-Richardson formula K-R20 while the reliability coefficient of the essay type of the AAT was obtained as 0.874 using Kendall coefficient of concordance. The reliability coefficient of MII was determined as 0.753 using Cronbach Alpha (α). The mean, standard deviation and the Analysis of Covariance (ANCOVA) were the instruments used for data analysis. The results of the data analysis indicated showed there was no significant interaction effect of treatment and gender on students' interest score. The study above is similar to the present study to some extent as they both investigated students' achievement and interest in mathematics in relation to gender. Moreover, both studies used achievement test and interest scale as instruments for data collection, and the same statistical tools for data analyses. However, the two studies differ in area, design, content area of mathematics, instructional strategies, the type of achievement test administered and the level of students investigated.

Iji, Okoronkwo and Anyor (2017**)** carried a study on improving junior secondary students' interest in algebraic word problems using Igbo language as a medium of instruction in Abia state, Nigeria. The purpose of the study is to investigate how the use of Igbo Language as a medium of instruction improved junior secondary school students' interest in algebraic word problems. Quasi- experimental design of pretest posttest control group type was adopted. Population of the study was 2,114 with sample size of 212 Junior Secondary One Students from four secondary schools, in Umuahia metropolis, Abia State. The instrument used for data collection was Algebraic Word Problem Interest questionnaire (AWPIQ). The reliability coefficient of the AWPIQ was obtained as 0.99 using Cronbach Alpha (α). The mean and standard deviation were used to answer the research questions while the analysis of covariance (ANCOVA) was used to test the hypotheses at 0.5 level of significance. The findings showed there was no significant difference between mean interest ratings of male and female students. Iji, Okoronkwo and Anyor’s study is somewhat similar to the present study as they all explored gender and students’ interest in mathematics. However, the study above differs from the present study in the content area of mathematics studied, area and design of the study. Moreover, while Iji, Okonkwo and Anyor investigated students’ the use of Igbo language as a medium of instruction in improving junior secondary students’ interest in algebraic word problems, this study investigated the efficacy of study questions and concept maps as advance organizers on SS III students’ achievement and interest in differential calculus.

Allahnana, Akande, Vintseh, Usman and Alaku (2018) conducted a study on assessment of gender and interest in mathematics achievement in Keffi Local Government Area of Nasarawa State, Nigeria. The purpose of the study was to assess gender and interest in mathematics achievement. The study adopted ex-post facto research design. The sample of this study consisted of 361 SS 3 students (182 male and 179 female) drawn from the target population of 3548 SS 3 students' in 2017/2018 academic session from 24 Senior Secondary Schools. The researchers developed a single Performa as instrument for data collection. Logical validity index of 0.89 and 0.82 were obtained with the rational consensus of 0.85. Descriptive statistics of mean and standard deviation were used to answers research questions while inferential statistics of biserial correlation was used to test the formulated hypotheses at the 0.05 level of significance. The finding showed that male students showed more interest in mathematics than female students that is to say. The study above shared one similarity with the present study as they all explored gender and SS III students’ interest in mathematics. However, the study differs from the present study in the type of research and design, instruments for data collection and the statistical tools used for data analysis.

Summary of Literature Review

In this chapter, literatures on the variables and previous studies that are relevant to the present study were examined. These literatures were reviewed under conceptual and theoretical framework. Under the conceptual framework it was revealed that calculus forms a very solid foundation for further studies in different fields. Despite this importance of calculus, students' achievement in calculus and mathematics in general, remain unimpressive.

One factor that was reviewed to have contributed to the students' poor achievement in mathematics is the continual use of traditional teaching strategy. However, the use of advance organizer strategy has been revealed to be very helpful in teaching of some topics in mathematics and other mathematics related subjects. Based on this, literatures were reviewed on the concept of advance organizer, types and different forms of advance organizers including concept mapping and study questions which are very fundamental to the present study. Furthermore, concepts of achievement and interest which are the key variables in this study were reviewed. It was revealed that students' interest in mathematics is strongly connected to their achievement in mathematics. Students' interest in mathematics is dampened when teachers adopt inappropriate instructional approach/strategy and lack of interest leads to poor achievement in the long run. In order to enhance students' interest in mathematical concepts, innovative teaching strategies should be employed, hence this study. Gender as it affects mathematics achievement was also reviewed.

Theoretically, this study discussed Ausubel's subsumption theory. This theory was chosen because it supports meaningful learning which promotes how students learn and retain information. Ausubel suggests that advance organizers might foster meaningful learning by prompting the student regarding preexisting super- ordinate concepts that are already in the student's cognitive structure, and by otherwise providing a context of general concepts into which the student can incorporate progressively differentiated details. Therefore, the use of advance organizer in teaching promotes critical thinking, decision making, problem-solving and reflection. Hence, the present study will support this theory by examining the effect of study questions and concept maps as advance organizer on students' achievement and interest in differential calculus.

On the empirical studies, findings on advance organizers as they affect students' achievement were reviewed. Also, studies on students' achievement in mathematics were reviewed alongside with the instructional/strategies employed. This shows that a lot of efforts have been made or are still being made in the quest for improvement in instruction; this study is aimed at complementing these efforts. The review shows that the use of advance organizers in the form of concept maps, mind maps, behavioural objectives, mathematical games and instructional games can help to enhance students' achievement but there was no empirical evidence of any study on the effect of study questions and concept maps as advance organizers strategy on the senior secondary school students' achievement and interest in differential calculus which the present study explored. Concluding the empirical studies, gender and mathematics achievement and interest were reviewed. From the review on gender as related to mathematics, it is clear that the issue of gender disparity or parity has not been resolved as different contrasting findings were revealed. This gender controversy informs the need for further investigation to explore gender related differences with respect to mathematics achievement and interest, hence the call for this study.

From the available literature, it has been shown that the use of advance organizers in the form of concept maps, mind maps, behavioural objectives, mathematical games and instructional analogy enhance students' achievement but to the best knowledge of the researcher, there was no empirical evidence of any study in Nigeria on the efficacy of study questions and concept maps as advance organizers strategy on the senior secondary school students' achievement and interest in differential calculus in Nsukka Local Government Area of Enugu State. It is on this basis that the present study was set out to investigate the efficacy of study questions and concept maps as advance organizers on students' achievement and interest in differential calculus.

CHAPTER THREE

RESEARCH METHOD

This chapter discussed the procedures for carrying out this research under the following subheadings: design of the study, area of the study, the population of the study, sample and sampling techniques, instrument for data collection, validation of instrument, reliability of instrument, experimental procedures, control of extraneous variables and method of data analysis.

Design of the Study

This study adopted a 2×2 factorial research design. The 2×2 factorial design is the type of design that allows researchers to examine the main effects of two individual independent variables simultaneously and to detect interactions among variables (Davis & Buskist, 2008). This study meets the criteria for a 2×2 factorial design because it investigated effects of two independent variables: advance organizers strategies and gender on two dependent variables, achievement and interest and interaction effects of the advance organizer strategies and gender on students' achievement and interest. The 2×2 factorial design for this study is shown below:

		Treatment	
		E_1 (SQAOS)	**E_2** (CMS)
Gender	Male	Group 1	Group 2
	Female	Group 1	Group 2

Where:

E_1 = treatment with study questions as advance organizers strategy (SQAOS)

E_2 = treatment with concept mapping strategy (CMS)

Area of the Study

This study was carried out in Nsukka Local Government Area of Enugu State. There are 22 co-education secondary schools in Nsukka Local Government Area. People in Nsukka Local Government Area are well known for trade and education pursuit. The choice of Nsukka Local Government Area was made because of their students' poor achievement in mathematics in external examinations such as West African Senior Secondary Examination Certificate (WAEC) and perceived lack of interest in the subject, particularly in topics such as calculus

Population of the Study

The population of the study is one thousand, eight hundred (1800) SS III mathematics students in the co-education schools in Nsukka Local Government Area consisting of eight hundred and twenty-eight (828) males and nine hundred and seventy-two (972) females. (Planning, research and Statistics Unit, Post Primary School Management Board (PPSMB) Nsukka Zonal Office 2018) (see appendix A_1, page 89). The SSIII students were chosen for this study because differential calculus, a newly introduced topic in the general mathematics curriculum, can be found only in SSIII syllabus.

Sample and Sampling Techniques

The sample of this study consisted of hundred and eighty four (184) SS III mathematics students (90 males and 94 females). The simple random sampling technique with replacement was used to select 4 out of the 22 co-education schools used for the study. The simple random sampling technique was used in order to ensure that all the schools had equal chance of being selected for the study. The names of all the 22 schools were written on a paper, folded and put in a bowl by the researcher. After thorough reshuffling, the researcher randomly selected two schools that were in the SQAOS group and then replaced them back in the bowl. The researcher then reshuffled again and selected randomly, another set of two schools that were in the CMS group. The SS III mathematics students in these groups were used to form the sample size for the study (see appendix A_2, page 89).

Instrument for Data Collection

Two instruments were used in collecting data for this study: Differential Calculus Achievement Test (DCAT) and Differential Calculus Interest Scale (DCIS). The development of the Differential Calculus Achievement Test (DCAT) was guided by a test blueprint or table of specification (see appendix C_6 page 230) constructed by the researcher using the 5 units taught during the period of the study. These units were drawn based on Senior Secondary School Mathematics Curriculum for SSIII (Federal Ministry of Education, (FME), 2016). The construction of the test blue print was guided by the objectives of the content area covered in this study.

Since the subjects of this study are SSIII students, the items of the instruments were written to cover both the lower and the higher order questions. However, only four out of the six levels of the cognitive domain of the modified Bloom's taxonomy of education were applied in the construction of the table of specification. The six levels of Bloom's taxonomy are: remembering, understanding, applying, analyzing, evaluating and creating. Evaluating and creating were not used for the construction of the table of specification. The Differential Calculus Achievement Test (DCAT) have two sections- section A which contains the following students' personal information: name of student, school, class and gender and section B which consists of forty (40) objective questions developed from the following content areas in differential calculus: differentiation from the first principle, differentiation of polynomials, and product and quotient rules of differentiation, chain rule of differentiation and application of differentiation. Each question was awarded one mark to get a total of 40 marks which was converted to percentages for easy interpretation using the formula:

$$Student'\,final\,score = \frac{student's\,mark}{total\,mark} \times \frac{100}{1}.$$

The Differential Calculus Interest Scale (DCIS) consisting of twenty (20) items was developed by the researcher for this study. The instrument was designed to assess students' interest in differential calculus. The responses of the students were scored on a 4-point scale options namely; strongly Agree (SA) = 4, Agree (A) = 3, Disagree (D) = 2 and Strongly Disagree (SD) = 1 for positively cued questions and the reverse order of Strongly Disagree (SD) = 4, Disagree (D) = 3, Agree (A) = 2 and strongly Agree (SA) = 1 for the negatively cued questions. The DCIS has two sections, A and B. Section A was filled by the students to provide their personal data: gender, class and age; while section B was filled by the students to express their level of interest in differential calculus.

Validation of Instruments

The Differential Calculus Achievement Test (DCAT) and Differential Calculus Interest Scale (DCIS) were validated by specialists in Science Education Department (Measurement and Evaluation unit and Mathematics Education unit), University of Nigeria, Nsukka. Copies of the SQAOS and CMS lesson plans, for study questions advance organizer (SQAOS) and concept maps advance organizers for concept Mapping Strategy (CMS) advance organizers, test blue print, Differential Calculus Achievement Test (DCAT) and Differential Calculus Interest Scale (DCIS) together with the title of the study, purpose of the study, research questions and hypotheses were sent to the specialists.

The specialists were requested to validate the DCAT instruments for clarity, relevance and content validity. In order to ascertain the content validity of the DCAT instrument, the researcher developed a test blue print or table of specification. This is to ensure that the items in the DCAT were exhaustive enough to cover the objectives stated based on the content area covered by the researcher. The specialists were also requested to check the clarity, appropriateness and relevance of the 30- item DCIS instrument developed by the researcher. Furthermore, the experts were also requested to make their own input on any useful information that would help to ensure the validity of the instruments. Based on the corrections, inputs and recommendations made by the experts, the DCIS was modified and increased to thirty-five (35) items (see appendix E, page 239). The items were then subjected to factor analysis to ensure construct validity of the DCIS instrument. The results of the factor analysis were interpreted based on Meredith (1969) recommendations as follows: items that loads 0.35 and above were accepted as Factorially pure, FP, and hence valid while items with factor loadings of less than 0.35 are Factorially impure, FI (invalid) and hence rejected. Furthermore, items which loaded up to 0.35 in more than one factors (i.e. those items that are factorially complex, FC) were rejected. The result from the factor analysis on the DCIS shows that twenty-three (23) items- 1, 4, 5, 6, 7, 9, 10, 12, 13,15, 17, 18, 19, 20, 21, 24, 26, 27, 29, 30, 33, 34 and 35 were factorially pure (FP), eight (8) items - 2, 8, 11, 14, 22, 23, 28, 31 and and four (4) items - 3, 16, 25, and 32 were factorially impure (FI) and factorially complex (FC) respectively (see appendix E, page 241). These items, that is, factorially impure (FI) and factorially complex (FC) items were discarded leaving the researcher with only twenty-three (23) items that were factorially pure (FI). These 23 items were then taken back to the specialists who recommended that items 29, 33 and 35 should be discarded. The remaining twenty (20) items were then selected by the researcher for the study.

Reliability of Instruments

In order to ascertain the reliability, both the DCAT and DCIS were trial tested. To achieve this, the researcher adopted the following procedures. The instruments were administered to 30 SSIII students in Community Secondary School, Ihe in Awgu Local Government Area which is outside the area of the study. From the data obtained on DCAT, the internal consistency reliability was determined through the use of Kuder Richardson 20 (K-R 20) formula. The Kuder Richardson (K-R 20) formula was used because the test items were dichotomously scored. The internal consistency reliability coefficient of 0.81 was obtained for DCAT (see appendix D_1, page 236).

The internal consistency of DCIS was determined using Cronbach Alpha. Cronbach Alpha was used because the items of the instrument were polytomously scored. The reliability index was estimated to be 0.92 (see appendix D_2, page 238) indicating that the instrument is reliable.

Experimental Procedure

The experiment was carried out strictly by the mathematics teachers in the sampled schools who used the study questions and concept maps as advance organizers strategies in teaching differential calculus subtopics.

Training of Research Assistants

The mathematics teachers who assisted the researcher in carrying out the experiment were trained by the researcher for one working week as follows:

Day 1: On this day, the researcher introduced the topic and purpose of the research to the mathematics teachers for the study questions as an advance organizer (SQAOS) group. The teacher then enlightened the teachers on the meaning and the importance of the advance organizer strategies with emphasis on study questions as advance organizers

Day 2: The researcher introduced the topic and purpose of the research to the mathematics teachers for the concept maps as an advance organizer (CMS) group. The teacher then enlightened the teachers on the meaning and the importance of concept maps as an advance organizer.

Day 3: Workshop on lesson plans with study questions as an advance organizer was organized by the researcher. The researcher explains to the teachers in SQAOS group how to devise a study questions advance organizers plan pointing out how, where and when to incorporate it in the teaching/learning of differential calculus.

Day 4: Workshop on lesson plans with concept maps as advance organizers was organized by the researcher. The researcher explains to the teachers in CMS group how to devise a concept maps as an advance organizer lesson plan pointing out how, where and when to incorporate it in the teaching/learning of differential calculus.

Day 5: On this day, all the Mathematics teachers met with the researcher. Each of the mathematics teachers were asked to present a micro teaching on the differential calculus subtopics using the study questions and concept maps as advance organizers as the researcher makes corrections where necessary.

Before the commencement of the experiment, students in the experimental groups were given pre-test on differential calculus achievement test (Pre-DCAT) and differential calculus interest scale (Pre-DCIS) by their regular mathematics teachers. After the pre-test, the trained teachers commenced the experiments in their respective schools while following strictly the lesson plans developed by the researcher. Two sets of lesson plans were developed by the researcher to guide the teachers-one set of lesson plans was developed for the study question as advance organizer strategy (SQAOS) and the other set of lesson plans for concept mapping strategy (CMS). Both the SQAOS and CMS lesson plans (see appendix C_3, page 107 and appendix C_4, page 172) highlighted the same content, objectives and materials used. Under the same experimental conditions, students in the SQAOS group were taught each of the topics with study questions as advance organizer while the students in CMS group were taught the topics with the concept mapping strategy.

The mathematics teachers in the study questions as advance organizer (SQAOS) group after introducing the lesson, shared the study questions (which are related to what the students are about to learn) to the students in the group and asked them to study them in 15 minutes. After the 15 minutes, the teacher asked the students to stop and then continued his teaching while linking the study questions advance organizer with what he was teaching.

Similarly, the mathematics teacher in concept mapping strategy (CMS) group after introducing the lesson shared the concept maps to the students and asked them to study them in 15 minutes. The teacher then continued teaching them while making references to the advance organizer. The concepts and examples presented as study questions were the same as those ones developed as concept maps. At the end of the third week after the treatment, post CAT and post CIS were administered to both groups at the same time by the students' respective mathematics teachers. Students' response were scored and recorded by the researcher.

Control of Extraneous Variables

The researcher took the following steps to control the extraneous variables that may mar the findings of this study:

- Reactive Arrangement: In order to avoid the threat associated with participants behaving in a typical manner because they are aware of being in the study, the researcher ensured that the mathematics teachers for the respective SSIII classes chosen for the study teach them. Additionally, the DCAT and DCIS were also administered by the same teachers.
- Subjects Interaction: In order to prevent threats associated with the interaction of the SS III students in the experimental groups with the SS III students in the same class that were not involved in the study, the researcher administered a placebo effect so that all the students will appear to be treated the same.
- Inter Group Variable: In order to correct the initial differences among the subjects, the analysis of covariance (ANCOVA) was used to control the inter group variable that was introduced as a result of using intact classes.

Method of Data Analysis

The data obtained from the instruments (DCAT and DCIS) were analyzed using the mean, standard deviation and analysis of covariance (ANCOVA). The mean and standard deviations were used to answer the research questions while the analysis of covariance (ANCOVA) was used to test the formulated hypotheses. The analysis of covariance (ANCOVA) was chosen because of the lack of initial equivalence in the groups as a result of using intact classes. The ANCOVA was also chosen to control the effect of the pretest on post test.

CHAPTER FOUR

RESULTS

In this chapter, data collected were analyzed and presented in tables based on the research questions and hypotheses that guided the study as follows:

Research Question 1: What are the mean achievement scores of students taught differential calculus with study questions as advance organizers and those taught with the concept mapping strategy?

Table 1: Mean and standard deviation of pretest and posttest achievement scores for Study questions as an advance organizer strategy (SQAOS) and Concept mapping strategy (CMS) Groups

Groups		Pretest		Posttest		
	N	Mean	SD	Mean	SD	Mean Gain
SQAOS Group	92	44.71	16.82	73.45	12.10	28.74
CMS Group	92	37.89	13.19	54.89	15.74	17.00

The result on Table 1 shows that the students taught differential calculus with study questions as an advance organizer strategy (SQAOS) had mean in the pretest as 44.71 with standard deviation as 16.82 and mean in the posttest as 73.45 with standard deviation as 12.10. The difference between the pretest and posttest mean achievement score for the SQAOS group was 28.74. On the other hand, the students taught differential calculus with the concept mapping strategy (CMS) had mean in the pretest as 37.89 and mean in the posttest as 54.89 with standard deviation as 15.74. The difference between the pretest and posttest mean achievement score for the CMS group was 17.00. The mean gain of 28.74 and 17.00 for SQAOS and CMS groups respectively indicate that there was an improvement in students' achievement due to the advance organizer strategies.

Research Question 2: What are the mean achievement scores of male and female students in differential calculus?

Table 2: Mean and standard deviation of pretest and posttest achievement scores of male and female students in differential calculus

Groups		**Pretest**		**Posttest**		
	N	**Mean**	**SD**	**Mean**	**SD**	**Mean Gain**
Male	90	43.58	16.74	73.83	10.28	30.25
Female	94	39.12	13.85	54.91	16.67	15.79

The result on Table 2 shows that the mean achievement scores of male in the pretest was 43.58 with standard deviation of 16.74 while the mean achievement scores of female in the pretest is 39.12 with standard deviation of 13.85. The mean achievement scores of male in the posttest was 73.83 with a standard deviation of 10.28 while the mean achievement scores of female in the posttest was 54.91 with standard deviation of 16.67. The difference between the pretest and posttest mean achievement score of male students was 30.25 while difference between the pretest and posttest mean achievement score of female students was 15.79 respectively. The mean gain of 30.25 and 15.79 for the male and female students indicate that students' achievement irrespective of gender improved due to the advance organizer strategies.

Research Question 3: What is the interaction effect of advance organizers strategies and gender on students' achievement in differential calculus?

Table 3: Mean and standard deviation of pretest and posttest achievement Due to Interaction of the Strategies and Gender in differential calculus

Groups			**Pretest**		**Posttest**		
		N	Mean	SD	Mean	SD	**Mean Gain**
SQAOS Group	Male	45	46.73	16.80	79.89	8.31	33.16
	Female	47	44.71	16.78	73.45	11.99	28.74
CMS Group	Male	45	40.42	16.25	67.78	8.34	27.36
	Female	47	37.89	13.19	54.89	15.74	17.00

The result on Table 3 shows that the male students in the study question as an advance organizer strategy (SQAOS) group had a mean of 46.73 in the pretest and mean of 79.89 with mean gain of 33.16 while the female students in the SQAOS group had a mean of 44.71 in the pre-test and 73.45 in the post-test with a mean gain of 28.74. On the other hand, the male students in the concept mapping strategy (CMS) group had a mean of 37.89 in the pre-test and a mean of 54.89 in the post-test with a mean gain of 27.36 while the female students in the CMS group had a mean of 37.89 in the pre-test and 54.89 in the post-test with a mean gain of 17.00.

Figure 2: Profile Plots for Interaction Effect of Gender and Advance Organizers Strategies on Students' Achievement

The figure above shows the interaction effect of the advance organizers strategies and gender on students' achievement in differential calculus. The plot suggests an ordinal interaction of the advance organizers strategies and gender on students' achievement in differential calculus since there is no appreciable crossing of lines connecting the means.

Research Question 4: What are the mean interest ratings of students taught differential calculus with study questions as advance organizers and those taught using the concept mapping strategy?

Table 4: Mean and standard deviation of pretest and posttest interest ratings for Study questions as an advance organizer strategy (SQAOS) and Concept mapping strategy (CMS) Groups

Groups		**Pretest**		**Posttest**		
	N	**Mean**	**SD**	**Mean**	**SD**	**Mean Gain**
SQAOS Group	92	2.00	0.55	3.16	0.38	1.16
CMS Group	92	2.07	0.63	2.25	0.50	0.18

The result on Table 4 shows that the mean interest rating in the pretest for the study questions as an advance organizer (SQAOS) group was 2.00 with a standard deviation of 0.55 while the mean interest rating in the pretest for concept mapping strategy (CMS) group was 2.07 with standard deviation of 0.63. The mean interest rating in the posttest for the SQAOS group was 3.16 with a standard deviation of 0.38 while the mean interest rating in the posttest for CMS group was 2.25 with a standard deviation of 0.50. The pretest and posttest mean differences of SQAOS and CMS groups were 1.16 and 0.18 respectively with mean gain difference of 0.69.

Research Question 5: What are the mean interest ratings of male and female students in differential calculus?

Table 5: Mean and standard deviation of pretest and posttest interest ratings of male and female students in differential calculus

Groups		Pretest		Posttest		
	N	Mean	SD	Mean	SD	**Mean Gain**
Male	90	2.12	0.60	2.77	0.70	0.65
Female	94	1.96	0.57	2.65	0.58	0.69

The result on Table 5 shows that the mean interest rating in the pretest for the male students was 2.12 with a standard deviation of 0.60 while the mean interest rating in the pretest for the female students was 1.96 with standard deviation of 0.57. The mean interest rating in the posttest for the male is 2.77 with a standard deviation of 0.70 while the mean interest rating in the posttest for the female is 2.65 with a standard deviation of 0.58. The pretest and posttest mean differences of male and female students are 0.65 and 0.69 respectively with mean gain of 0.67

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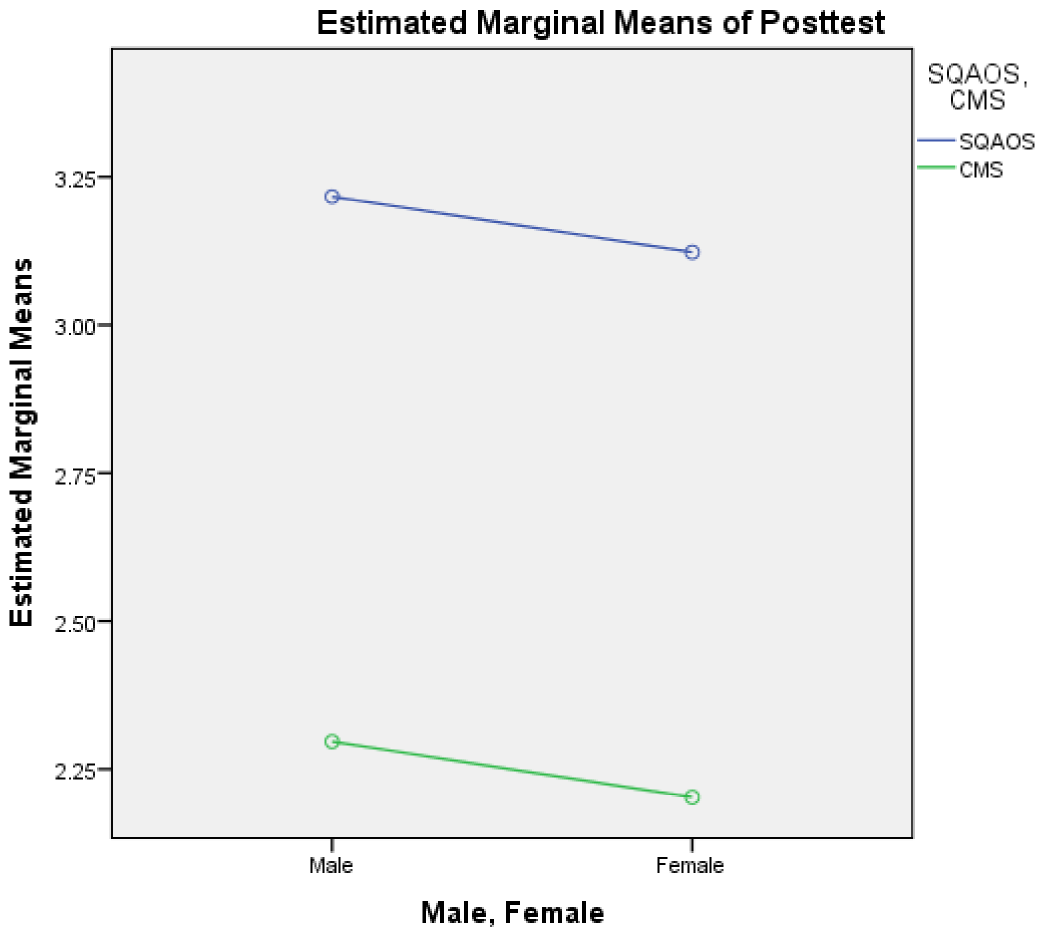
Research Question 6: What is the interaction effect of advance organizers strategies and gender on students' interest in differential calculus?

Table 6: Mean and standard deviation of pretest and posttest Interest Ratings of the Students Due to Interaction of the Strategies and Gender in differential calculus

Groups			**Pretest**		**Posttest**		
		N	**Mean**	**SD**	**Mean**	**SD**	**Mean Gain**
SQAOS Group	Male	45	2.28	0.55	3.28	0.29	1.00
	Female	47	1.73	0.41	3.05	0.42	1.32
CMS Group	Male	45	1.96	0.61	2.24	0.56	0.28
	Female	47	2.18	0.62	2.26	0.43	0.08

The result on Table 6 shows that the male students in the SQAOS group had a mean interest rating of 2.28 in the pretest and mean interest rating of 3.28 in the post-test with mean gain of 1.00 while the female students in the SQAOS group had a mean interest rating of 1.73 in the pre-test and 3.05 in the post-test with a mean gain of 1.32. On the other hand, the male students in the CMS group had a mean interest rating of 1.96 in the pre-test and a mean interest rating of 2.24 in the post-test with a mean gain of 0.56 while the female students in the CMS group had a mean interest rating of 2.18 in the pre-test and 2.26 in the post-test with a mean gain of 0.08.

Figure 3: Profile Plots for Interaction Effect of Advance Organizers Strategies and Gender on Students' Interest



The figure above explains the interaction effect of the advance organizers strategies and gender on students' interest in differential calculus. The parallel lines suggest that there was no interaction effect of advance organizers strategies and gender on students' interest in differential calculus.

Hypothesis 1

There is no significant difference in the mean achievement scores of students taught differential calculus with study questions as advance organizers strategy and those students taught with the concept mapping strategy.

Table 7: Analysis of Covariance (ANCOVA) of Students' Achievement Scores in Differential Calculus

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.
Corrected Model	35258.754[a]	4	8814.689	95.992	.000
Intercept	69478.028	1	69478.028	756.619	.000
Pretest	1138.221	1	1138.221	12.395	.001
Gender	14848.457	1	14848.457	161.700	.000
Groups	13043.170	1	13043.170	142.041	.000
Gender * Groups	1780.378	1	1780.378	19.388	.000
Error	16437.023	179	91.827		
Total	809333.000	184			
Corrected Total	51695.777	183			

R Squared = .682 (Adjusted R Squared = .675)

Result on Table 7 shows that the advance organizers strategies with respect to students' achievement in differential calculus had main effect of 142.041 with associated probability value of 0.000. The associated probability value of 0.000 is less than the level of significance set at 0.05, the null hypothesis was rejected.

Hypothesis 2

There is no significant difference between the mean achievement scores of male and female students in differential calculus.

Result on Table 7 was also used in testing hypothesis 2. The result shows that the F-value for male and female students was 161.700 with significance of F at 0.000. This value is less than the already set significance level of 0.05. Hence, the null hypothesis was rejected.

Hypothesis 3

There is no significant interaction effect of advance organizers strategies and gender on students' achievement in differential calculus.

The result on Table 7 was also used in testing hypothesis 3. The result shows that an F-ratio of 19.388 with associated probability value of 0.000 was obtained for the interaction between the advance organizers strategies and gender on students' achievement in differential calculus. Since the probability value of 0.000 is less than the level of significance set at 0.05, the null hypothesis was rejected.

Hypothesis 4

There is no significant difference between the mean interest ratings of students taught differential calculus with study questions as advance organizers strategy and those students taught with the concept mapping strategy

Table 8: Analysis of Covariance (ANCOVA) of Students' Interest Ratings in Differential Calculus

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.
Corrected Model	40.004[a]	4	10.001	52.784	.000
Intercept	80.462	1	80.462	424.666	.000
Pretest	.360	1	.360	1.901	.170
Gender	.438	1	.438	2.310	.130
Groups	38.812	1	38.812	204.845	.000
Gender * Groups	.392	1	.392	2.070	.152
Error	33.915	179	.189		
Total	1423.663	184			
Corrected Total	73.919	183			

a. R Squared = .541 (Adjusted R Squared = .531)

The result on Table 8 shows that the advance organizers strategies had a main effect of 204.845 on students' interest in differential calculus with associated probability value of 0.000. Since the probability value of 0.000 is less than the level of significance set at 0.05, the null hypothesis was rejected.

Hypothesis 5

There is no significant difference between the mean interest ratings of male and female students in differential calculus.

The result on Table 8 was also used to test hypothesis 5. The result shows that the F-value for male and female students with respect to students' interest ratings on differential calculus was found to be 2.310 with associated probability value of 0.130. Since the associated probability value of 0.130 is higher than the significant value set at 0.05, the null hypothesis was not rejected.

HO_6: There is no significant interaction effect of strategies and gender on students' interest in differential calculus.

The result on Table 8 was also used in testing hypothesis 6. The result shows that the F-value for the interaction effect of advance organizers strategies and gender on students' interest was found to be 2.070 with associated probability value of 0. 152. The probability value of 0. 152 was higher than the significant level set at 0.05. As a result, the null hypothesis was not rejected.

Summary of Findings

The findings of this study are summarized as follows:

1. Students' achievement in differential calculus improved due to the advance organizers strategies. However, there was significant difference in the mean achievement scores of students taught differential calculus with study questions as an advance organizer strategy (SQAOS)and the students taught with concept mapping strategy (CMS)
2. The mean achievement score of male students in differential calculus is higher than the mean achievement score of the female students. There was significant difference in the mean achievement scores of male and female students in differential calculus.
3. There was interaction effect of the advance organizer strategies and gender on students' achievement in differential calculus.
4. Students' interest in differential calculus improved due to the advance organizers strategies. However, there was significant difference in the mean interest ratings of students taught differential calculus with SQAOS and students those students taught with CMS strategy

5. The mean interest rating of the male students in differential calculus is higher than the mean interest ratings of the female students. However, there was no significant difference in the mean interest ratings of male and female students in differential calculus.
6. There was no significant interaction effect of gender and advance organizer strategies on students' interest in differential calculus.

CHAPTER FIVE

DISCUSSION OF FINDINGS, CONCLUSIONS, IMPLICATIONS, RECOMMENDATIONS AND SUMMARY

This chapter was organized under the following headings: discussion of findings, conclusion, educational implications of the study, recommendations, limitations of the study, suggestions for further studies and summary of the study.

Discussion of Findings

Discussion of findings was presented under the following headings:

- Students’ achievement in differential calculus
- Influence of gender on students’ achievement in differential calculus
- Interaction effect of gender and instructional strategies on students’ achievement in differential calculus
- Students’ interest in differential calculus
- Influence of gender on students’ interest in differential calculus
- Interaction effect of gender and instructional strategies on students’ interest in differential calculus

Students’ Achievement in Differential Calculus

Result from Table 1 shows that both the use of study questions as an advance organizer strategy (SQAOS) and concept mapping strategy (CMS) are both effective in enhancing students’ achievement in differential calculus. However, the mean achievement scores of students in the SQAOS group is higher than the mean achievement score of the in the CMS group. This result was analyzed by the ANCOVA in Table 7 result and shows that there is significant difference in the mean achievement scores of students taught differential calculus with study questions as advance organizers and students taught with concept maps as advance organizers. This showed that the use of study questions as advance organizers strategy in teaching differential calculus is more effective in improving students' achievement in differential calculus than the use of concept maps as advance organizers.

The higher achievement in differential calculus may have resulted due to the use of study questions as advance organizers strategy adopted by the teachers. The use of study questions as advance organizers encouraged the learners to learn independently by provoking deeper critical thinking of the solutions to the questions. This ensured active participation of the students in the learning process. The study questions as advance organizers strategy enabled them to understand the direction of the upcoming lesson and what is expected of them. It also enabled them to establish a cognitive structure which the teacher impressed upon as the lesson progressed. This strategy have given rise to meaningful learning which Ausubel's theory advocated as opposed to rote learning have accounted for the observed significant effect on the students' achievement in differential calculus. This finding supports earlier finding by Githua and Nyabwa (2008) who reported that students taught using analogies as advance organizers had significantly higher scores in Mathematics achievement test (MAT) than those taught in the conventional way.

Influence of Gender on Students' Achievement in Differential Calculus

The finding from Table 2 reveals that male students in the experimental groups recorded a higher achievement score than the female students in the POST DCAT. This is further confirmed by Table 7 of ANCOVA result which reveals that there is significant difference between the mean achievement scores of male and female students in differential calculus. This implies that gender has significant effect on students' achievement on differential calculus. These findings seem to disagree with that of Okigbo and Agu (2010) and Ajai and Imoko (2015) who reported a no significant difference in the achievement of male and female students in mathematics. However, the result of this study agrees with Nekang (2013), Owolabi and Adaramati (2015) who reported that male achieved higher in mathematics than the female.

Interaction effect of gender and advance organizer strategies on students' achievement in differential calculus

Result from Table 3 shows that the achievement of both the male female students in differential calculus improved as a result of the instructional strategies used. However, both the male and female students that were taught differential calculus with study questions as advance organizers achieved higher than the male and female students that were taught with concept maps as advance organizers. This implies that there is interaction effect of the strategies and gender on the students' achievement in differential calculus. This is confirmed in the ANCOVA result of Table 7 that shows that there is significant interaction effect of advance organizers strategies and gender on students' achievement in differential calculus. This means that advance organizers strategies and gender produced a combined effect on the mean achievement scores obtained by the students in the differential calculus test (DCAT). This finding is in line with Iweka (2006) who revealed that there is significant interaction effect of treatment and gender on students' achievement in mathematics.

Students' Interest in Differential Calculus

Result from Table 4 shows that both the use of study questions and concept maps as advance organizers are both effective in enhancing students' interest in differential calculus although the mean interest ratings of the students in the SQAOS group is higher than the mean interest rating of the students in the CMS group. This is further analyzed by the ANCOVA result of Table 8 which indicates that there is significant difference in the mean interest ratings of students taught differential calculus with study questions as an advance organizer strategy and the students taught with concept mapping strategy. These findings imply that the use of study questions and concept maps as advance organizers strategy enhances students' interest in differential calculus. This is in line with Akinoso (2011) who submitted that the use of appropriate instructional strategies enhance students' interest in mathematics learning.

The use of study questions and concept maps allowed the students to generate all or most of the logical relationships in the material to be learnt and point out relationships between familiar and less familiar material. This is in line with some researchers such as Cakir, Simsek and Tezcan (2009); Areelu and Dawodu (2015) who asserted that if the mathematics contents are phrased to students in a personalized way, the success, attitude, motivation and interest of the students are raised. The high interest rating recorded in differential calculus resulted due to the use of study questions and concept maps as advance organizers. This report lends credence to earlier findings by Nekang (2004) and Imoko, (2005) who recorded that concept mapping strategy is more effective in enhancing students' interest in mathematics than the conventional strategy. However, the findings of this research show that the interest rating recorded for the students who were taught differential calculus with study questions as advance organizers strategy is higher than the interest rating of the students taught with concept mapping strategy. This implies that the use of study questions as advance organizers is superior to the use of concept maps as advance organizers in enhancing students' interest in mathematics.

Influence of Gender on Students' Interest in Differential Calculus

Result from Table 5 shows that the mean interest rating of the male students is a little higher than the interest rating of the female students. The ANCOVA result from Table 8 shows that there is no significant difference in the mean interest ratings of male and female students in differential calculus. This implies that gender has no significant effect on students' interest in differential calculus supporting earlier findings of Akogwu (2015) who reported that gender has no significance influence on students' interest in mathematics. Hence, the use of innovative instructional strategies such as study questions and concept maps as advance organizers is capable of enhancing students' interest in mathematics irrespective of gender. Hence, gender has no influence on students' interest in differential calculus.

Interaction effect of gender and advance organizer strategies on students' interest in differential calculus

The finding from Table 6 shows that the interest of students (whether male or female) was improved due to the advance organizers strategies. However, the result of the ANCOVA in Table 8 shows that the interaction effect of the advance organizers strategies and gender is not statistically significance. This implies that the students showed consistent interest in differential calculus irrespective of gender. Hence, the effect of the advance organizers strategies (SQAOS and CMS) on students' interest in differential calculus does not depend on gender. This finding agrees with Areelu and Ladele (2018) who reported that there is no significant interaction effect of Jigsaw instructional strategy and gender on students' interest in mathematics.

Conclusion

Based on the findings of this research the following conclusions were drawn:

1. The use of study questions and concept maps as advance organizers strategies have positive effect on students' achievement in differential calculus. However, the study questions as advance organizers strategy is more efficacious in improving students' achievement in differential calculus than the concept mapping strategy.
2. Both the study questions and concept maps as advance organizers are efficacious in enhancing students' interest in differential calculus. However, the study questions as advance organizers strategy is more effective in improving students' interest than the concept mapping strategy.

3. The mean achievement score of the male students in differential calculus is higher than the mean achievement score of the female students. This result was further strengthened by the hypothesis result that there is significant difference in the mean scores of male and female students in differential calculus. This implies that gender has significance influence on students' achievement in differential calculus taught.
4. The mean interest rating of male students in differential calculus is higher than the mean interest rating of the female students in differential calculus. However, the difference in the mean interest ratings of male and female students in differential calculus is not statistically significant. Hence, gender has no significance influence on students' interest in differential calculus during the study.
5. There is significant interaction effect of advance organizers strategies and gender on students' achievement in differential calculus.
6. There is no significant interaction effect of advance organizers strategies and gender on students' interest in differential calculus knowledge.

Educational Implications of the study

The findings of this study have far- reaching implications to the teachers, curriculum planners and professional associations. Although the advance organizers strategies are effective in enhancing students achievement and interest in differential calculus, the study questions as advance organizers strategy is more effective than the more popular concept mapping strategy. Therefore, mathematics teachers should employ the study questions as advance organizers strategy more to compliment the concept mapping strategy in teaching mathematics. This is because the study questions help the students to establish a cognitive structure which the teacher strengthens as the lesson progressed. The logical structure of the questions with hints motivated the interest of the students and gave them sense of direction of the lesson. By comparing and cross-referencing of the lesson with the answers the students provided in the advance organizers meaningful learning for permanence was achieved as reflected in the students' achievement scores. This implies that a well structured and logical study questions as advance organizers can have a far reaching effect on students' achievement and interest in newly- introduced topics in mathematics curriculum such as calculus irrespective of gender. Hence, the overall goal of education for all and education for sustainable developments will be achieved.

The findings also show that study questions as advance organizers strategy can be effectively achieved within the normal mathematics class period. This calls for the curriculum planners or designers to make adequate provision for the incorporation of this great innovative and learner-centred instructional strategy into the curriculum. Mathematics textbook writers can also make provision for the study questions advance organizer prior to each unit. This will enable the classroom mathematics teachers to have a clue on how to develop study questions advance organizer as they prepare their lesson. This informs the need for government and professional associations such as Mathematical Association of Nigeria (MAN) and Science Teachers Association of Nigeria (STAN) to organize routine in-service training teachers, workshops and conferences for mathematics teachers.

Recommendations

Based on the findings of this research, the following recommendations are made:

1. Mathematics teachers should incorporate the study questions as advance organizers strategy in teaching and learning of mathematics since it enhances students' achievement and interest.
2. Mathematics textbook writers should include a well designed study questions advance organizer before any unit in their textbook.

3. Government in collaboration with professional associations should organize public lectures, seminars and workshops in order to create awareness and train teachers on the best way to use the study questions as advance organizers strategy.
4. Government through their education agencies should sponsor more researches into the use of study questions as advance organizers in other areas of mathematics and science fields

Limitations of the study

This research has the following limitations:

1. Students' interpretation ability of the study questions and concept maps advance organizers may have affected the findings of this research.

Suggestions for further study

Based on the researcher's observations, the following suggestions for further researches on the use of study questions as advance organizers in teaching and learning were made:

1. A replication of this study should be done on integral calculus.
2. Similar studies should be carried on the other newly-introduced topics in mathematics curriculum such as matrices and determinants, modulo arithmetic and binary operations.
3. A similar study can be conducted with considerations of the effects on school location, socio-economic status of learners.
4. Based on the students' cognitive ability of the students which differs, studies on the use of study questions and concept maps as advance organizers backed with cooperative learning strategies such as peer tutoring and think-pair share should be carried out.

Summary of the study

This study was carried out to determine the efficacy of study questions and concept maps as advance organizers on students' achievement and interest in differential calculus. Study questions are questions given to students ahead of a lesson to enable the learners to orient themselves with the topic they are about to learn. A concept map, on the other hand, is a graph structure that shows the relationship among the different concepts the learners are about to learn. Both the study questions and concept maps direct the learners' attention to what is important in the upcoming lesson. The study was delimited to SS III students in Nsukka Local Government Area of Enugu State, Nigeria and focused on some subtopics in differential calculus. Six research questions and six null hypotheses tested at 0.05 level of significance were formulated to guide the study.

Literatures related to the study were reviewed under conceptual and theoretical frameworks, related subthemes, and review of empirical studies and summary of literature review. Under conceptual framework the relationship among the variables in the study were shown diagrammatically. Paul David Ausubel's subsumption theory (1963) was reviewed because it is concerned with how learners connect their previous knowledge with what they are about to learn. Therefore, this study has lent credence to Ausubel's subsumption theory. The review of empirical studies showed that study questions as an advance organizers strategy is effective in enhancing students' achievement in some science fields. Furthermore, advance organizers inform of mathematical games, analogy and concept mapping strategy was reviewed to be effective in improving students' achievement and interest in some areas of mathematics such as trigonometry, probability, and commercial arithmetic. However, reports on efficacy of study questions and concept maps as advance organizers on students' achievement and interest in differential calculus in Nsukka Local Government Area where the researcher carried out this study were not available. This informed the need to carry out this research to complement earlier studies on advance organizers.

The study employed the 2×2 factorial design. A sample of one hundred and eighty-four (184) SS III students consisting of ninety (90) male and ninety-four (94) female from 4 co-educational schools in Nsukka Local Government Area of Enugu State through purposive sampling technique. Two schools each were randomly assigned to Study Questions as Advance Organizer Strategy (SQAOS) group and Concept Mapping Strategy (CMS) group . The classes in these schools were used for the experiments. The students in the SQAOS group were treated with study questions as advance organizers in differential calculus while the students in CMS group were treated with concept maps as advance organizers using the lesson plans developed by the researcher. All the groups received both the pre-test and post-test. Two instruments were used for data collection: differential calculus achievement test (DCAT) and differential calculus interest scale (DCIS). These instruments were validated by two specialists in Measurement and Evaluation unit and one specialist from Mathematics Education unit, all from the Department of Science Education; University of Nigeria, Nsukka. A test blue print was developed to ensure the content validity of the DCAT while the DCIS was subjected to factor analysis to ensure its construct validity. The reliability index of DCAT was established as 0.81 using Kudder Richardson (K-R 20) formula while the reliability index of DCIS was established as 0.92 using Cronbach Alpha

The data collected were analyzed with mean, standard deviation and analysis of covariance (ANCOVA). The mean and standard deviation were used to answer the research questions while analysis of covariance (ANCOVA) was used to test the null hypotheses at 0.05 level of significance.

The results of the analysis showed that both study questions and concept maps as advance organizers are efficacious in enhancing students' achievement and interest in differential calculus. However, study questions as an advance organizer is more efficacious than concept maps as advance organizers. Based on this, it was recommended that mathematics teachers should incorporate the study questions as advance organizers strategy in the teaching and learning of mathematics. It was also recommended that public lectures, seminars and workshop should be organized for mathematics teachers on the appropriate use of study questions as advance organizers.

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APPENDICES

APPENDIX A_1

POPULATION OF STUDENTS IN NSUKKA LOCAL GOVERNMENT AREA, ENUGU STATE

S/No	Name of School	Number of Male	Number of Female	Total
1	C.S.S ISIENU	50	93	143
2	OPI H.S OPI	9	11	20
3	C.S.S LEJJA	24	38	62
4	EDEM C.S.S EDEM	86	105	191
5	C.H.S UMABOR	35	25	60
6	C.S.S OKPUJE	20	30	50
7	C.S.S EHA-NDIAGU	24	20	44
8	C.S.S IBAGWA ANI	35	38	73
9	C.R.S EDE-OBALLA	103	98	201
10	C.S.S OBUKPA	86	87	173
11	C.S.S EZEBUNAGU	48	62	110
12	C.S.S NRU	7	13	20
13	MODEL S.S NSUKKA	50	61	111
14	C.S.S OPI-AGU	45	32	77
15	LEJJA H.S LEJJA	23	22	45
16	COMP. S.S OPI	20	37	57
17	C.S.S ALOR UNO	34	47	81
18	AGU UMABOR	13	20	33
19	C.S.S AKPOTORO-OBIMO	11	16	27
20	OKUTU C.S.S OKUTU	12	21	33
21	C.S.S BREME	61	49	110
22	C.H.S AJUONA-OBIMO	32	47	79
	TOTAL	828	972	1800

Source: Planning, research and Statistics Unit, PPSMB Nsukka Zonal Office 2019

APPENDIX A_2

SUMMARY OF SAMPLED SCHOOLS

Sampled school	Male	Female	Total
Model S.S School Nsukka	22	23	45
C. S. S Obukpa	23	24	47
C. S. S, Isienu	20	26	46
C. S. S Ede-Oballa	25	21	46
Total	90	94	184

APPENDIX B

INITIAL INSTRUMENTS

INTRODUCTION LETTER

Department of Science Education,
Faculty of Education,
University of Nigeria,
Nsukka.
8th October, 2018.

Dear Sir/Madam,

Request for Validation of Research Instruments and Lesson Plans

I am a postgraduate student of the above department carrying out a Master's degree (M.Ed.) Research on the topic: **Efficacy of Study Questions as Advance Organizers students' Achievement and Interest in Differential Calculus.** The following documents along with the purpose of the research, research questions and hypotheses have been attached: a test blue print (table of specification), Differential Calculus Achievement Test (DCAT), Differential Calculus Interest Scale (DCIS), lesson plans for both the experimental and control groups, the study questions advance organizers for experimental and the concept maps for the control group.

I hereby humbly request that you validate the instruments. Please, check the DCAT to see if they are appropriate, meaningful and in conformity with the test blue print and whether it is exhaustive enough to cover all the topics indicated in the lesson plan. The instruments should be checked for their clarity and suitability to the purpose of the study. I request that you remove all the ambiguous and irrelevant statements and suggest relevant information that may be vital to this study.

Many thanks for considering my request.

Yours faithfully,

Uzoigwe, Joseph Sylvester
PG/MED/16/80537

Purpose of the Study

The purpose of this study is to investigate the efficacy of study questions as advance organizer on students' interest and achievement in differential calculus.

Specifically, this study is set to determine:

i. The effect of study questions as advance organizers on students' achievement in differential calculus.
ii. The effect of study questions as advance organizers on the interest of students in differential calculus.
iii. Influence of gender on the achievement of students in differential calculus.
iv. Influence of gender on students' interest in differential calculus.
v. Interaction effect of the advance organizer strategies and gender on students' achievement in differential calculus.
vi. Interaction effect of the advance organizer strategies and gender on students' interest in differential calculus.

Research Questions

The following research questions guide the study:

i. What is the mean achievement score of students taught differential calculus with study questions as advance organizers and those students taught with the concept mapping strategy?
ii. What is the mean interest score of students taught differential calculus with study questions as advance organizers and those taught using the concept mapping strategy?
iii. What are the mean achievement scores of male and female students taught differential calculus with study questions as advance organizers?

iv. What are the mean interest scores of male and female students taught differential calculus with the study questions as advance organizers?

Research Hypotheses

The following null hypotheses were formulated to guide the study at 0.05 level of significance:

HO_1: There is no significant difference between the mean achievement scores of students taught differential calculus with study questions as advance organizers strategy and those students taught with the concept mapping strategy.

HO_2: There is no significant difference between the mean interest scores of students taught differential calculus with study questions as advance organizers strategy and those students taught with the concept mapping strategy.

HO_3: There is no significant difference between the mean achievement scores of male and female students taught differential calculus with study questions as advance organizer strategy.

HO_4: There is no significant difference between the mean interest scores of male and female students taught differential calculus using study questions as advance organizer.

HO_5: There is no significant interaction effect of the advance organizer strategies and gender on students' achievement in differential calculus.

HO_6: There is no significant interaction effect of the advance organizer strategies and gender on students' interest in differential calculus.

DIFFERENTIAL CALCULUS INTEREST SCALE (DCIS)

SECTION A: Demographic Data of the Respondents

Instruction: Please, fill in the gap your personal data.

Gender: Male ☐ Female ☐

Name of School: ……………………………………………………………………..

This instrument is developed to help you express your feelings towards differentiation (differential calculus). Please, read each statement carefully and tick against the point that best expresses your feelings or the extent to which you like the topic. I promise to treat all the IN.

SECTION B: Differential Calculus Interest Scale (DCIS)

Instruction: Please, tick against the point that best expresses your feelings

SA = Strongly Agree

A = Agree

D = Disagree

SD = Strongly Disagree

S/No	Item Statement	SA	A	D	SD
1	I love learning differential				
2	I get excited whenever it is time to learn differential calculus				
3	Using the rules of differentiation to solve problems is very interesting				
4	I ask questions during differential calculus lessons				
5	I make effort to discover why I fail a particular question on differential calculus and then take my correction				
6	I do not like missing lessons on differential calculus				
7	I enjoy differential calculus lessons more than other topics in mathematics				

8	Differential calculus is one of my best topics in mathematics				
9	The real life applications of differential calculus make it even more interesting to me				
10	I try on my own to derive rules of differentiation				
11	Differential calculus lessons are boring				
12	I study differential calculus ahead, in preparation for the next day's lesson				
13	I look for more challenging tasks on differential calculus				
14	I would want differential calculus to be removed from the mathematics syllabus				
15	I try to solve assignments on differential calculus by myself				
16	I always feel like going out of the class during lessons on differential calculus				
17	I ask a lot of questions to discover reasons during differential calculus lessons				
18	I solve more difficult tasks on differential calculus with my mates				
19	I like differential calculus because of my teacher's method of teaching				
20	I feel happy whenever an assignment is given on differential calculus				
21	Differential calculus is a very difficult topic				
22	I would like to attempt questions on differential calculus in external examinations such as WAEC and NECO				
23	I would like to study courses where differential calculus are applied in the university				
24	I enjoy differential calculus lessons when the teacher allows us to figure out solutions to problems by ourselves				
25	I often discuss differential calculus problems with my				

	mates during free periods				
26	I doze off during differential calculus classes				
27	I feel uncomfortable whenever it is time to learn differential calculus				
28	Rules of differentiation make it easier for me to solve problems in differential calculus				
29	Differential calculus lessons are real and practical				
30	I always feel that the teacher should continue teaching us differential calculus beyond the period allocated to it				

THE TEST DEVELOPMENT PROCESS

CONTENT DIMENSION

S/No	TOPIC	OBJECTIVES	PERCENTAGE (%)
1	Differentiation from the First Principle	3	15
2	Differentiation of Polynomials	3	15
3	Product and Quotient Rules of Differentiation	5	25
4	Differentiation by chain Rule	4	20
5	Application of Differentiation	5	25
TOTAL		20	100

COGNITIVE DIMENSION

Topic	Remembering 35%	Understanding 10%	Applying 40%	Analyzing 15%	Evaluating -	Creating -	Total 100%
Differentiation from the First Principle	I	I	I	-	-	-	3
Differentiation of Polynomials	I	-	II	-	-	-	3
Product and Quotient Rules of Differentiation	II	-	II	I	-	-	5
Differentiation by chain Rule	I	I	I	I			4
Applications of Differentiation	II	-	II	I			5
Total	7	2	8	3	-	-	20

TABLE OF SPECIFICATION FOR ACHIVEMENT FOR DIFFERENTIAL CALCULUS ACHIEVEMENT TEST (DCAT)

Content / Topic	Rememberi ng 35%	Understanding 10%	Applying 40%	Analyzin g 15%	Evalua ting -	Creatin g -	Total 100%
Differentiation from the First Principle **15%**	2	1	2	1	-	-	6
Differentiation of Polynomials **15%**	2	1	2	1	-	-	6
Differentiation by Product Rule **25%**	4	1	3	1	-	-	9
Differentiation by Quotient Rule **20%**	3	1	3	1	-	-	8
Differentiation by chain rule **25%**	4	1	4	2			11
Total	14	5	15	6	-	-	40

DIFFERENTIAL CALCULUS ACHIEVEMENT TEST (DCAT)

Section A: Personal Data of Students

Name of School ___________________________________**Gender:** Male ☐ Female ☐

Instructions: Answer all questions. Circle the correct answer from the options lettered A-D

1. The process of finding the differential coefficient of a function is called________(A) integration (B) differentiation (C) polynomials (D) evaluation
2. One of these is NOT associated with differential coefficient (A) gradient (B) slope (C) derivative (D) tangent
3. The differential coefficient of $y = f(x)$ from the first principle is given by ___________
   (A) $\frac{dy}{dx} = \lim \overset{\Delta x}{\to} 0 \, \frac{f(x+\Delta_x)-f(x)}{\Delta_x}$ (B) $\frac{dy}{dx} = \lim \overset{\Delta x}{\to} 0 \, \frac{f(x+\Delta_x)-f(x)}{x}$ (C) $\frac{dy}{dx} = \lim \overset{\Delta x}{\to} 0 \, \frac{f(x+\Delta_x)}{\Delta_x}$

   (D) $\frac{dy}{dx} = \lim \overset{\Delta x}{\to} 0 \, \frac{f(x+2x)-f(\Delta_x)}{\Delta_x}$
4. The differential coefficient of $y = f(x)$ at $x = a$ from the first principle is given by ___________

   (A) $\frac{dy}{dx} = \lim \overset{\Delta x}{\to} 0 \, \frac{f(a+2a)-f(\Delta_x)}{\Delta_x}$ (B) $\frac{dy}{dx} = \lim \overset{\Delta x}{\to} 0 \, \frac{f(a+\Delta_x)-f(a)}{a}$ (C) $\frac{dy}{dx} = \lim \overset{\Delta x}{\to} 0 \, \frac{f(a+\Delta_x)}{\Delta_x}$

   (D) $\frac{dy}{dx} = \lim \overset{\Delta x}{\to} 0 \, \frac{f(a+\Delta_x)-f(a)}{\Delta_x}$
5. Find the derivative of the function $y = 3x^2$ from the first principle (A) $2x^3$ (B) $6x$ (C) $6x^2$ (D) $3x$
6. Determine the derivative of $y = \frac{1}{x}$ from the first principle (A) x^2 (B) x^{-1} (C) $\frac{-1}{x^2}$ (D) $1 - x$
7. Given that $y = x^n$, derive the formula for the differentiation of y with respect to x (A) $\frac{dy}{dx} = nx^n$ (B) $\frac{dy}{dx} = x^{n-1}$ (C) $\frac{dy}{dx} = nx^{n-1}$ (D) $\frac{dy}{dx} = (n-1)x^n$
8. Differentiate $y = ax^n$ with respect to x A) $\frac{dy}{dx} = ax^n$ (B) $\frac{dy}{dx} = anx^{n-1}$ (C) $\frac{dy}{dx} = nx^{a-1}$ (D) $\frac{dy}{dx} = (n-1)x^{n-a}$
9. State the standard derivative for the function $y = tanx$ (A) $secx$ (B) $cosx$ (C) $sinx$ (D) sec^2x
10. The differential coefficient of e^x is_____________ (A) xe^x (B) e^{x-1} (C) e^x (D) $2e^x$
11. Find the derivative of $y = x^4 - 3x^3 + x + \frac{1}{x}$ at $x = -1$ (A) -13 (B) 5 (C) -3 (D) 25
12. Determine the derivative of $y = 6x^2 + 4x - 3sinx$ at $x = 0$ (A) 0 (B) 1 (C) 2 (D) 3

13. Given that $y = uv$ where u and v are functions of x then $\frac{dy}{dx}$ is equal to___________ (A) $u\frac{dv}{dx} + v\frac{du}{dx}$ (B) $\frac{dv}{dx} + \frac{du}{dx}$ (C) $u\frac{dv}{dx}$ (D) $u\frac{dv}{dx} - v\frac{du}{dx}$
14. The derivative $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ represents ____________rule of differentiation (A) chain (B) addition (C) quotient (D) product
15. Which of these rules of differentiation is best used to differentiate the product of two functions of x? (A) product (B) addition (C) quotient (D) chain
16. Differentiate $(3x + 1)(3x - 2)$ with respect to x (A) $18x - 3$ (B) $18x$ (C) $9x - 3$ (D) 9
17. If $y = \frac{u}{v}$ then $\frac{dy}{dx} =$ ___________ (A) $u\frac{dv}{dx} + v\frac{du}{dx}$ (B) $\frac{dv}{dx} + \frac{du}{dx}$ (C) $u\frac{dv}{dx} - v\frac{du}{dx}$ (D) $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
18. Find $\frac{dy}{dx}$ if $y = \frac{a(x)}{b(x)}$ (A) $a\frac{db}{dx} + b\frac{da}{dx}$ (B) $\frac{dy}{dx} = \frac{b\frac{du}{dx} - a\frac{dv}{dx}}{b^2}$ (C) $a\frac{db}{dx} - b\frac{da}{dx}$ (D) $\frac{db}{dx} + \frac{da}{dx}$
19. The derivative of $y = tanx$ can best be found by ____________ (A) quotient rule (B) product rule (C) chain rule (D) subtraction rule
20. Differentiate $y = \frac{2-x^2}{2+x^2}$ w.r.t x (A) $\frac{8x}{(2-x^2)^2}$ (B) $\frac{4x}{(2-x^2)^2}$ (C) $\frac{-8x}{(2+x^2)^2}$ (D) $\frac{8x}{(x^2)^2}$
21. Find the derivative of $y = cotx$ w.r.t x (A) sec^2x (B) cos^2x (C) $-cosec^2x$ (D) sin^2x
22. Which of these is not a function of function? (A) $y = (5x + 2)(4x + 2)$ (B) $y = e^{5x}$ (C) $y = \sin(2x + 1)$ (D) $y = (2 + x^2)^2$
23. The derivative of the function $y = cos5x$ with respect to x can best be solved by _____________ (A) quotient rule (B) product rule (C) chain rule (D) subtraction rule
24. If $y = f(u)$ and $u = f(x)$ then $\frac{dy}{dx}$ is given by___________(A) $u\frac{dv}{dx} + v\frac{du}{dx}$ (B) $\frac{dy}{du} \times \frac{du}{dx}$ (C) $u\frac{dv}{dx} - v\frac{du}{dx}$ (D) $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
25. If $y = \sin(3x - 1)$, find $\frac{dy}{dx}$ at $x = \frac{1}{3}$ (A) 0 (B) 1 (C) 2 (D) 3
26. Let $y = \ln(5x^2 + 2)$ then $\frac{dy}{dx}$ is__________ (A) $5x\ln(5x^2)$ (B) $\ln(5x^2)$ (C) $10x\ \ln(5x^2 + 2)$ (D) $\frac{10x}{5x^2+2}$
27. Find the derivative of $y = e^{(x+3)(x+2)}$ at $x = -2$ (A) 3 (B) 1 (C) 2 (D) 5
28. Which of these is NOT a rule for differentiation? (A) quotient rule (B) product rule (C) chain rule (D) subtraction rule
29. Which of these is NOT correct? (A) $u\frac{dv}{dx} + v\frac{du}{dx}$ (B) $\frac{dy}{du} \times \frac{du}{dx}$ (C) $u\frac{dv}{dx} - v\frac{du}{dx}$ (D) $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
30. Differentiation is applied in problems involving the following areas except (A) dynamics (B) slope of a curve (C) Economics (D) Law
31. Find the third derivative of the function $y = 6x^2 + 7x - 6$ (A) 0 (B) $12x + 7$ (C) $12x - 6$ (D) $7x - 6$

32. Find the second derivative of the function $y = xe^x$ (A) e^x (B) $2e^x$ (C) $e^x(x+2)$ (D) $2xe^x$
33. One of these is NOT a stationary point (A) maximum point (B) minimum point (C) yielding point (D) point of inflexion
34. If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$ at $x = a$ then we have ______________ (A) maximum point (B) minimum point (C) yielding point (D) point of inflexion
35. At what value of x does the function $f(x) = x^3 - 3x^2 - 2x$ have a point of inflexion? (A)3 (B) 1 (C) 4 (D) 8
36. The necessary condition for $y = f(x)$ to have a maximum point is that______________ (A) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$ (B) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$ (C) $\frac{dy}{dx} = \frac{d^2y}{dx^2}$(D) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$
37. Let $s = f(t)$ be a distant function where s is the displacement and t is the time. Then the velocity and acceleration are defined as ___________ and _____ respectively (A) $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ (B) $\frac{ds}{dt}$ and $\frac{d^2t}{dx^2}$ (C) $\frac{ds}{dt}$ and $\frac{d^2s}{dt^2}$ (D) $\frac{dt}{ds}$ and $\frac{d^2t}{ds^2}$
38. If a particle with the distance function $s = f(t)$ travelling with a velocity is momentarily at rest, then___________ (A) $\frac{d^2t}{dx^2} = 0$ (B) $\frac{ds}{dt} = 0$ (C) $\frac{ds}{dt} = \frac{d^2s}{dt^2}$ (D) $\frac{dt}{ds} - \frac{d^2t}{ds^2}$

The motion of a particle along a straight line is described by the distance function

$$s = \frac{t^3}{3} - 2t^2 - 3t$$

Use the information above to answer questions 38 and 39.

39. What is the velocity of the particle? (A) $t^2 - 4t - 3$ (B) $4t - 3$ (C) $t^2 - 4t$(D) $9t - 12$
40. A body is defined by the distance function find the acceleration of the body at $t = 3\ secs$. (A) $5ms^{-2}$ (B) $3ms^{-2}$ (C) $2ms^{-2}$ (D) $1ms^{-2}$

MARKING SCHEME FOR DIFFERENTIAL CALCULUS ACHIEVEMENT TEST

Item Number	Correction Options
1	B
2	D
3	A
4	B
5	B
6	C
7	C
8	B
9	D
10	C
11	A
12	D
13	A
14	D
15	A
16	A
17	D
18	B
19	A
20	C
21	C
22	A
23	C
24	B
25	A
26	D
27	D
28	D
29	C
30	D
31	A
32	C
33	C
34	B
35	B
36	D
37	B
38	B
39	A
40	C

APPENDIX C

FINAL DRAFT OF INSTRUMENTS

APPENDIX C_1

INTRODUCTION LETTER

Department of Science Education,
Faculty of Education,
University of Nigeria,
Nsukka.
8th October, 2018.

Dear Sir/Madam,

Request for Validation of Research Instruments and Lesson Plans

I am a postgraduate student of the above department carrying out a Master's degree (M.Ed.) Research on the topic: **Efficacy of Study Questions as Advance Organizers on students' Achievement and Interest in Differential Calculus.** The following documents along with the purpose of the research, research questions and hypotheses have been attached: a test blue print (table of specification), Differential Calculus Achievement Test (DCAT), Differential Calculus Interest Scale (DCIS), lesson plans for both the experimental and control groups, the study questions advance organizers for experimental and the concept maps for the control group.

I hereby humbly request that you validate the instruments. Please, check the DCAT to see if they are appropriate, meaningful and in conformity with the test blue print and whether it is exhaustive enough to cover all the topics indicated in the lesson plan. The instruments should be checked for their clarity and suitability to the purpose of the study. I request that you remove all the ambiguous and irrelevant statements and suggest relevant information that may be vital to this study.

Many thanks for considering my request.

Yours faithfully,

Uzoigwe, Joseph Sylvester
PG/MED/16/80537

Purpose of the Study

The purpose of this study is to investigate the efficacy of study questions as advance organizer on students' interest and achievement in differential calculus.

Specifically, this study is set to determine:

i. the effect of study questions as advance organizers on students' achievement in differential calculus.
ii. the effect of study questions as advance organizers on the interest of students in differential calculus.
iii. influence of gender on the achievement of students in differential calculus.
iv. influence of gender on students' interest in differential calculus.
v. interaction effect of the advance organizer strategies and gender on students' achievement in differential calculus.
vi. interaction effect of the advance organizer strategies and gender on students' interest in differential calculus.

Research Questions

The following research questions guide the study:

i. what is the mean achievement score of students taught differential calculus with study questions as advance organizers and those students taught with the concept mapping strategy?
ii. what is the mean interest score of students taught differential calculus with study questions as advance organizers and those taught using the concept mapping strategy?
iii. what are the mean achievement scores of male and female students taught differential calculus with study questions as advance organizers?

iv. what are the mean interest scores of male and female students taught differential calculus with the study questions as advance organizers?

Research Hypotheses

The following null hypotheses were formulated to guide the study at 0.05 level of significance:

HO_1: There is no significant difference between the mean achievement scores of students taught differential calculus with study questions as advance organizers strategy and those students taught with the concept mapping strategy.

HO_2: There is no significant difference between the mean interest scores of students taught differential calculus with study questions as advance organizers strategy and those students taught with the concept mapping strategy.

HO_3: There is no significant difference between the mean achievement scores of male and female students taught differential calculus with study questions as advance organizer strategy.

HO_4: There is no significant difference between the mean interest scores of male and female students taught differential calculus using study questions as advance organizer.

HO_5: There is no significant interaction effect of the advance organizer strategies and gender on students' achievement in differential calculus.

HO_6: There is no significant interaction effect of the advance organizer strategies and gender on students' interest in differential calculus.

APPENDIX C2

DIFFERENTIAL CALCULUS INTEREST SCALE (DCIS)

SECTION A: Demographic Data of the Respondents

Instruction: Please, fill in the gap your personal data.

Gender: Male [ ] Female [ ]

Name of School: ………………………………...

This instrument is developed to help you express your feelings towards differentiation (differential calculus). Please, read each statement carefully and tick against the point that best expresses your feelings or the extent to which you like the topic. I promise to treat all the information you will provide with utmost confidentiality.

SECTION B: Differential Calculus Interest Scale (DCIS)

Instruction: Please, tick against the point that best expresses your feelings

SA = Strongly Agree

A = Agree

D = Disagree

SD = Strongly Disagree

S/No	Item Statement	SA	A	D	SD
1	I like learning differential calculus through group discussion				
2	I get excited whenever it is time to learn differential calculus				
3	Differential calculus is a very interesting topic				
4	I ask questions during differential calculus lessons				
5	I make effort to discover why I fail a particular question				

	on differential calculus and then take my correction				
6	I do not like missing lessons on differential calculus				
7	I enjoy differential calculus lessons more than other topics in mathematics				
8	Differential calculus is one of my best topics in mathematics				
9	I try on my own to derive rules of differentiation				
10	I study differential calculus ahead, in preparation for the next day's lesson				
11	I look for more challenging tasks on differential calculus				
12	I try to solve assignments on differential calculus by myself				
13	I always feel like going out of the class during lessons on differential calculus				
14	I ask a lot of questions to discover reasons during differential calculus lessons				
15	I feel happy whenever an assignment is given on differential calculus				
16	Using the rules of differentiation in solving problems is interesting				
17	I would like to attempt questions on differential calculus in external examinations such as WAEC and NECO				
18	I often discuss differential calculus problems with my mates during free periods				
19	I doze off during differential calculus classes				
20	I feel uncomfortable whenever it is time to learn differential calculus				

APPENDIX C_3

LESSON PLAN FOR STUDY QUESTIONS ADVANCE ORGANIZERS

(SQAOS GROUP)

WEEEK ONE

Subject: Mathematics

Class: SSS III

Duration: 40 minutes

Content: Differentiation from the First Principle

Specific Objectives: At the end of the lesson, the students should be able to:

(i) define and explain the concept of differentiation

(ii) state the differential coefficient of the function $y = f(x)$ from the first principle

(iii) find the derivative of functions from the first principle

Instructional Materials: Markerboard illustrations

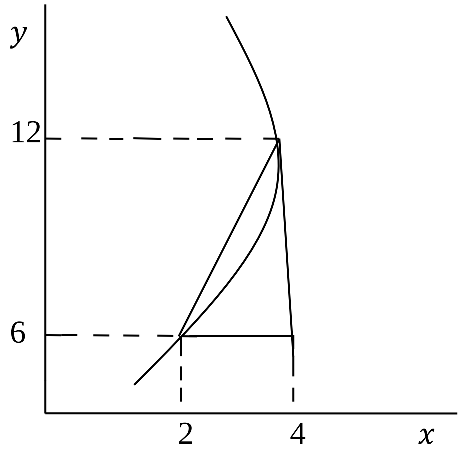
Entry Behaviour: The students can:

i. state the formula for determining the slope of a curve.

ii. determine the slope of a curve.

Test on Entry Behaviour:

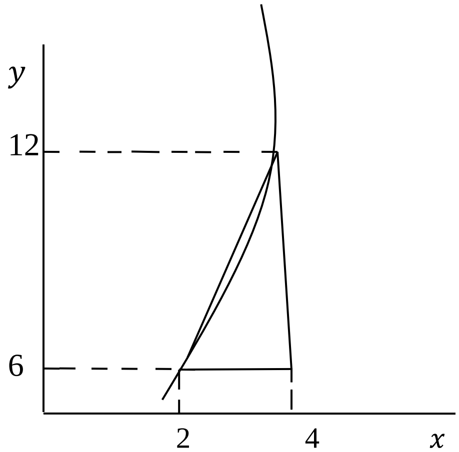
1. state the formula for finding the slope of the graph below?
2. determine the slope of the graph below

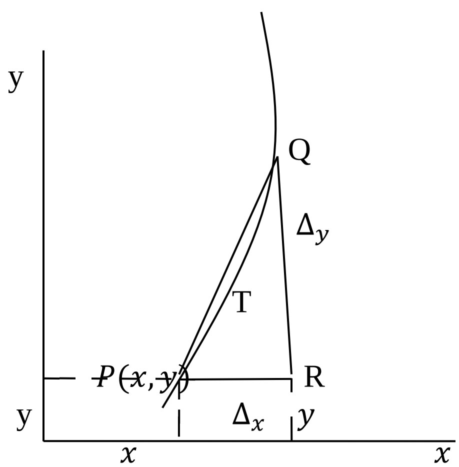


Instructional Techniques: Set induction, questioning, reinforcement, illustration, explanation, non verbal clues, repetition and use of examples

Insructional Procedure:

Steps	Contents Development (CD)	Teachers' Activities	Students' Activities	Strategies
1	Introduction	The teacher arouses the students' interest by asking the following questions: use the graph below to answer the following questions	The students provide the answers to the questions thus: $Slope = \frac{y_2-y_1}{x_2-x_1}$ $Slope = \frac{12-6}{4-2} = \frac{6}{2} =$ 3	Set induction Questioning and reinforcement

		 1. What is the formula for finding the slope of the graph above? 2. Determine the slope of the graph		
2	The concept of Differentiation and Differentiation from the First Principle	The teacher first explains to the students the objectives of the lesson and what is expected of them. The teacher then gives each student the study questions advance organizer and asks them to study them carefully and provide answers to the questions sequentially in 15 minutes (see the last page of this lesson plan for the study questions advance organizers). The teacher explains to the students the need to	Each student studies the questions independently and provides the answers to them. The expected answers are: 1. (A) increment of x and y 2. (B) $m_1 = \frac{\Delta_y}{\Delta_x}$ 3. (B) Δ_x tends to 0 4. (A) $\lim_{\Delta x \to 0}$ 5. (A) $m_2 = \lim_{\Delta x \to 0} \frac{\Delta_y}{\Delta_x}$ 6. (A) $\frac{dy}{dx} =$	Explanation, non verbal clues and repetition

		provide the answers sequentially and instructs the students not to jump or mix up the numbers as each question is a hint to the succeeding question. After the 15 minutes, the teacher asks the students to stop and then begins to explain the concept of differentiation thus:  The slope $m_1 = \frac{RQ}{RP} = \frac{\Delta_y}{\Delta_x}$ Let point P be fixed and let Q approach P so that we now have a new line PT. As point Q approaches the point on the curve, Δ_x becomes smaller and smaller until it eventually becomes zero. Then the	$\lim_{\Delta x \to 0} \frac{\Delta_y}{\Delta_x}$ 7. (B) $\frac{dy}{dx} = 2x$ 8. (A) $y + \Delta_y = f(x + \Delta_x)$ 9. (B) $\Delta_y = f(x + \Delta_x) - f(x)$ 10. (B) $\frac{\Delta_y}{\Delta_x} = \frac{f(x+\Delta_x) - f(x)}{\Delta_x}$ 11. (A) $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta_x) - f(x)}{\Delta_x}$ 12. (D) integral of y 13. (B) $f(x) = x^2$ and $f(x + \Delta_x) = (x + \Delta_x)^2$ 14. (A) $\frac{dy}{dx} = 2x$ 15. (A) $\frac{dy}{dx} = 4$ The students then listen attentively to the teacher. The students compares what the teacher is teaching with the answers they provide in the advance organizer	

		limiting value m_2 of m_1 as Δ_x tends to zero can be written as $m_2 = \lim \xrightarrow{\Delta x} 0\ m_1 = \lim \xrightarrow{\Delta x} 0\ \frac{\Delta_y}{\Delta_x}$ and is represente by $\frac{dy}{dx}$ which is read as dee y dee x . That is, $\frac{dy}{dx} = \lim \xrightarrow{\Delta x} 0\ \frac{\Delta_y}{\Delta_x}$ $\frac{dy}{dx}$ is called differential coefficient or derivative of y with respect to x. We also use $f'(x)$ or y' read f prime of x or y prime to mean $\frac{dy}{dx}$. The process of determining the differential coefficient or derivatives is called differentiation. Terms often associated with differentiation are: gradient or slope of a curve, rate of change. Now consider $y = f(x)$ then $y + \Delta_y = f(x + \Delta_x)$ $\Delta_y = f(x + \Delta_x) - y$, i.e $\Delta_y = f(x + \Delta_x) - f(x)$		

		Divide both sides by Δ_x $\frac{\Delta_y}{\Delta_x} = \frac{f(x+\Delta_x)-f(x)}{\Delta_x}$ $\lim \overset{\Delta x}{\rightarrow} 0 \frac{\Delta_y}{\Delta_x} = \lim \overset{\Delta x}{\rightarrow} 0 \frac{f(x+\Delta_x)-f(x)}{\Delta_x}$ But $\frac{dy}{dx} = \lim \overset{\Delta x}{\rightarrow} 0 \frac{\Delta_y}{\Delta_x}$ $\therefore \frac{dy}{dx} = \lim \overset{\Delta x}{\rightarrow} 0 \frac{f(x+\Delta_x)-f(x)}{\Delta_x}$ when $x = a$, then $f'(a)$ $= \lim \overset{\Delta x}{\rightarrow} 0 \frac{f(a+\Delta_x)-f(a)}{\Delta_x}$		
3	Finding the Derivative from the First Principle	The teacher explains how to find derivative from the first Principle with the following examples Example: Find from the first principle the derivative of each of the following functions with respect to x (a) $y = x$ (b) $y = x^2$ Solution (a) $y = x$ $f(x) = x$ $f(x+\Delta_x) = x + \Delta_x$ Apply the formula:	The students listen attentively and take note. The students make reference to the advance organizer. The stuents ask questions if any	Use of examples and explanation

		$\frac{dy}{dx} = \lim \overset{\Delta x}{\rightarrow} 0 \frac{f(x+\Delta_x)-f(x)}{\Delta_x}$ $= \lim \overset{\Delta x}{\rightarrow} 0 \frac{x+\Delta_x \ -x}{\Delta_x}$ $= \lim \overset{\Delta x}{\rightarrow} 0 \frac{\Delta_x}{\Delta_x}$ $= \lim \overset{\Delta x}{\rightarrow} 0 \ 1$ $= 1$ $\therefore \frac{dy}{dx} = 1$ (b) $y = x^2$ $f(x) = x^2$ $f(x + \Delta_x) = (x + \Delta_x)^2$ $\frac{dy}{dx} = \lim \overset{\Delta x}{\rightarrow} 0 \frac{f(x+\Delta_x)-f(x)}{\Delta_x}$ $= \lim \overset{\Delta x}{\rightarrow} 0 \frac{(x + \Delta_x)^2 - x^2}{\Delta_x}$ $= \lim$ $\overset{\Delta x}{\rightarrow} 0 \frac{x^2 + 2x\Delta_x - (\Delta_x)^2 - x^2}{\Delta_x}$ $= \lim \overset{\Delta x}{\rightarrow} 0 \frac{2x\Delta_x - (\Delta_x)^2}{\Delta_x}$ $= \lim \overset{\Delta x}{\rightarrow} 0 \ 2x - \Delta_x$ $= 2x$ $y = \frac{1}{x}$ $f(x) = \frac{1}{x}$ $f(x + \Delta_x) = \frac{1}{x+\Delta_x}$		

<table>
<tr><td></td><td></td><td>Apply the formula:
$\frac{dy}{dx} = \lim \overset{\Delta x}{\to} 0 \frac{f(x+\Delta_x)-f(x)}{\Delta_x}$
$= \lim \overset{\Delta x}{\to} 0 \frac{\frac{1}{x+\Delta_x} - \frac{1}{x}}{\Delta_x}$
$= \lim \overset{\Delta x}{\to} 0 \frac{\frac{x-(x+\Delta_x)}{x(x+\Delta_x)}}{\Delta_x}$
$= \lim \overset{\Delta x}{\to} 0 \frac{\frac{-\Delta_x}{x^2+x\Delta_x}}{\Delta_x}$
$\lim \overset{\Delta x}{\to} 0 \frac{-\Delta_x}{\Delta_x(x^2+x\Delta_x)}$
$\lim \overset{\Delta x}{\to} 0 \frac{-1}{(x^2+x\Delta_x)}$
$= \frac{-1}{(x^2+0)} = \frac{-1}{(x^2)}$
$\therefore \frac{dy}{dx} = \frac{-1}{(x^2)}$</td><td></td><td></td></tr>
<tr><td>4</td><td>Evaluation</td><td>The teacher evaluates the lesson by asking the students to:
i. Define and explain the concept of differentiation
ii. State the differential coefficient of the function $y = f(x)$ from the first principle
iii. Find the derivative of the function $y = x^3$ from the first</td><td>The students give answers to the questions thus:
$y = x^3$
$f(x) = x^3$
$f(x+\Delta_x)$
$= (x+\Delta_x)^3$
$\frac{dy}{dx} = \lim$
$\overset{\Delta x}{\to} 0 \frac{f(x+\Delta_x)-f(x)}{\Delta_x} =$
$\lim$
$\overset{\Delta x}{\to} 0 \frac{(x+\Delta_x)^3 - x^3}{\Delta_x} =$</td><td>Questioning and non-verbal clues</td></tr>
</table>

		principle	$\lim_{\Delta x \to 0} \frac{3x^2\Delta_x + 3x(\Delta_x)^2 + (\Delta_x)^3}{\Delta_x}$ $= \lim_{\Delta x \to 0} 3x^2 + 3x\Delta_x = 3x^2$	

Summary: The teacher summarizes the lesson by highlighting the key points of the lesson thus:

1. The process of finding the differential coefficient or derivative of a function is called differential calculus.
2. The derivative of the function $y = f(x)$ from the first principle is given by

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta_x) - f(x)}{\Delta_x}$$

Assignment

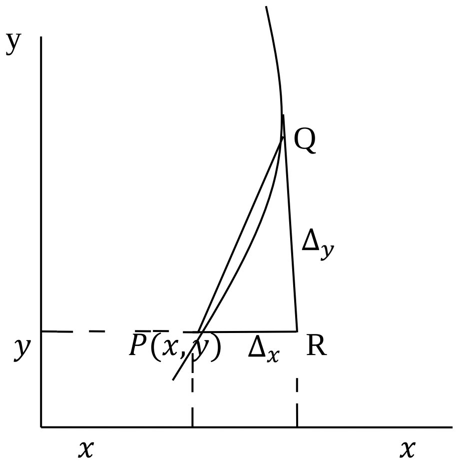
1. Find, from the first principle, the differential coefficient of $y = 4x^2$
2. Determine the derivative of $y = \frac{1}{x^2}$ with respect to x from the first principle.

THE STUDY QUESTIONS ADVANCE ORGANIZER FOR DIFFERENTIATION FROM THE FIRST PRINCIPLE

Let y be a function of x denoted by

$$y = f(x).$$

Study the graph of $y = f(x)$ below and answer the questions that follow. Each of the questions provides a hint to the succeeding questions.



1. Δ_x and Δ_y represent __________________ respectively (A) increment of x and y (B) decrease in x and y
2. The slope m_1 of the line PQ is given by _________________ (A) $m_1 = \frac{\Delta_x}{\Delta_y}$ (B) $m_1 = \frac{\Delta_y}{\Delta_x}$
3. Let point P be fixed and let Q approach P so that we now have a new line PT. What happens to Δ_x as point Q keeps approaching point P along the curve as shown in the figure above (A) Δ_x remains the same (B) Δ_x tends to 0
4. Number 3 above can be written in terms of limit as _____________ (A) $\lim_{\Delta x \to 0}$ (B) $\xrightarrow{\Delta x} 0$
5. The new slope m_2, the limiting value of m_1 is given by _______________ (A) $m_2 = \lim_{\Delta x \to 0} \frac{\Delta_y}{\Delta_x}$ (B) $m_2 = \lim_{\Delta x \to 0} \frac{\Delta_x}{\Delta_y}$
6. If $\frac{dy}{dx}$ is used instead of m_2, then____________ (A) $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta_y}{\Delta_x}$ (B) $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta_x}{\Delta_y}$
7. If $\frac{dy}{dx} = \lim_{\Delta x \to 0} 2x + \Delta_x,$ what is the value of $\frac{dy}{dx}$? (A) $\frac{dy}{dx} = \Delta_x$ (B) $\frac{dy}{dx} = 2x$
8. Let $y = f(x)$, then _______________ (A) $y + \Delta_y = f(x + \Delta_x)$ (B) $y + \Delta_y = f(x)$

9. From 8 above what is Δ_y? (A) $\Delta_y = f(x) - f(x)$ (B) $\Delta_y = f(x + \Delta_x) - f(x)$

10. Obtain an expression for $\frac{\Delta_y}{\Delta_x}$ from 9. (A) $\frac{\Delta_y}{\Delta_x} = \frac{f(x)-f(x)}{\Delta_x} \frac{\Delta_y}{\Delta_x}$ (B) $= \frac{f(x+\Delta_x)-f(x)}{\Delta_x}$

11. $\frac{dy}{dx}$ from expression 10 becomes_______________ (A) $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta_x)-f(x)}{\Delta_x}$ (B) $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x)-f(x)}{\Delta_x}$

12. $\frac{dy}{dx}$ (read 'dee y dee x') represent the following expressions except (A) slope or gradient of the tangent to the curve $y = f(x)$ (B) differential coefficient of $y = f(x)$ (C) the derivative of y with respect to x (D) integral of y

13. Given that $y = x^2$, then ______________ (A) $f(x) = x^2$ and $f(x + \Delta_x) = (\Delta_x)^2$ (B) $f(x) = x^2$ and $f(x + \Delta_x) = (x + \Delta_x)^2$

14. Find the differential coefficient of the function $y = x^2$ using the formula $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta_x)-f(x)}{\Delta_x}$. (A) $\frac{dy}{dx} = 2x$ (B) $\frac{dy}{dx} = x$

15. What is the value of $\frac{dy}{dx}$ at $x = 2$? (A) $\frac{dy}{dx} = 4$ (B) $\frac{dy}{dx} = 2$

LESSON PLAN FOR STUDY QUESTIONS ADVANCE ORGANIZERS
(SQAOS GROUP)
WEEK TWO

Subject: Mathematics

Class: SS III

Duration: 40 minutes

Content: Differentiation of Polynomials

Specific Objectives: At the end of the lesson, the students should be able to:

(i) Derive the formula for the differentiation of the polynomial $y = x^n$ from the first principle

(ii) Find the derivatives of polynomials with respect to x

(iii)State the standard derivatives of some functions

Instructional Materials: Markerboard illustrations

Instructional Techniques/ Strategies: Set induction, questioning, reinforcement, illustration, explanation, non verbal clues, repetition and use of examples

Entry Behaviour: The students can:

(i) find the derivative of functions with respect to x.

Test on Entry Behaviour:

(ii) find the derivative of $y = 3x^2$

Instructional Procedure:

Steps	Content Development (CD)	Teachers' Activities	Students' Activities	Strategies
1	Introducion	The teacher introduces the lesson by asking the students to find the derivative of $y = 3x^2$ with respect to x using the first principle:	The students solve the problems and provide the answers to the questions. Students are expected to solve the questions thus: $y = x^2$ $f(x) = x^2$ $f(x + \Delta_x) = (x + \Delta_x)^2$ $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta_x)-f(x)}{\Delta_x} = \lim_{\Delta x \to 0} \frac{(x+\Delta_x)^2-x^2}{\Delta_x}$ $= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta_x - (\Delta_x)^2}{\Delta_x}$ $= \lim_{\Delta x \to 0} \frac{2x\Delta_x - (\Delta_x)^2}{\Delta_x}$ $= \lim_{\Delta x \to 0} \frac{\Delta_x(2x - \Delta_x)}{\Delta_x}$	Set induction and Questioning

			$= \lim_{\Delta x \to 0} 2x - \Delta_x$ $= 2x$	
2	Deriving the Formula for Differentiating polynomials	The teacher first explains to the students the objectives of the lesson and what is expected of them. The teacher then gives each student the study questions advance organizer and asks them to study them carefully and provide answers to the questions sequentially in 15 minutes (see the last page of this lesson plan for the study questions advance organizers). The teacher explains to the students the need to provide the answers sequentially and instructs the students not to jump or mix up the numbers as each question is a hint to the succeeding question. After the	Each student studies the questions independently and provides the answers to them. 1. (A)$y + \Delta_y = (x + \Delta_x)^n$ 2. (A) $\Delta_y = (x + \Delta_x)^n - x^n$ 3. (A)$x^n + nx^{n-1}y + \frac{n(n-1)}{1.2}x^{n-1}y^2 + \cdots + y^n$ 4. (A)$\Delta_y = x^n + nx^{n-1}\Delta_x + \frac{n(n-1)}{1.2}x^{n-1}(\Delta_x)^2 \cdots (\Delta_x)^n - x^n$ 5. (A) $\frac{\Delta_y}{\Delta_x} = nx^{n-1} + \frac{n(n-1)}{1.2}x^{n-1}\Delta_x + \cdots (\Delta_x)^{n-1}$ 6. (A) $\frac{dy}{dx} = nx^{n-1}$ 7. (A) $\frac{dy}{dx} = 3x^2$ 8. (B) $\frac{dy}{dx} = 6x^2$ 9. (A) $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ 10. (A) $\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$ 11. (B) $cosx$	Explanation, questioning and non-verbal clues

		15 minutes, the teacher asks the students to stop and then begins to explain how to derive the formula used for the differentiation of polynomials thus: Let $y = x^n$, then $y + \Delta_y = (x + \Delta_x)^n$ $\Delta_y = (x + \Delta_x)^n - y$ $\Delta_y = (x + \Delta_x)^n - x^n$ Expanding $(x + \Delta_x)^n$ using the binomial expansion $(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$ $= x^n + nx^{n-1}y + \frac{n(n-1)}{1.2} x^{n-1} y^2 + \cdots + y^n$ We have $(x + \Delta_x)^n = x^n + nx^{n-1}\Delta_x + \frac{n(n-1)}{1.2} x^{n-1} (\Delta_x)^2 + \cdots (\Delta_x)^n$ $\Delta_y = x^n + nx^{n-1}\Delta_x + \frac{n(n-1)}{1.2} x^{n-1} (\Delta_x)^2 + \cdots (\Delta_x)^n - x^n$ $\frac{\Delta_y}{\Delta_x} = nx^{n-1} + \frac{n(n-1)}{1.2} x^{n-1}\Delta_x + \cdots (\Delta_x)^{n-1}$	12. (B) $-sinx$ 13. (A) e^x 14. (A) 0 15. (A) sec^2x The students then listen attentively to the teacher and compare each step of the teaching with the answers they gave in the advance organizer	

		As $\Delta_x \rightarrow 0$, then $\frac{dy}{dx} = nx^{n-1}$ Also, (i) if $y = ax^n$, then $\frac{dy}{dx} = anx^{n-1}$ if u and v are functions of x then $\frac{d(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx}$		
3	Differentiation of polynomial	The teacher explains differentiation of polynomials with the following examples <u>Example:</u> Differentiate the following with respect to x (a) $y = x^3$ (b) $y = 3x^4$ (c) $y = x^4 + 2x^3 - 4x^{-2} + 2x - 1$ at $x = 1$ <u>Solution</u> (a) $y = x^3$ $\frac{dy}{dx} = 3x^{3-1} = 3x^2$ (b) $y = 3x^4$ $\frac{dy}{dx} = 4 \times 3x^{4-1} = 12x^3$ (c) $y = x^4 + 2x^3 - 4x^{-2} + 2x - 1$	The students listen attentively to the teacher while making reference to the advance organizer. The students write the example in their note	Use of examples and explanation

		$\frac{dy}{dx} = \frac{d(x^4)}{dx} + \frac{d(2x^3)}{dx} - \frac{d(4x^{-2})}{dx} +$ $\frac{d(2x)}{dx} - 1$ $\frac{dy}{dx} = 4x^3 + 6x^2 + 8x^{-3} + 2 + 0$ $\frac{dy}{dx} = 4x^3 + 6x^2 + 8x^{-3} + 2$ $\frac{dy}{dx} = 4x^3 + 6x^2 + \frac{8}{x^3} + 2$ At $x = 1$ $\frac{dy}{dx} = 4(1)^3 + 6(1)^2 + \frac{8}{(1)^3} + 2$ $\frac{dy}{dx} = 4 + 6 + 8 + 2 = 20$		
4	Standard Derivatives of Some Basic Functions	The teacher states the derivatives of some basic functions thus: 1. If $y = sinx$, then $\frac{dy}{dx} = \frac{d(sinx)}{dx} = cosx$ 2. If $y = cosx$, then $\frac{dy}{dx} = \frac{d(cosx)}{dx} = -sinx$ 3. If $y = e^x$, then $\frac{dy}{dx} = \frac{d(e^x)}{dx} = e^x$ 4. If $y = c$, then $\frac{dy}{dx} = \frac{d(c)}{dx} = 0$ 5. If $y = \log_e x$ or $\ln x$, then $\frac{dy}{dx} = \frac{d(\log_e x)}{dx}$ or $\frac{d(\ln x)}{dx} = \frac{1}{x}$	The students write the standard derivatives in their notes as they compare the standard derivatives with what their answers in the advance organizer	Explanation

		6. If $y = tanx$, then $\frac{dy}{dx} = \frac{d(tanx)}{dx} = sec^2x$		
5	Evaluation	The teacher evaluates the lesson by asking the students to: i. Derive the formula for the differentiation of the polynomial $y = x^n$ from the first principle ii. Find the derivatives of the following polynomials with respect to x: $y = 3x^4 + x^3 - 2x^2 + 5x - 7$ iii. State the standard derivatives of the following functions: x^n, $sinx$, $cosx$, $tanx$ and e^x	The students give answers to the questions thus: 1. Let $y = x^n$, then $y + \Delta_y = (x + \Delta_x)^n$ $\Delta_y = (x + \Delta_x)^n - y$ $\Delta_y = (x + \Delta_x)^n - x^n$ Expanding $(x + \Delta_x)^n$ using the binomial expansion $(x + y)^n =$ $\sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$ $= x^n + nx^{n-1}y +$ $\frac{n(n-1)}{1.2} x^{n-1} y^2 +$ $\cdots + y^n$ We have $(x + \Delta_x)^n = x^n$ $+ nx^{n-1}\Delta_x$ $+ \frac{n(n-1)}{1.2} x^{n-1} (\Delta_x)^2$ $+ \cdots (\Delta_x)^n$ $\Delta_y = x^n + nx^{n-1}\Delta_x +$	Questioning and reinforcement

			$\frac{n(n-1)}{1.2}x^{n-1}(\Delta_x)^2 +$ $\cdots(\Delta_x)^n \; - x^n$ $\frac{\Delta_y}{\Delta_x} \quad = nx^{n-1} +$ $\frac{n(n-1)}{1.2}x^{n-1}\Delta_x +$ $\cdots(\Delta_x)^{n-1}$ As $\Delta_x \rightarrow 0$, then $\frac{dy}{dx} \;\; = nx^{n-1}$ i. (a) $y = 3x^4 + x^3 - 2x^2 + 5x - 7$ $\frac{dy}{dx} \quad = 4 \times 3x^{4-1} +$ $3x^{3-1} + 2 \times 2x^{2-1} +$ $5 - 0$ $\frac{dy}{dx} \quad = 12x^3 + 3x^2 +$ $4x + 5 - 0$ $\frac{dy}{dx} \quad = 12x^3 + 3x^2 +$ $4x + 5$ The standard derivatives of x^n, $sinx$, $cosx$, $tanx$ and e^x are respectively: $\frac{dy}{dx} \quad = nx^{n-1}$, $\frac{dy}{dx} \quad =$ $cosx$, $\frac{dy}{dx} \quad = -sinx$, $\frac{dy}{dx} = sec^2x$ and $\frac{dy}{dx}$ $= e^x$	

Summary: The teacher summarizes the lesson by highlighting the main points of the lesson thus:

i. If $y = x^n$, then $\frac{dy}{dx} = nx^{n-1}$

ii. If $y = ax^n$, then $\frac{dy}{dx} = anx^{n-1}$

iii. If u and v are functions of x then $\frac{d(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

iv. If $y = sinx$, then $\frac{dy}{dx} = \frac{d(sinx)}{dx} = cosx$

v. If $y = cosx$, then $\frac{dy}{dx} = \frac{d(cosx)}{dx} = -sinx$

vi. If $y = e^x$, then $\frac{dy}{dx} = \frac{d(e^x)}{dx} = e^x$

vii. If $y = c$, then $\frac{dy}{dx} = \frac{d(c)}{dx} = 0$

viii. If $y = \log_e x$ or $\ln x$, then $\frac{dy}{dx} = \frac{d(\log_e x)}{dx}$ or $\frac{d(\ln x)}{dx} = \frac{1}{x}$

ix. If $y = tanx$, then $\frac{dy}{dx} = \frac{d(tanx)}{dx} = sec^2x$

THE STUDY QUESTIONS ADVANCE ORGANIZER FOR DIFFERENTIATION OF POLYNOMIALS

1. If $y = x^n$, then ________________ (A) $y + \Delta_y = (x + \Delta_x)^n$ (A) $y + \Delta_y = (\Delta_x)^n$
2. Δ_y from 1 is given by __________________ (A) $\Delta_y = (x + \Delta_x)^n - x^n$ (A) $\Delta_y = (\Delta_x)^n - x^n$
3. The binomial expansion $\sum_{k=0}^{n}\binom{n}{k} x^{n-k}y^k$ of $(x + y)^n$ is given by __________ (A)$x^n + nx^{n-1}y + \frac{n(n-1)}{1.2}x^{n-1}y^2 + \cdots y^n$ (B) $nx^{n-1}y + \frac{n(n-1)}{1.2}x^{n-1}y^2 + \cdots$ (B) What is the expansion of $(x + \Delta_x)^n$? (A) $x^n + nx^{n-1}\Delta_x + \frac{n(n-1)}{1.2}x^{n-1}(\Delta_x)^2 + \cdots (\Delta_x)^n$ (B) $nx^{n-1}\Delta_x + \frac{n(n-1)}{1.2}x^{n-1}(\Delta_x)^2 + \cdots$
4. What is Δ_y from 2 and 3? (A) $\Delta_y = x^n + nx^{n-1}\Delta_x + \frac{n(n-1)}{1.2}x^{n-1}(\Delta_x)^2 + \cdots (\Delta_x)^n \; - x^n$ (B) $\Delta_y = nx^{n-1}\Delta_x + \frac{n(n-1)}{1.2}x^{n-1}(\Delta_x)^2 + \cdots \; - x^n$
5. Obtain an expression for $\frac{\Delta_y}{\Delta_x}$. (A) $\frac{\Delta_y}{\Delta_x} = nx^{n-1} + \frac{n(n-1)}{1.2}x^{n-1}\Delta_x + \cdots (\Delta_x)^{n-1}$ (B) $\frac{\Delta_y}{\Delta_x} = nx^{n-1}\Delta_x + \frac{n(n-1)}{1.2}x^{n-1}\Delta_x \; + \cdots x^n$
6. As $\Delta_x \to 0$, then___________ (A) $\frac{dy}{dx} = nx^{n-1}$ (B) $\frac{dy}{dx} = x^n$
7. If $y = x^3$, find $\frac{dy}{dx}$. (A) $\frac{dy}{dx} = 3x^2$ (B) $\frac{dy}{dx} = x^2$
8. Differentiate $y = 2x^3$ with respect to x. (A) $\frac{dy}{dx} = 3x^2$ (B) $\frac{dy}{dx} = 6x^2$
9. If $y = u + v$ where u and v are functions of x then ____________ (A) $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ (B) $\frac{dy}{dx} = \frac{duv}{dx}$
10. If $y = u - v$ where u and v are functions of x then ____________ (A) $\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$ (B) $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$
11. If $y = sinx$, then $\frac{dy}{dx} =$ __________________ (A) $-cosx$ (B) $cosx$
12. If $y = cosx$, then $\frac{dy}{dx} =$ __________________ (A) $sinx$ (B) $-sinx$
13. If $y = e^x$, then $\frac{dy}{dx} =$ __________________ (A) e^x (B) xe^{x-1}
14. If $y = c$, then $\frac{dy}{dx} =$ __________________ (A) 0 (B) cx
15. If $y = \log_e x$ or $\ln x$, then $\frac{dy}{dx} =$ __________________ (A) $-\mathrm{x}\log_e x$ (B) $\frac{1}{x}$
16. If $y = tanx$, then $\frac{dy}{dx} =$ __________________ (A) sec^2x (B) $secx$

LESSON PLAN FOR STUDY QUESTIONS ADVANCE ORGANIZERS
(SQAOS GROUP)
WEEK THREE

Subject: Mathematics

Class: SS III

Duration: 80 minutes

Content: Rules of Differentiation: Product and Quotient Rule

Specific Objectives: At the end of the lesson, the students should be able to:

(i) state the product rule of differentiation

(ii) find the derivatives of polynomials with respect to x using the product rule

(iii) state the quotient rule of differentiation

(iv) find the derivatives of function with respect to x by the quotient rule

(v) solve problems involving trigonometric functions using product and quotient rule

Instructional Materials: Markerboard illustrations

Instructional Techniques: Set induction, questioning, reinforcement, illustration, explanation, non verbal clues, repetition and use of examples

Entry Behaviour: The students can:

i. find the derivative of polynomials with respect to x.

Test on Entry Behaviour:

i. Find the derivative of the polynomial with respect to x:

$y = 2x^4 - x^3 + 3x^2$ at $x = 2$

Instructional Procedure:

Steps	Content Development (CD)	Teacher's Activities	Students' Activities	Strategies
1	Introduction	The teacher introduces the lesson by asking the students to find the derivative of the polynomial $y = 2x^4 - x^3 + 3x^2$ at $x = 2$ with respect to x	The students solve the problems and provide the answers to the questions. Students are expected the questions thus: $\frac{dy}{dx} = \frac{d(2x^4)}{dx} - \frac{d(x^3)}{dx} + \frac{d(3x^2)}{dx}$ $\frac{dy}{dx} = 8x^3 - 3x^2 + 6x$ At $x = 2$ $\frac{dy}{dx} = 8(2)^3 - 3(2)^2 + 6(2)$ $\frac{dy}{dx} = 8 \times 8 - 3 \times 4 + 12$ $\frac{dy}{dx} = 64 - 12 + 12 = 64$	Questioning and Reinforcement
2	Differentiation by Product Rule	The teacher first explains to the students the objectives of the lesson and what is expected of them. The teacher then gives each	Each student studies the questions independently and provides the answers to them. The students are expected to	Explanation, questioning and non verbal clues

		student the study questions advance organizer and asks them to study them carefully and provide answers to the questions sequentially in 15 minutes (see the last page of this lesson plan for the study questions advance organizers). The teacher explains to the students the need to provide the answers sequentially and instructs the students not to jump or mix up the numbers as each question is a hint to the succeeding question. After the 15 minutes, the teacher asks the students to stop and then begins to explain how to derive the product rule of differentiation Let $y = uv$, where u and v are functions of x, then	provide the following answers: 1.(A)$y + \Delta_y = (u + \Delta_u)(v + \Delta_v)$ 2. (A)$\Delta_y = uv + u\Delta_v + v\Delta_u + \Delta_u\Delta_v - uv$ 3.(A)$\frac{\Delta_y}{\Delta_x} = u\frac{\Delta_v}{\Delta_x} + v\frac{\Delta_u}{\Delta_x}$ 4. (A)$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ 5. (A) $u = 3x + 2$ and $v = x^2 + 2$ 6. (B) 3 7. (A) $2x$ 8. (A)$\frac{dy}{dx} = 9x^2 + 4x + 6$ 9. (A) $y + \Delta_y = \frac{u+\Delta_u}{v+\Delta_v}$ 10.(A) $\Delta_y = \frac{u+\Delta_u}{v+\Delta_v} - \frac{u}{v}$ 11.(B)$\Delta_y = \frac{uv+v\Delta_u-uv-u\Delta v}{v(v+\Delta_v)}$ 12.(A) $\Delta_y = \frac{v\Delta_u-u\Delta v}{v(v+\Delta_v)}$ 13.(A)$\frac{\Delta_y}{\Delta_x} = \frac{\frac{v\Delta_u}{\Delta_x}-\frac{u\Delta_v}{\Delta_x}}{v(v+\Delta_v)}$	

		$y+\Delta_y = (u+\Delta_u)(v+\Delta_v)$ $\Delta_y = (u+\Delta_u)(v+\Delta_v) \ - y$ $\Delta_y = (u+\Delta_u)(v+\Delta_v) \ - uv$ $= vu + u\Delta_v + v\Delta_u + \Delta_u\Delta_v$ $- uv$ $= u\Delta_v + v\Delta_u + \Delta_u\Delta_v$ $\frac{\Delta_y}{\Delta_x} = \frac{u\Delta_v}{\Delta_x} + \frac{v\Delta_u}{\Delta_x} + \frac{\Delta_u\Delta_v}{\Delta_x}$ At this point the teacher should make reference to the advance while asking the students to make comparisons with the advance organizer. As $\Delta x \to 0$, $\Delta u \to 0$, $\Delta v \to 0$ $\lim \overset{\Delta x}{\to} 0 \frac{\Delta_y}{\Delta_x} = \frac{dy}{dx} = u\frac{dv}{dx} +$ $v\frac{du}{dx} + 0$ $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ Therefore, if $y = uv$, where u and v are functions of x, then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$	14.(B) $\lim_{\Delta x \to 0} \frac{\Delta_y}{\Delta_x} = \frac{dy}{dx} =$ $\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v(v+0)}$ 15.(B) $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ 16. (A) $\frac{dy}{dx} = \frac{4x}{(1-x^2)^2}$ The students then listen attentively to the teacher. The students make comparisons of the teaching with the advance organizer as they try to discover reasons and relationships	

3	Derivatives of Polynomials by Product Rule	The teacher teaches the students how to find the derivatives of polynomials by product rule with this example <u>Example</u> Differentiate $y = (3x+2)(x^2+2)$ w.r.t x at $x = \frac{1}{3}$ <u>Solution</u> $y = (3x+2)(x^2+2)$ $u = (3x+2), v = (x^2+2)$ $\therefore y = uv$ $\frac{du}{dx} = 3,\ \frac{dv}{dx} = 2x$ $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ $\frac{dy}{dx} = (3x+2)(2x)$ $+ (x^2+2)(3)$ It can be noticed that to differentiate product of two functions keep the first, differentiate the second plus keep the second, differentiate the first. Now,	The students listen attentively and write down the example. The students compare the steps with the study questions	Use of examples and explanation

		$\frac{dy}{dx} = 6x^2 + 4x + 3x^2 + 6$ $= 9x^2 + 4x + 6$ At $x = \frac{1}{3}$, $\frac{dy}{dx} = 9(\frac{1}{3})^2 + 4(\frac{1}{3}) + 6$ $= 9(\frac{1}{9}) + 4(\frac{1}{3}) + 6$ $= 1 + \frac{4}{3} + 6$ $= \frac{25}{3}$		
4	Differentiation by Quotient Rule	The teacher explain how to derive the quotient rule of differentiation Let $y = \frac{u}{v}$, where u and v are functions of x, then $y + \Delta_y = \frac{u + \Delta_u}{v + \Delta_v}$ $\Delta_y = \frac{u + \Delta_u}{v + \Delta_v} - y$ $\Delta_y = \frac{u + \Delta_u}{v + \Delta_v} - \frac{u}{v}$ $\Delta_y = \frac{uv + v\Delta_u - uv - u\Delta v}{v(v + \Delta_v)}$ $\Delta_y = \frac{v\Delta_u - u\Delta v}{v(v + \Delta_v)}$	The students listen attentively to the teacher. The students make comparisons of the teaching with the advance organizer as they try to discover reasons and relationships. The students ask questions	Explanation, repetition and questioning

		$$\frac{\Delta_y}{\Delta_x} = \frac{v\Delta_u - u\Delta v}{\Delta_x v(v+\Delta_v)}$$ $$\frac{\Delta_y}{\Delta_x} = \frac{\frac{v\Delta_u}{\Delta_x} - \frac{u\Delta_v}{\Delta_x}}{v(v+\Delta_v)}$$ As $\Delta u \to 0$ and $\Delta v \to 0$ $\lim \overset{\Delta x}{\to} 0 \frac{\Delta_y}{\Delta_x} = \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v(v+0)}$ $$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$ Therefore, if $y = \frac{u}{v}$ where u and v are functions of x, then $$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$		
5	Finding Derivative of Functions by Quotient Rule	The teacher explains how to find the derivative of functions using the quotient rule with the following example: <u>Example1</u> Differentiate $y = \frac{1+x^2}{1-x^2}$ w.r.t x at $x = -2$ <u>Solution</u> $$y = \frac{1+x^2}{1-x^2}$$ $u = 1 + x^2, \qquad v = 1 - x^2$ $\therefore y = \frac{u}{v}$	The students listen attentively to the teacher and write the example in their notes. The students ask questions as they make comparisons with the study questions.	Use of examples and explanation

		$\frac{du}{dx} = 2x,\ \frac{dv}{dx} = -2x$ $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ $\frac{dy}{dx} = \frac{(1-x^2)(2x)-(1+x^2)(-2x)}{(1-x^2)^2}$ Notice that in order to differentiate quotient of two functions of x, keep the second function differentiate the first second minus keep the first differentiate the second all over the second squared. Now, $\frac{dy}{dx} = \frac{2x - 2x^3 + 2x + 2x^3}{(1-x^2)^2}$ $= \frac{4x}{(1-x^2)^2}$ At $x = -2$ $\frac{dy}{dx} = \frac{4(-2)}{(1-(-2)^2)^2}$ $= \frac{-8}{(1-4)^2}$ $= \frac{-8}{(-3)^2}$ $= -\frac{8}{9}$		

6	Problems Involving Trigonometry	The teachers teaches the students how to solve problems involving trigonometry with the following examples: <u>Example 1</u> If $y = x^3 sinx$ <u>Solution</u> $u = x^3, v = \sin x$ $\frac{du}{dx} = 3x^2,\ \frac{dv}{dx} = cosx$ Using the product rule $$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$ we have $\frac{dy}{dx} = x^3 cosx\ + sinx.3x^2$ $\frac{dy}{dx} = x^2(xcosx\ + 3sinx)$ <u>Example 2</u> If $y = \frac{sinx}{x^2+cosx}$, find $\frac{dy}{dx}$ when $x = 0$ <u>Solution</u> $$y = \frac{sinx}{x^2 + cosx}$$ The first function is	The students listen attentively and write the examples in their notebook	Use of examples explanation and questioning

		$u = sinx$ and the second function is $v = x^2 + cosx$ $\frac{du}{dx} = cosx,\ \frac{dv}{dx} = 2x - sinx$ $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ $\frac{dy}{dx} =$ $\frac{(x^2+cosx)(cosx)-sinx(2x-sinx\ )}{(x^2+cosx)^2}$ $\frac{dy}{dx} = \frac{x^2+cos^2x-2xsinx+sin^2x\ )}{(x^2+cosx)^2}$ $\frac{dy}{dx} = \frac{x^2-2xsinx+cos^2x+sin^2x}{(x^2+cosx)^2}$ Recall from trigonometry that $cos^2x + sin^2x = 1$ $\therefore \frac{dy}{dx} = \frac{x^2 - 2xsinx + 1}{(x^2 + cosx)^2}$ At $x = 0$, $\frac{dy}{dx} = \frac{0^2 - 2(0)sin0 + 1}{(0^2 + cos0)^2}$ $\frac{dy}{dx} = \frac{0 - 0 + 1}{(0 + 1)^2}$ $\frac{dy}{dx} = \frac{1}{(1)^2} = \frac{1}{1} = 1$		

7	Evaluations	The teacher evaluates the lesson by asking the students to: i. state the product rule of differentiation ii. Find the derivatives of $y = (7x^2 + 2x)(4x + 5)$ w.r.t x using the product rule iii. state the quotient rule of differentiation iv. Find the derivatives of function $y = \frac{x^2+4}{5-x^2}$ with respect to x by the quotient rule v. Solve the following problems involving trigonometric functions (a) Differentiate $y = x^2 cosx$ at using product (b) Find the derivative of $y = \frac{sinx}{x^3+cosx}$ at $x = 0$ using quotient rule	The students give answers to the questions thus: (i)The product rule states that If $y = uv$, where u and v are functions of x, then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ (ii)$y = (7x^2 + 2x)(4x + 5)$ $u = (7x^2 + 2x), v = (4x + 5)$ $\frac{du}{dx} = 14x + 7,\ \frac{dv}{dx} = 4$ $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ $\frac{dy}{dx} = (7x^2 + 2x)(4) + (4x + 5)(14x + 7)$ $\frac{dy}{dx} = 56x^2 + 28x + 70x + 35 = 56x^2 + 98x + 35$ (iii)The quotient rule of differentiation of the function of the form $y = \frac{u}{v}$ where u and v are functions of x, is given by	Questioning and reinforcement

			$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ (iv) a. $y = x^2 cosx$ $u = x^2, v = cos\ x$ $\frac{du}{dx} = 2x,\ \frac{dv}{dx} = -\sin x$ Using the product rule $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ we have $\frac{dy}{dx} = -x^2 sinx\ +$ $cosx. 2x$ $\frac{dy}{dx} = -x^2 sinx\ +$ $2xcosx$ $\frac{dy}{dx} = 2xcosx - x^2 sinx$ $\frac{dy}{dx} = x(2cosx\ + xsinx)$ $v.\ y = \frac{sinx}{x^3 + cosx}$ $u = sinx$ $v = x^3 + cosx$ $\frac{du}{dx} = cosx,\ \frac{dv}{dx} = 3x^2 -$ $sinx$ $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ $\frac{dy}{dx} =$	

			$\frac{x^3+cosx(cosx)-sinx(3x^2-sinx\,)}{(x^3+cosx)^2}$ $\frac{dy}{dx} =$ $\frac{x^3+cos^2x-3x^2sinx+sin^2x\,)}{(x^3+cosx)^2}$ $\frac{dy}{dx} =$ $\frac{x^3-3x^2sinx+cos^2x+sin^2x}{(x^3+cosx)^2}$ Recall from trigonometry that $cos^2x + sin^2x = 1$ $\therefore \frac{dy}{dx} = \frac{x^3-3x^2sinx+1}{(x^3+cosx)^2}$ At $x = 0$, $\frac{dy}{dx} = \frac{0^3-3(0)^2sin0+1}{(0^3+cos0)^2}$ $\frac{dy}{dx} = \frac{0-0+1}{(0+1)^2}$ $\frac{dy}{dx} = \frac{1}{(1)^2} = \frac{1}{1} = 1$	

Summary: The teacher summarizes the lesson thus:

i. If $y = uv$ where u and v are functions of x, then the product rule is

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

that is: to differentiate product of two functions keep the first, differentiate the second plus keep the second, differentiate the first

ii. The quotient rule of differentiation of the function of the form $y = \frac{u}{v}$ where u and v are functions of x, is given by

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

That is: put down the denominator (differentiate the numerator) – put down the numerator (differentiate the denominator) all over the denominator squared

Assignment:

1. Differentiate the following functions with respect to x:
   a. $y = (4sinx)(4x + 5)$
   b. $y = \frac{7x^2}{x+sinx}$

THE STUDY QUESTIONS ADVANCE ORGANIZER FOR DIFFERENTIATION BY PRODUCT AND QUOTIENT RULE

1. Let $y = uv$ where u and v are all functions of x. Then_________ (A) $y + \Delta_y = (u + \Delta_u)(v + \Delta_v)$ (B) $y + \Delta_y = \left(uv + \Delta_{u\Delta_v}\right)$
2. Expanding the RHS of 1 above we have_______ (A) $\Delta_y = uv + u\Delta_v + v\Delta_u + \Delta_u\Delta_v - uv$ (B) $\Delta_y = uv + \Delta_{u\Delta_v} - uv$
3. Write an expression for $\frac{\Delta_y}{\Delta_x}$ (A) $\frac{\Delta_y}{\Delta_x} = u\frac{\Delta_v}{\Delta_x} + v\frac{\Delta_u}{\Delta_x}$ (B) $\frac{\Delta_y}{\Delta_x} = \frac{uv + \Delta_{u\Delta_v} - \mathrm{uv}}{\Delta_x}$
4. If $\Delta_x \rightarrow 0$, $\Delta_u \rightarrow 0$ and $\Delta_v \rightarrow 0$ then $\frac{dy}{dx}$ of $y = uv$ is given by _______________ (A) $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ (B) $\frac{dy}{dx} = \frac{uv + dudv - \mathrm{uv}}{dx}$
5. Given that $y = (3x + 2)(x^2 + 2)$, then ___________ (A) $u = 3x + 2$ and $v = x^2 + 2$ (B) $u = 3x$ and $v = x^2$
6. What is the value of $\frac{du}{dx}$? (A) 1 (B) 3
7. What is the value of $\frac{dv}{dx}$? (A) $2x$ (B) 2
8. Using the formula in 4 above differentiate $y = (3x + 2)(x^2 + 2)$ with respect to x (A) $\frac{dy}{dx} = 9x^2 + 4x + 6$ (B) $\frac{dy}{dx} = 4x + 5$

Let $y = \frac{u}{v}$ where u and v are functions of x.

Use the information above to answer questions 9-16.

9. What is the expression for $y + \Delta_y$? (A) $y + \Delta_y = \frac{u+\Delta_u}{v+\Delta_v}$ (B) $y + \Delta_y = \frac{u+\Delta_y}{v+\Delta_x}$
10. From 1 above obtain an expression for Δ_y (A) $\Delta_y = \frac{u+\Delta_u}{v+\Delta_v} - \frac{u}{v}$ (B) $\Delta_y = \frac{u+\Delta_u}{v+\Delta_v} - \frac{y}{x}$
11. Simplifying the expression in 2 above we have (A) $\Delta_y = \frac{yv + v\Delta_u - xy - u\Delta v}{x(v+\Delta_v)}$ (B) $\Delta_y = \frac{uv + v\Delta_u - uv - u\Delta v}{v(v+\Delta_v)}$
12. This reduces Δ_y to__________ (A) $\Delta_y = \frac{v\Delta_u - u\Delta v}{v(v+\Delta_v)}$ (B) $\Delta_y = \frac{x\Delta_u - y\Delta v}{x(v+\Delta_v)}$
13. If we divide both sides by of 4 by Δ_x then get______ (A) $\frac{\Delta_y}{\Delta_x} = \frac{\frac{v\Delta_u}{\Delta_x} - \frac{u\Delta_v}{\Delta_x}}{v(v+\Delta_v)}$ (B) $\frac{\Delta_y}{\Delta_x} = \frac{\Delta_y - \Delta_x}{v(v+\Delta_v)}$
14. As $\Delta u \rightarrow 0$ and $\Delta v \rightarrow 0$, then______ (A) $\lim_{\Delta x\rightarrow 0}\frac{\Delta_y}{\Delta_x} = \frac{dy}{dx} = \frac{v\frac{du}{dx} - 0}{v(v+\Delta v)}$ (B) $\lim_{\Delta x\rightarrow 0}\frac{\Delta_y}{\Delta_x} = \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v(v+0)}$
15. If u and v are functions of x and $y = \frac{u}{v}$, then_________(A) $\frac{dy}{dx} = \frac{v\frac{du}{dx}}{v^2}$ (B) $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
16. Use the formula above to find the derivative of the function $y = \frac{1+x^2}{1-x^2}$ w.r.t x (A) $\frac{dy}{dx} = \frac{4x}{(1-x^2)^2}$ (B) $\frac{dy}{dx} = \frac{4x}{1-x^2}$

LESSON PLAN FOR STUDY QUESTIONS ADVANCE ORGANIZERS

(SQAOS GROUP)

WEEK FOUR

Subject: Mathematics

Class: SS III

Duration: 40 minutes

Content: Differentiation by Chain Rule

Specific Objectives: At the end of the lesson, the students should be able to:

(i) Identify functions of function

(ii) State the chain rule of differentiation

(iii) Find the derivatives of function with respect to x by the chain rule

(iv) Solve problems involving differentiation by chain rule

Instructional Materials: Markerboard illustrations

Instructional Techniques/ Stategies: Set induction, questioning, reinforcement, illustration, explanation, non verbal clues, repetition and use of examples

Entry Behaviour: The students can

i. find the derivative of a function with respect to x.

Test on Entry Behaviour:

i. find the derivative of $y = (4 + x^3)(x^2 + 1)$ with respect to x

Instructional Procedure:

Steps	Content Development (CD)	Teacher's Activities	Students' Activities	Strategies
1	Introduction	The teacher introduces the lesson by asking the students to find the derivative of $y = (4 + x^3)(x^2 + 1)$ with respect to x	The students solve the problems and provide the answers to the questions. Students are expected the questions thus: $y = (4 + x^3)(x^2 + 1)$ $u = 4 + x^3$, $v = x^2 + 1$ $\therefore y = uv$ $\frac{du}{dx} = 3x^2$, $\frac{dv}{dx} = 2x$ $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ $\frac{dy}{dx} = (4 + x^3)(2x) +$ $(x^2 + 1)(3x^2)$ $= 8x + 2x^4 + 3x^4 +$ $3x^2$ $= 5x^4 + 3x^2 + 8x$	Questioning and Reinforcement

2	Function of Functions	The teacher first explains to the students the objectives of the lesson and what is expected of them. The teacher then gives each student the study questions advance organizer and asks them to study them carefully and provide answers to the questions sequentially in 15 minutes (see the last page of this lesson plan for the study questions advance organizers). The teacher explains to the students the need to provide the answers sequentially and instructs the students not to jump or mix up the numbers as each question is a hint to the succeeding question. After the 15 minutes, the teacher asks the students to stop and	Each student studies the questions independently and provides the answers to them. The students are expected to give the following answers: 1.(B) a function of a function of x 2. (B) $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ 3. (A) composite function 4.(A) $u = 2x^3 + 1$ 5. (A) $y = (u)^3$ 6. (B) $\frac{dy}{du} = 3u^2$ and $\frac{du}{dx} = 6x^2$ 7. (A)$\frac{dy}{dx} = 3u^2 \times 6x^2$ 8. (A)$\frac{dy}{dx} = 18x^2(2x^3 + 1)^2$ The students then listen attentively to the teacher.	Explanation, question and non-verbal clues

		then begins to explain function of function thus: Let $y = f(u)$ and $u = h(x)$. This means that y is a function of u and u is a function of x. Then $$y = f(h(x))$$ is said to be a function of a function of x or a composite function. Examples of functions of functions are: $y = (2x^3 + 1)^3$. $y = e^{(3x+5)}$		
3	Chain Rule for Differentiation	The teacher explains the chain for differentiation thus: Given that $y = f(u)$ and $u = h(x)$, then derivative of y w.r.t u is $\frac{dy}{du}$ The derivative of u w.r.t x is $\frac{du}{dx}$ $$\therefore \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$ This is called the chain rule of differentiation.	The students listen attentively to the teacher	Explanation, repetition and questioning

4	Differentiation of functions of functions by chain rule	The teacher explains how to differentiation of functions of functions by chain rule with the following example: Example Find the derivative of $y = (2x^3 + 1)^3$ w.r.t x Solution $y = (2x^3 + 1)^3$ $u = 2x^3 + 1$ Then $y = u^3$ $\frac{dy}{du} = 3u^2$ $\frac{du}{dx} = 6x^2$ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $\frac{dy}{dx} = 3u^2 \times 6x^2$ $= 18x^2u^2$ $= 18x^2\,(2x^3 + 1)^2$ At this point the teacher make reference to the study questions advance organizer as he asks the students to compare the solution with the advance organizer and ask questions	The students listen attentively to the teacher and write down the examples in their notebooks. The students make comparisons with the study questions and ask questions.	Use of example and explanation

5	Further problems involving Chain rule	The teacher solves further problems involving differentiation by chain rule with the following examples: <u>Example</u> Differentiate the following functions with respect to x: i. $y = tan(4x + 1)$ ii. $y = e^{(4x^2 + 5)}$ <u>Solution</u> $y = tan(4x + 1)$ $u = 4x + 1$ Then $y = \tan u$ $\frac{dy}{du} = sec^2\text{u}$, $\frac{du}{dx} = 4$ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $\frac{dy}{dx} = sec^2\text{u} \times 4$ $= 4sec^2\text{u}$ But $u = 4x + 1$ $\therefore \frac{dy}{dx} = 4sec^2(4x + 1)$. $y = e^{(4x^2 + 5)}$ Let	The students listen attentively to the teacher	Use of examples and explanation

		$u = 4x^2 + 5$, then $y = e^u$ $\frac{dy}{du} = e^u$, $\frac{du}{dx} = 8x$ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $\frac{dy}{dx} = e^u \times 8x$ $\frac{dy}{dx} = 8xe^u$ But $u = 4x^2 + 5$ $\therefore \frac{dy}{dx} = 8xe^{(4x^2+5)}$ A quick way to differentiate function of a function is to differentiate the function inside the bracket, multiply it by the derivative of the main function and then simply		
6	Evaluation	The teacher evaluates the lesson by asking the students to: Identify which of the following is (are) function(s) of function: $e^{(4x)}$, $7sinx$, $14x^3$, $(2x^3+1)^3$ .State the chain rule of	The students give answers to the questions thus: $e^{(4x)}$, $(2x^3+1)^3$ The chain rule is given by $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $y = cos2x^2$ $u = 2x^2$	Questioning

		differentiation i.Find the derivatives of function $cos2x^2$ with respect to x by the chain rule ii.Solve this problem involving differentiation by chain rule: $y = e^{sin(x+5)}$	Then $y = \cos u$ $\frac{dy}{du} = -sin\text{u}, \quad \frac{du}{dx} = 4x$ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $\frac{dy}{dx} = sin\text{u} \times 4x$ $= 4xsin\text{u}$ But $u = 2x^2$ $\therefore \frac{dy}{dx} = -4xsin2x^2$ $y = e^{sin(x+5)}$ $u = sin(x + 5)$ $y = e^u$ $\frac{du}{dx} = \cos(x + 5)$ $\frac{dy}{du} = e^u$ $\frac{dy}{dx} = e^u \times \cos(x + 5)$ $= \cos(x + 5)e^u$ $\frac{dy}{dx} = \cos(x + 5)e^{sin(x+5)}$	

Summary: The teacher summarizes the lesson thus:

i. If $y = f(u)$ and $u = h(x)$, then $y = f(h(x))$ is said to be a function of a function of x

ii. Examples of function of functions are: $y = (2x^3 + 1)^3$, $y = e^{(3x+5)}$

iii. The chain rule: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ is used for the differentiation of function of a function.

Assignments:

1. Differentiate the following with respect to x

   a. $y = ln(5x^2 + 2)$

   b. $y = e^{(3x+5)(4x-1)}$

THE STUDY QUESTIONS ADVANCE ORGANIZER FOR DIFFERENTIATION BY PRODUCT CHAIN RULE

1. Let $y = f(u)$ and $u = h(x)$. Then$y = f(h(x))$. Then y can be said to be __________ (A) a function of h (B) a function of a function of x
2. Which of these is true of the derivative of y with respect to x? (A) $\frac{dy}{dx} = \frac{dy}{dx} \times \frac{du}{dx}$ (B) $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
3. $y = f(h(x))$ can also be called a_______ (A) composite function (B) complex function

Let $y = (2x^3 + 1)^3$

Use the information above to answer questions 4-5

4. What is the value of u when the function $y = (2x^3 + 1)^3$ is compared with the expression 1 above (A) $u = 2x^3 + 1$ (B) $u = 2x^3$
5. What then will y from 4 above? (A) $y = (u)^3$ (B) $y = (x)^3$
6. Find the expression for the derivatives of u and y from 4 and 5 above. (A)) $\frac{dy}{du} = 6x^2$ and $\frac{du}{dx} = x^2$ (B) $\frac{dy}{du} = 3u^2$ and $\frac{du}{dx} = 6x^2$
7. Using the formula in 2 above, the derivative of $y = (2x^3 + 1)^3$ with respect to x can be expressed as__________(A) $\frac{dy}{dx} = 3u^2 \times 6x^2$ (B) $\frac{dy}{dx} = 6x^2 \times x^2$
8. The derivative of $y = (2x^3 + 1)^3$ with respect to x when simplified is___________________(A) $\frac{dy}{dx} = 18x^2(2x^3 + 1)^2$ (B) $\frac{dy}{dx} = 6x^4$

LESSON PLAN FOR STUDY QUESTIONS ADVANCE ORGANIZERS
(SQAOS GROUP)
WEEK FIVE

Subject: Mathematics

Class: SS III

Duration: 80 minutes

Content: Applications of Differentiation

Specific Objectives: At the end of the lesson, the students should be able to:

(i) Find higher derivatives of the function $y = f(x)$

(ii) Define and identify the maximum point, minimum point and point of inflexion of a curve of $y = f(x)$

(iii) solve problems involving maximum and minimum points

(iv) define velocity and acceleration

(v) apply differentiation to problems involving motion of a body

Instructional Materials: Markerboard illustrations

Instructional Techniques/ Strategies: Set induction, questioning, reinforcement, illustration, explanation, non verbal clues, repetition and use of examples

Entry Behaviour: The students can:

i. find the derivative of a function of x

Test on Entry Behaviour:

i. find the derivative of the function $y = 4x^6$

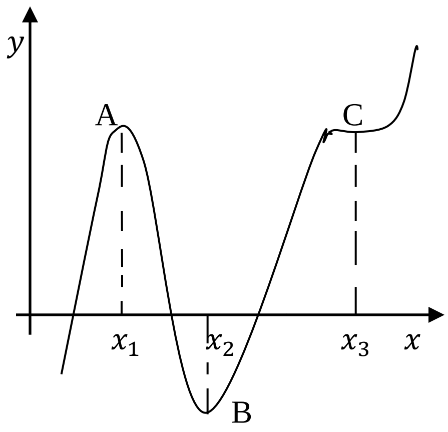
Instructional Procedure:

Steps	Content Development (CD)	Teachers' Activities	Students' Activities	Strategies
1	Introduction	The teacher introduces the lesson by asking the students to find the derivative of the function $y = 4x^6$	The students are expected to solve the problem thus: $\frac{dy}{dx} = 4 \times 6x^{6-1}$ $\frac{dy}{dx} = 24x^5$	Questioning and Reinforcement

2	Higher Derivatives of Functions	The teacher first explains to the students the objectives of the lesson and what is expected of them. The teacher then gives each student the study questions advance organizer and asks them to study them carefully and provide answers to the questions sequentially in 15 minutes (see the last page of this lesson plan for the study questions advance organizers). The teacher explains to the students the need to provide the answers sequentially and instructs the students not to jump or mix up the numbers as each question is a hint to the succeeding question. After the 15 minutes, the teacher asks the students to stop and then begins to explain how to find the higher derivatives of the functions thus: Let	Each student studies the questions independently and provides the answers to them. The expected answers to the questions are as following: 1.(A) $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ 2.(B) 14 3.(A) $6x$ 4. (A) At point A where $x = x_1$ 5. (B) At point B where $x = x_2$ 6.(A) At point C where $x = x_3$ 7.(A) there is no gradient because the tangent at the points is	Explanation, questioning and non-verbal clues

		$y = f(x)$ Then $\frac{dy}{dx}$ is the first derivative. Differentiating $\frac{dy}{dx}$ we have $$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$ $\frac{d^2y}{dx^2}$ is the second derivative of y with respect to x $\frac{d^3y}{dx^3}$ is the third derivative of y with respect to x, and so on. In general, $\frac{d^ny}{dx^n}$ is nth derivative of y with respect to x. <u>Example</u> Find the first, second and the third derivatives of each of the following functions with respect to x (a) $y = 3x^2 + 5x + 6$ (b) $y = 2xsinx$ <u>Solution</u> $(a)\ y = 3x^2 + 5x + 6$ First derivative $\frac{dy}{dx} = 6x + 5$ Second derivative, $\frac{d^2y}{dx^2} = 6$	parallel to x 8. (A) $\frac{dy}{dx} = 0$ 9. (B) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$ 10. (A) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$ 11.(A) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$ 12. (B) velocity 13.(A)acceleration 14. (A) $v = \frac{ds}{dt}$ 15. (A) $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}$ 16.(B) $v = 18t + 27$ 17. (A) $a = 18$ The students then listen attentively to the teacher. The students compare the teacher's	

		(b) $y = 2xsinx$ Applying the product rule, we have $\frac{dy}{dx} = 2x(cosx) + sinx(2)$ $\frac{dy}{dx} = 2xcosx + 2sinx$ $\frac{d^2y}{dx^2} = 2x(-sinx) + cosx(2) + 2cosx$ $\frac{d^2y}{dx^2} = -2xsinx + 2cosx + 2cosx$ $\frac{d^2y}{dx^2} = -2xsinx + 4cosx$	explanations with the answers they answer in the advance organizer	

3	Maximum and Minimum Point of a Curve	The teacher explains maximum and minimum point of a curve thus:  Consider the graph above of the function $y = f(x)$. At point A i.e. $x = x_1$, a maximum value occurs. Here there is a change in sign from $+ve$ to $-ve$ as we turn At point B i.e. $x = x_2$, a minimum value occurs. Here there is a change in sign from $-ve$ to $+ve$ as we turn. The maximum point A and minimum point B are called turning points. At point C i.e. $x = x_3$, the curve flattens and goes up again. We call this, point of inflexion. Points A, B and C are called	The students listen attentively and take note	Explanation and illustration

		stationary points. There, gradient (or slope) is zero since the tangents at these points are parallel to x-axis. That is, $\frac{dy}{dx} = 0$ at A, B and C		

4	Problems Involving Maximum and Minimum Points	The teacher explains how to solve problems involving stationary points thus: the necessary and sufficient conditions for stationary points are as follows: 1. If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2}$ $at\ x = a > 0$ then there is a maximum point. 2. If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0\ at\ x = a$ then there is a minimum point. 3. If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0\ at\ x = a$ then there is a point of inflexion. The teacher at this point asks the students to compare the stationary points with what they have in the study questions advance organizer <u>Example 1</u> Find the stationary point on the	The students listen attentively to the teacher. The students make comparisons with the advance organizer	Explanation, Repetition and use of examples

		curve $$y = x^3 - 5x^2 + 12x - 8$$ And distinguish between them. <u>Solution</u> $$y = x^3 - 5x^2 + 12x - 8$$ $\frac{dy}{dx} = 3x^2 - 12x + 12$. At stationary point $\frac{dy}{dx} = 0$. So, $$3x^2 - 12x + 12 = 0$$ $$x^2 - 4x + 4 = 0$$ $$(x - 2)^2 = 0$$ $$x = 2$$ $$\frac{d^2y}{dx^2} = 6x - 12$$ At $x = 2$, $$\frac{d^2y}{dx^2} = 6(2) - 12 = 12 - 12$$ $$= 0$$ There is a point of inflexion since $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$ at $x = 2$ <u>Example 3</u> Find the nature of the turning point of the function $$\frac{x^3}{3} + x^2 - 3x + 4$$		

		Solution $$\frac{dy}{dx} = x^2 + 2x - 3$$ At turning point $\frac{dy}{dx} = 0$. So, $$x^2 + 2x - 3 = 0$$ $$x^2 + 3x - x - 3 = 0$$ $$x(x+3) - 1(x+3) = 0$$ $(x-1)(x+3) = 0$ $x = 1,\ x = -3$ $$\frac{d^2y}{dx^2} = 2x + 2$$ At $x = 1$, $$\frac{d^2y}{dx^2} = 2(1) + 2 = 2 + 2 = 4$$ $$> 0$$ At $x = 1$, there is a minimum point. At $x = -3$, $$\frac{d^2y}{dx^2} = 2(-3) + 2 = -6 + 2$$ $$= -4 < 0$$ At $x = -3$, there is a maximum point.		

5	Definition of Velocity and Acceleration	The teacher defines velocity and acceleration thus: The motion of a particle (body) along straight line can be specified by the function $$s = f(t)$$ Where s is the displacement and t is the time. The velocity of the body is therefore the rate of change of the displacement with respect to time. It is given by $$v = \frac{ds}{dt}$$ If the body is momentarily at rest, then $$\frac{ds}{dt} = 0$$ The acceleration of the body is the rate of change of velocity with respect to time given by $$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}$$ At constant velocity, $\frac{dv}{dt} = 0$ If $\frac{dv}{dt} < 0$, we have retardation or deceleration	The students pays attention and write down notes	Explanation and questioning

6	Application of Differentiation to Motion	The teacher explains application of differentiations to motion with the following examples: Example 1 The motion of a particle along a straight line is described by the distance function $s = t^3 - 9t^2 + 27t - 10$ Find: (a) the velocity and acceleration after 2 secs. (b) the distance when the particle is momentarily at rest Solution $s = t^3 - 9t^2 + 27t - 10$ Velocity $v = \frac{ds}{dt} = 3t^2 - 18t + 27$ When $t = 2$ secs. $v = 3(2)^2 - 18(2) + 27$ $v = 12 - 36 + 27$ $v = 3ms^{-1}$ $a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 6t - 18$	The students listen attentively to the teacher	Explanation and use of examples

		At $t = 2$, $a = 6 \times 2 - 18 = 12 - 18$ $a = -6ms^{-2}$ which is a retardation. (b) If the body is momentarily at rest, then $\frac{ds}{dt} = 0$ That is, $3t^2 - 18t + 27 = 0$ $t^2 - 6t + 9 = 0$ $t^2 - 6t + 9 = 0$ $t^2 - 3t - 3t + 9 = 0$ $t(t-3) - 3(t-3) = 0$ $(t-3)(t-3) = 0$ $\therefore t = 3\ secs$ Hence, $s = t^3 - 9t^2 + 27t - 10$ $s = t^3 - 9t^2 + 27t - 10$ $s = 27 - 9 \times 9 + 27(3) - 10$ $s = 27 - 81 + 81 - 10$ $s = 27 - 10$ $s = 17m$		

7	Evaluation	The teacher evalautes the lesson asking the students to: i. Find third derivative of the function $y = 5x^3 - 6x^2 + 7x - 10$ w.t. x ii. Define and identify the maximum point, minimum point and point of inflexion of a curve of $y = f(x)$ iii. Solve the following problems involving maximum and minimum points: Find the nature of the turning point of the function $\frac{x^3}{3} + x^2 - 3x + 4$ iv. Define velocity and acceleration v. Apply differentiation to solve the following problems involving motion of a body: Find the velocity and acceleration of the function $s = 9t^2 + 27t$	The students give answers to the questions. The students are expected to provide the following answers: (i) $y = 5x^3 - 6x^2 + 7x - 10$ $\frac{dy}{dx}$ $= 3 \times 5x^{3-1} - 2 \times 6x^{2-1} + 7 - 0$ $\frac{dy}{dx} = 15x^2 - 12x + 7$ $\frac{d^2y}{dx^2} = 30x - 12$ $\therefore \frac{d^3y}{dx^3} = 30$ (ii) If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2}$ $at\ x = a > 0$ then there is a maximum point. If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0\ at\ x = a$	Questioning and Reinforcement

		at $t = 5s$ the teacher asks the answer they get in (v) with question 16 in the study questions advance organizer	then there is a minimum point. If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0 \; at \; x = a$ then there is a point of inflexion. (iii) $y = \frac{x^3}{3} + x^2 - 3x + 4$ For turning point, $\frac{dy}{dx} = 0$ $\frac{dy}{dx} = x^2 + 2x - 3$ $= 0$ $x^2 + 2x - 3 = 0$ $x^2 + 3x - x - 3$ $= 0$ $x(x + 3) - 1(x$ $+ 3) = 0$ $(x + 3)(x - 1) = 0$ $x + 3 = 0, \;\; x - 1 =$ 0 $x = -3, x = 1$ $\frac{d^2y}{dx^2} = 2x + 2$	

			$\frac{d^2y}{dx^2}$ at $x = -3$ $\frac{d^2y}{dx^2} = 2(-3) + 2$ $= -6 + 2 = -4$ (iv) The velocity of a body with the displacement $s = f(t)$ is given by $v = \frac{ds}{dt}$. The acceleration of the body is given by $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}$. (iv) $s = t^2 - 3t + 4$ at $t = 5s$ $v = \frac{ds}{dt} = 2t - 3$ At $t = 5s$ $v = 2(5) - 3$ $10 - 3 = 7m/s$ $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$ $= 2.$ At $t = 5s$ $a = 2ms^{-2}$.	

Summary: The teacher summarizes the lesson thus:

i. To find the second derivatives $\frac{d^n y}{dx^n}$ of the function $y = f(x)$ is we differentiate the function n times

ii. The stationary points have the following conditions:

a. If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2}\ at\ x = a > 0$ then there is a maximum point.

b. If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0\ at\ x = a$ then there is a minimum point.

c. If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0\ at\ x = a$then there is a point of inflexion.

iii. The velocity of a body with the displacement $s = f(t)$ is given by $v = \frac{ds}{dt}$. If the body is momentarily at rest, then $\frac{ds}{dt} = 0$

iv. The acceleration of the body is then given by $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}$. At constant velocity, $\frac{dv}{dt} = 0$. If $\frac{dv}{dt} < 0$, we have retardation or deceleration.

Assignment

1. Given the distance function

$$s = 4t^3 + 7t - 10$$

Find:

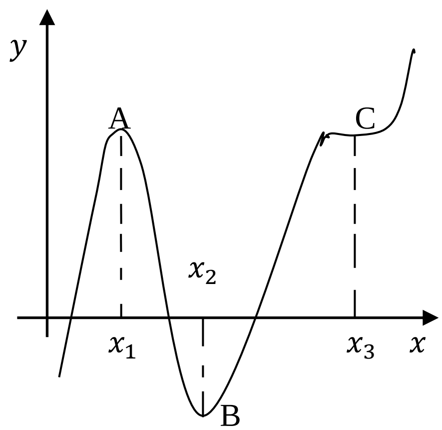
(a) the velocity and acceleration after 2 secs.

(b) the distance assuming the particle is momentarily at rest

THE STUDY QUESTIONS ADVANCE ORGANIZER FOR APPLICATION OF DIFFERENTIATION

1. If the first derivative of the function $y = f(x)$ is $\frac{dy}{dx}$, then the second and the third derivatives are__________ respectively (A) $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ (B) $\frac{dx^2}{d^2y}$ and $\frac{dx^3}{d^3y}$
2. What is the second derivative of $y = 7x^2$ _______ (A) $14x$(B) 14
3. Find the third derivative of $y = x^3 + 4x$ with respect to x (A) $6x$ (B) $3x^2 + 4$

Consider the graph below of the function $y = f(x)$



Consider the graph above carefully and answer the following questions

4. At what point does the maximum point occur? (A) At point A where $x = x_1$ (B) At point B where $x = x_2$
5. Where does the minimum point occur? (A) At point A where $x = x_1$ (B) At point B where $x = x_2$
6. At what point do we have the point of inflexion? (A) At point C where $x = x_3$ (B) At point B where $x = x_2$
7. What happens to the gradient of the curve at points A, B and C? (A) there is no gradient because the tangent at the points is parallel to x (B) the gradient increases
8. The gradient of the curve at points A, B and C is given by_______ (A) $\frac{dy}{dx} = 0$ (B) $\frac{dy}{dx} = f(x) + c$
9. The necessary and sufficient condition for the curve to have maximum point $at\ x = a$ is that______(A) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$ (B) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$

10. For the curve to have minimum point $at\ x = a$ then________ (A) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$ (B) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$
11. The necessary and sufficient condition for the curve to be a point of inflexion $at\ x = a$ is that______(A) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$ (B) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$
12. The rate of change of displacement of a particle with respect to time is called_____ (A) speed (B) velocity
13. __________ is the rate of change of velocity with respect to time (A) acceleration (B) speed
14. If the motion of a body is defined by the function $s = f(t)$ where s is displacement and t is the time taken, then the velocity of the body is given by_________ (A) $v = \frac{ds}{dt}$ (B) $v = \frac{dt}{ds}$
15. The acceleration of the same body in 14 above is __________ (A) $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}$ (B) $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dt}{ds}\right) = \frac{d^2t}{ds^2}$

The motion of a particle along a straight line is described by the distance function

$$s = 9t^2 + 27t$$

Use the information to answer question to answer questions 16 to 17

16. What is the velocity of the body? (A) $s = 18t$ (B) $s = 18t + 27$
17. Find the acceleration of the body (A) $s = 18$ (B) $s = 18t$

APPENDIX C4

LESSON PLAN FOR CONCEPT MAPPING STRATEGY

(CMS GROUP)

WEEK ONE

Subject: Mathematics

Class: SSS III

Duration: 40 minutes

Content: Differentiation from the First Principle

Specific Objectives: At the end of the lesson, the students should be able to:

(i) define and explain the concept of differentiation

(ii) state the differential coefficient of the function $y = f(x)$ from the first principle

(iii) find the derivative of functions from the first principle

Instructional Materials: Markerboard illustrations

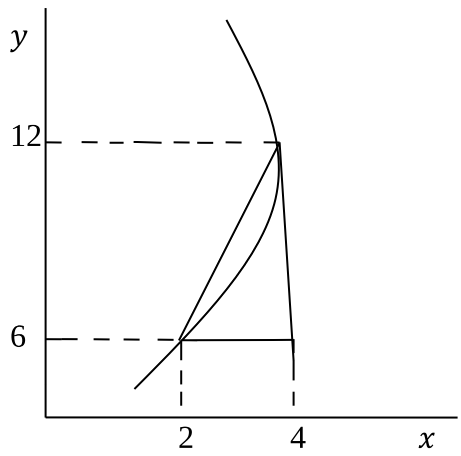
Entry Behaviour: The students can:

i. state the formula for determining the slope of a curve.

ii. determine the slope of a curve.

Test on Entry Behaviour:

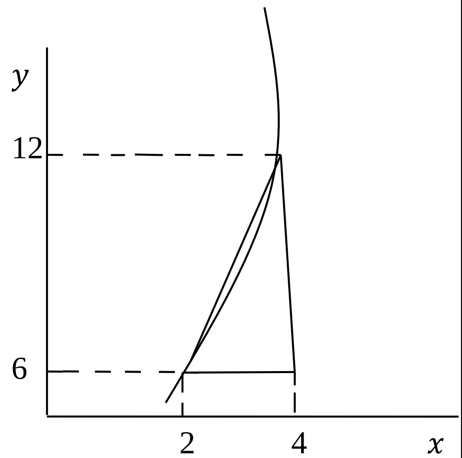
1. State the formula for finding the slope of the graph below?
2. Determine the slope of the graph below

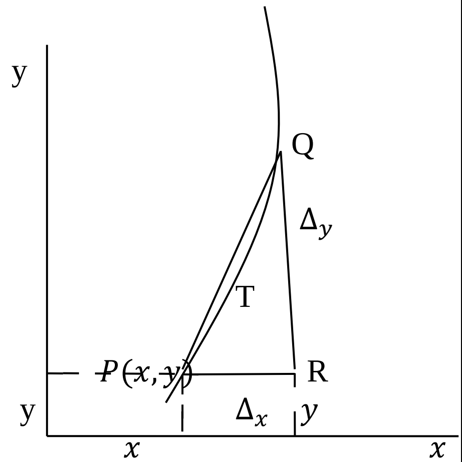


Instructional Techniques: Set induction, questioning, reinforcement, illustration, explanation, non verbal clues, repetition and use of examples

Insructional Procedure:

Steps	Contents Development (CD)	Teachers' Activities	Students' Activities	Strategies
1	Introduction	The teacher arouses the students' interest by asking the following questions: use the graph below to answer the following questions:	The students provide the answers to the questions thus: $Slope = \frac{y_2 - y_1}{x_2 - x_1}$ $Slope = \frac{12-6}{4-2} = \frac{6}{2} = 3$	Set induction Questioning and reinforcement

		 What is the formula for finding the slope of the graph above? Determine the slope of the graph		
2	The concept of Differentiation and Differentiation from the First Principle	The teacher first explains to the students the objectives of the lesson and what is expected of them. The teacher then gives each student the concept map advance organizer (see the back page of this lesson plan) and asks them to study it in 15 minutes. The teacher guides the students on how to follow the concept map with the arrows directions. After the 15 minutes, the teacher	Each student studies the concept map independently. The students then listen attentively to the teacher. The students refer to the concept map while answering the teacher's question	Explanation, non verbal clues and repetition

		asks the students to stop and then begins to explain the concept of differentiation thus:  The slope $m_1 = \frac{RQ}{RP} = \frac{\Delta_y}{\Delta_x}$ Let point P be fixed and let Q approach P so that we now have a new line PT. As point Q approaches the point on the curve, Δ_x becomes smaller and smaller until it eventually becomes zero. Then the limiting value m_2 of m_1 as Δ_x tends to zero can be written as $m_2 = \lim \overset{\Delta x}{\rightarrow} 0\; m_1 = \lim \overset{\Delta x}{\rightarrow} 0\; \frac{\Delta_y}{\Delta_x}$ and is represented by $\frac{dy}{dx}$ which is read as dee y dee x . That is, $\frac{dy}{dx} = \lim \overset{\Delta x}{\rightarrow} 0\; \frac{\Delta_y}{\Delta_x}$ $\frac{dy}{dx}$ is called differential coefficient or derivative of y with		

		respect to x. We also use $f'(x)$ or y' read f prime of x or y prime to mean $\frac{dy}{dx}$. The process of determining the differential coefficient or derivatives is called differentiation. Terms often associated with differentiation are gradient or slope of a curve, rate of change (at this point the teacher makes reference to the concept by asking the students to mention words that are used instead of differentiation) Now consider $y = f(x)$ then $y + \Delta_y = f(x + \Delta_x)$ $\Delta_y = f(x + \Delta_x) - y$, i.e $\Delta_y = f(x + \Delta_x) - f(x)$ Divide both sides by Δ_x $\frac{\Delta_y}{\Delta_x} = \frac{f(x+\Delta_x)-f(x)}{\Delta_x}$ $\lim \overset{\Delta x}{\rightarrow} 0 \frac{\Delta_y}{\Delta_x} = \lim \overset{\Delta x}{\rightarrow} 0 \frac{f(x+\Delta_x)-f(x)}{\Delta_x}$ But $\frac{dy}{dx} = \lim \overset{\Delta x}{\rightarrow} 0 \frac{\Delta_y}{\Delta_x}$ $\therefore \frac{dy}{dx} = \lim \overset{\Delta x}{\rightarrow} 0 \frac{f(x+\Delta_x)-f(x)}{\Delta_x}$ This is called the differential coefficient of $f(x)$ from the first		

		principle. At this point, the teacher should ask the students to compare the formula above with what it is in the concept map when $x = a$, then $f'(a)$ $= \lim \overset{\Delta x}{\rightarrow} 0 \frac{f(a+\Delta_x) - f(a)}{\Delta_x}$		
3	Finding the Derivative from the First Principle	Example: Find from the first principle the derivative of each of the following functions with respect to x i. $y = x$ ii. $y = x^2$ Solution i. $y = x$ $f(x) = x$ $f(x+\Delta_x) = x + \Delta_x$ Apply the formula: $\frac{dy}{dx} = \lim \overset{\Delta x}{\rightarrow} 0 \frac{f(x+\Delta_x)-f(x)}{\Delta_x}$ $= \lim \overset{\Delta x}{\rightarrow} 0 \frac{x+\Delta_x - x}{\Delta_x}$ $= \lim \overset{\Delta x}{\rightarrow} 0 \frac{\Delta_x}{\Delta_x} = 1$ $\frac{dy}{dx} = 1$ ii. $y = x^2$	The students listen attentively and ask questions as they make comparisons with concept map advance organizer.	Use of examples and explanation

		$f(x) = x^2$ $f(x+\Delta_x) = (x+\Delta_x)^2$ $\frac{dy}{dx} = \lim \overset{\Delta x}{\rightarrow} 0 \, \frac{f(x+\Delta_x)-f(x)}{\Delta_x}$ $= \lim \overset{\Delta x}{\rightarrow} 0 \, \frac{(x+\Delta_x)^2 - x^2}{\Delta_x}$ $= \lim$ $\overset{\Delta x}{\rightarrow} 0 \, \frac{x^2 + 2x\Delta_x - (\Delta_x)^2 - x^2}{\Delta_x}$ $= \lim \overset{\Delta x}{\rightarrow} 0 \, \frac{2x\Delta_x - (\Delta_x)^2}{\Delta_x}$ $= \lim \overset{\Delta x}{\rightarrow} 0 \; 2x - \Delta_x$ $= 2x$ at this point, the teacher asks the students to make comparisons with the organizer and ask questions $y = \frac{1}{x}$ $f(x) = \frac{1}{x}$, $f(x+\Delta_x) = \frac{1}{x+\Delta_x}$ Apply the formula: $\frac{dy}{dx} = \lim \overset{\Delta x}{\rightarrow} 0 \, \frac{f(x+\Delta_x)-f(x)}{\Delta_x}$ $= \lim \overset{\Delta x}{\rightarrow} 0 \, \frac{\frac{1}{x+\Delta_x} - \frac{1}{x}}{\Delta_x}$ $= \lim \overset{\Delta x}{\rightarrow} 0 \, \frac{\frac{x-(x+\Delta_x)}{x(x+\Delta_x)}}{\Delta_x}$ $= \lim \overset{\Delta x}{\rightarrow} 0 \, \frac{\frac{-\Delta_x}{x^2 + x\Delta_x}}{\Delta_x}$ $\lim \overset{\Delta x}{\rightarrow} 0 \, \frac{-\Delta_x}{\Delta_x(x^2 + x\Delta_x)}$		

		$\lim \overset{\Delta x}{\to} 0 \dfrac{-1}{(x^2 + x\Delta_x)}$ $= \dfrac{-1}{(x^2+0)} = \dfrac{-1}{(x^2)}$ $\therefore \frac{dy}{dx} = \frac{-1}{(x^2)}$		
4	Evaluation	The teacher evaluates the lesson by asking the students to: i. Define and explain the concept of differentiation ii. State the differential coefficient of the function $y = f(x)$ from the first principle iii. Find the derivative of the function $y = x^3$ from the first principle	The students give answers to the questions The students give answers to the questions thus: $y = x^3$ $f(x) = x^3$ $f(x + \Delta_x) = (x + \Delta_x)^3$ $\frac{dy}{dx} = \lim \overset{\Delta x}{\to} 0 \frac{f(x+\Delta_x) - f(x)}{\Delta_x} =$ $\lim \overset{\Delta x}{\to} 0 \frac{(x+\Delta_x)^3 - x^3}{\Delta_x} =$ $\lim \overset{\Delta x}{\to} 0 \frac{3x^2\Delta_x + 3x(\Delta_x)^2 + (\Delta_x)^3}{\Delta_x}$ $= \lim \overset{\Delta x}{\to} 0\ 3x^2 + 3x\Delta_x = 3x^2$	Questioning and non-verbal clues

Summary: The teacher summarizes the lesson by highlighting the key points of the lesson thus:

i. The process of finding the differential coefficient or derivative of a function is called differential calculus.

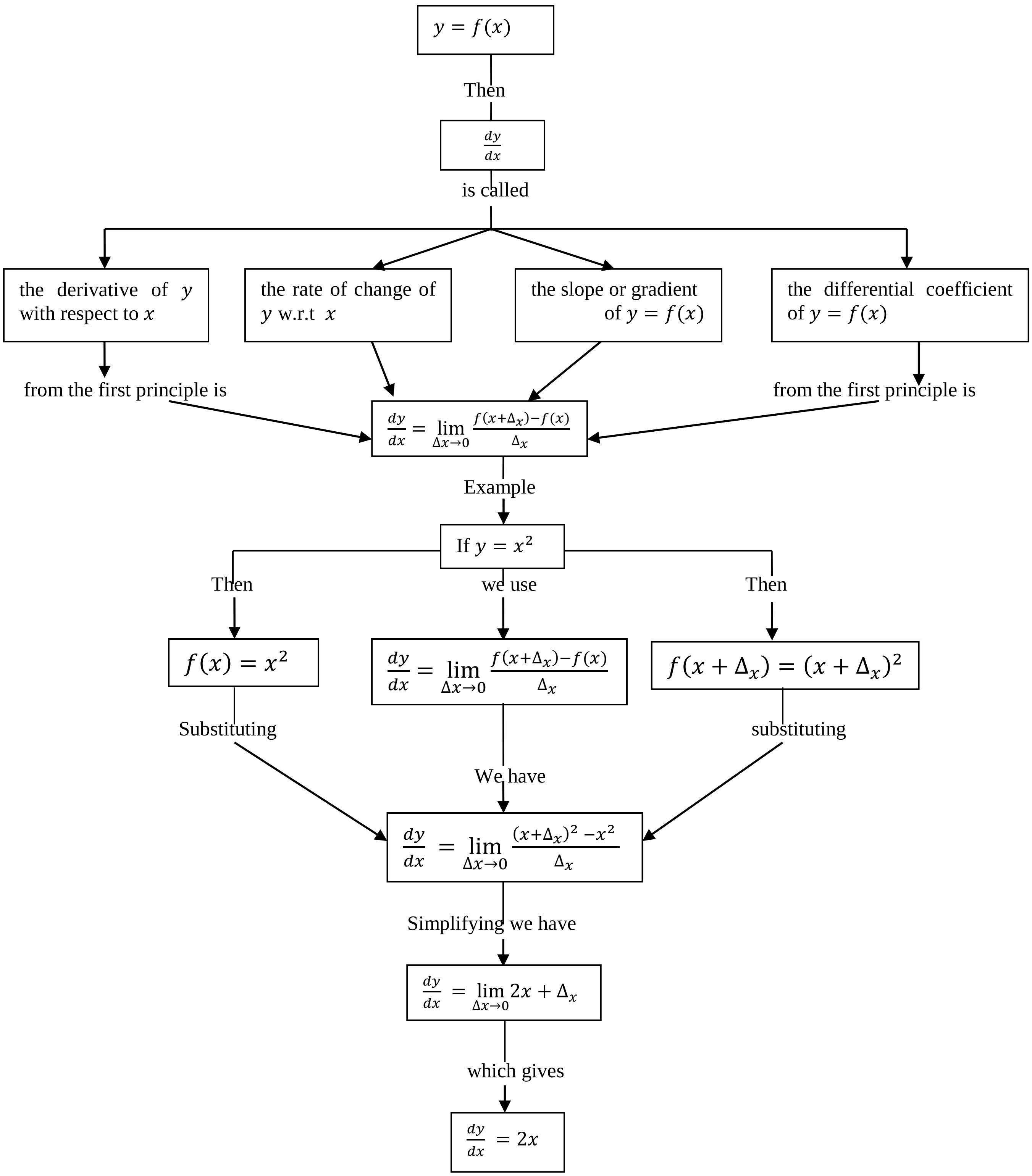
ii. The derivative of the function $y = f(x)$ from the first principle is given by

$$\frac{dy}{dx} = \lim \overset{\Delta x}{\rightarrow} 0 \, \frac{f(x + \Delta_x) - f(x)}{\Delta_x}$$

Assignment

1. Find, from the first principle, the differential coefficient of $y = 4x^2$
2. Determine the derivative of $y = \frac{1}{x^2}$ with respect to x from the first principle.

THE CONCEPT MAP ON DIFFERENTIATION FROM THE FIRST PRINCIPLE



LESSON PLAN FOR CONCEPT MAPPING STRATEGY
(CMS GROUP)
WEEK TWO

Subject: Mathematics

Class: SS III

Duration: 40 minutes

Content: Differentiation of Polynomials

Specific Objectives: At the end of the lesson, the students should be able to:

(i) derive the formula for the differentiation of the polynomial $y = x^n$ from the first principle

(ii) find the derivatives of polynomials with respect to x

(iii) state the standard derivatives of some functions

Instructional Materials: Markerboard illustrations

Instructional Techniques/ Strategies: Set induction, questioning, reinforcement, illustration, explanation, non verbal clues, repetition and use of examples

Entry Behaviour: The students can:

i. find the derivative of functions with respect to x.

Test on Entry Behaviour:

1. find the derivative of $y = 3x^2$

Instructional Procedure:

Steps	Content Development (CD)	Teachers' Activities	Students' Activities	Strategies
1	Introducion	The teacher introduces the lesson by asking the students to find the derivative of $y = 3x^2$ with respect to x using the first principle:	The students solve the problems and provide the answers to the questions. Students are expected to solve the questions thus: $y = x^2$ $f(x) = x^2$ $f(x + \Delta_x) = (x + \Delta_x)^2$ $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta_x) - f(x)}{\Delta_x} = \lim_{\Delta x \to 0} \frac{(x+\Delta_x)^2 - x^2}{\Delta_x}$ $= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta_x - (\Delta_x)^2}{\Delta_x}$ $= \lim_{\Delta x \to 0} \frac{2x\Delta_x - (\Delta_x)^2}{\Delta_x} = \lim_{\Delta x \to 0} \frac{\Delta_x(2x - \Delta_x)}{\Delta_x}$ $= \lim_{\Delta x \to 0} 2x - \Delta_x$ $= 2x$	Set induction and Questioning

2	Deriving the Formula for Differentiating polynomials	The teacher first explains to the students the objectives of the lesson and what is expected of them. The teacher then gives each student the concept map advance organizer (see the back page of this lesson plan) and asks them to study it in 15 minutes. The teacher guides the students on how to follow the concept map with the arrows directions. After the 15 minutes, the teacher asks the students to stop and then begins to explainhow to derive the formula used for the differentiation of polynomials thus: Let $y = x^n$, then $y + \Delta_y = (x + \Delta_x)^n$ $\Delta_y = (x + \Delta_x)^n - y$ $\Delta_y = (x + \Delta_x)^n - x^n$ Expanding $(x + \Delta_x)^n$ using the binomial expansion $(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$	Each student studies the concept map independently. The students then listen attentively to the teacher while making reference to the advance organizer to make comparisons	Explanation, questioning and non-verbal clues

		$= x^n + nx^{n-1}y + \frac{n(n-1)}{1.2}x^{n-1}y^2 +$
		$\cdots + y^n$
		We have
		$(x + \Delta_x)^n = x^n + nx^{n-1}\Delta_x$
		$+ \frac{n(n-1)}{1.2}x^{n-1}(\Delta_x)^2 + \cdots (\Delta_x)^n$
		$\Delta_y = x^n + nx^{n-1}\Delta_x +$
		$\frac{n(n-1)}{1.2}x^{n-1}(\Delta_x)^2 + \cdots (\Delta_x)^n \; - x^n$
		$\frac{\Delta_y}{\Delta_x} \quad = nx^{n-1} + \frac{n(n-1)}{1.2}x^{n-1}\Delta_x +$
		$\cdots (\Delta_x)^{n-1}$
		As $\Delta_x \to 0$, then
		$\frac{dy}{dx} \; = nx^{n-1}$
		Also,
		if $y = ax^n$, then
		$\frac{dy}{dx} \; = anx^{n-1}$
		if u and v are functions of x then
		$\frac{d(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx}$		

3	Differentiation of polynomial	<u>Example:</u> Differentiate the following with respect to x 1. $y = x^3$ 2. $y = 3x^4$ 3. $y = x^4 + 2x^3 - 4x^{-2} + 2x - 1$	The students listen attentively to the teacher and write the example in their notes. The students	Use of examples and explanation

		at $x = 1$ Solution 1. $y = x^3$ $\frac{dy}{dx} = 3x^{3-1} = 3x^2$ 2. $y = 3x^4$ $\frac{dy}{dx} = 4 \times 3x^{4-1} = 12x^3$ 3. $y = x^4 + 2x^3 - 4x^{-2} + 2x - 1$ $\frac{dy}{dx} = \frac{d(x^4)}{dx} + \frac{d(2x^3)}{dx} - \frac{d(4x^{-2})}{dx} + \frac{d(2x)}{dx} - 1$ $\frac{dy}{dx} = 4x^3 + 6x^2 + 8x^{-3} + 2 + 0$ $\frac{dy}{dx} = 4x^3 + 6x^2 + 8x^{-3} + 2$ $\frac{dy}{dx} = 4x^3 + 6x^2 + \frac{8}{x^3} + 2$ At $x = 1$ $\frac{dy}{dx} = 4(1)^3 + 6(1)^2 + \frac{8}{(1)^3} + 2$ $\frac{dy}{dx} = 4 + 6 + 8 + 2 = 20$	ask questions.	
4	Standard Derivatives of Some Basic Functions	The teacher states the standard derivatives of some basic functions thus: 1. If $y = sinx$, then $\frac{dy}{dx} = \frac{d(sinx)}{dx} = cosx$ 2. If $y = cosx$, then $\frac{dy}{dx} = \frac{d(cosx)}{dx} = -sinx$	The students compare the standard derivatives with what are in the concept map. The students then write the standard in their	Explanation

		3. If $y = e^x$, then $\frac{dy}{dx} = \frac{d(e^x)}{dx} = e^x$ 4. If $y = c$, then $\frac{dy}{dx} = \frac{d(c)}{dx} = 0$ 5. If $y = \log_e x$ or $\ln x$, then $\frac{dy}{dx} = \frac{d(\log_e x)}{dx}$ or $\frac{d(\ln x)}{dx} = \frac{1}{x}$ 6. If $y = tanx$, then $\frac{dy}{dx} = \frac{d(tanx)}{dx} = sec^2x$ At this point, the teacher should allow the students to compare the standard derivatives with what are in the concept maps as they write the standard derivatives in their notes	notes	
5	Evaluation	The teacher evaluates the lesson by asking the students to: i. Derive the formula for the differentiation of the polynomial $y = x^n$ from the first principle ii. Find the derivatives of the following polynomials with respect to x: (a) $y = 4x^3 + 3x + 6$ (b) $y = 3x^4 + x^3 - 2x^2 + 5x - 7$	The students give answers to the questions thus: 1. Let $y = x^n$, then $y + \Delta_y = (x + \Delta_x)^n$ $\Delta_y = (x + \Delta_x)^n - y$ $\Delta_y = (x + \Delta_x)^n - x^n$ Expanding $(x + \Delta_x)^n$ using the binomial expansion	Questioning and reinforcement

		iii. State the standard derivatives of the following functions: x^n, $sinx$, $cosx$, $tanx$ and e^x	$(x+y)^n =$ $\sum_{k=0}^{n}\binom{n}{k} x^{n-k}y^k$ $= x^n + nx^{n-1}y +$ $\frac{n(n-1)}{1.2}x^{n-1}y^2 +$ $\cdots + y^n$ We have $(x+\Delta_x)^n = x^n$ $+ nx^{n-1}\Delta_x$ $+ \frac{n(n-1)}{1.2}x^{n-1}(\Delta_x)^2$ $+ \cdots (\Delta_x)^n$ $\Delta_y = x^n + nx^{n-1}\Delta_x +$ $\frac{n(n-1)}{1.2}x^{n-1}(\Delta_x)^2 +$ $\cdots (\Delta_x)^n - x^n$ $\frac{\Delta_y}{\Delta_x} = nx^{n-1} +$ $\frac{n(n-1)}{1.2}x^{n-1}\Delta_x +$ $\cdots (\Delta_x)^{n-1}$ As $\Delta_x \to 0$, then $\frac{dy}{dx} = nx^{n-1}$ ii. (a) $y = 3x^4 + x^3 - 2x^2 + 5x - 7$ $\frac{dy}{dx} = 4 \times 3x^{4-1} +$ $3x^{3-1} + 2 \times 2x^{2-1} +$ $5 - 0$ $\frac{dy}{dx} = 12x^3 + 3x^2 +$	

			$4x + 5 - 0$ $\frac{dy}{dx} = 12x^3 + 3x^2 + 4x + 5$ The standard derivatives of x^n, $sinx$, $cosx$, $tanx$ and e^x are respectively: $\frac{dy}{dx} = nx^{n-1}$, $\frac{dy}{dx} = cosx$, $\frac{dy}{dx} = -sinx$, $\frac{dy}{dx} = sec^2x$ and $\frac{dy}{dx} = e^x$	

Summary: The teacher summarizes the lesson by highlighting the main points of the lesson thus:

i. If $y = x^n$, then $\frac{dy}{dx} = nx^{n-1}$

ii. If $y = ax^n$, then $\frac{dy}{dx} = anx^{n-1}$

iii. If u and v are functions of x then $\frac{d(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

iv. If $y = sinx$, then $\frac{dy}{dx} = \frac{d(sinx)}{dx} = cosx$

v. If $y = cosx$, then $\frac{dy}{dx} = \frac{d(cosx)}{dx} = -sinx$

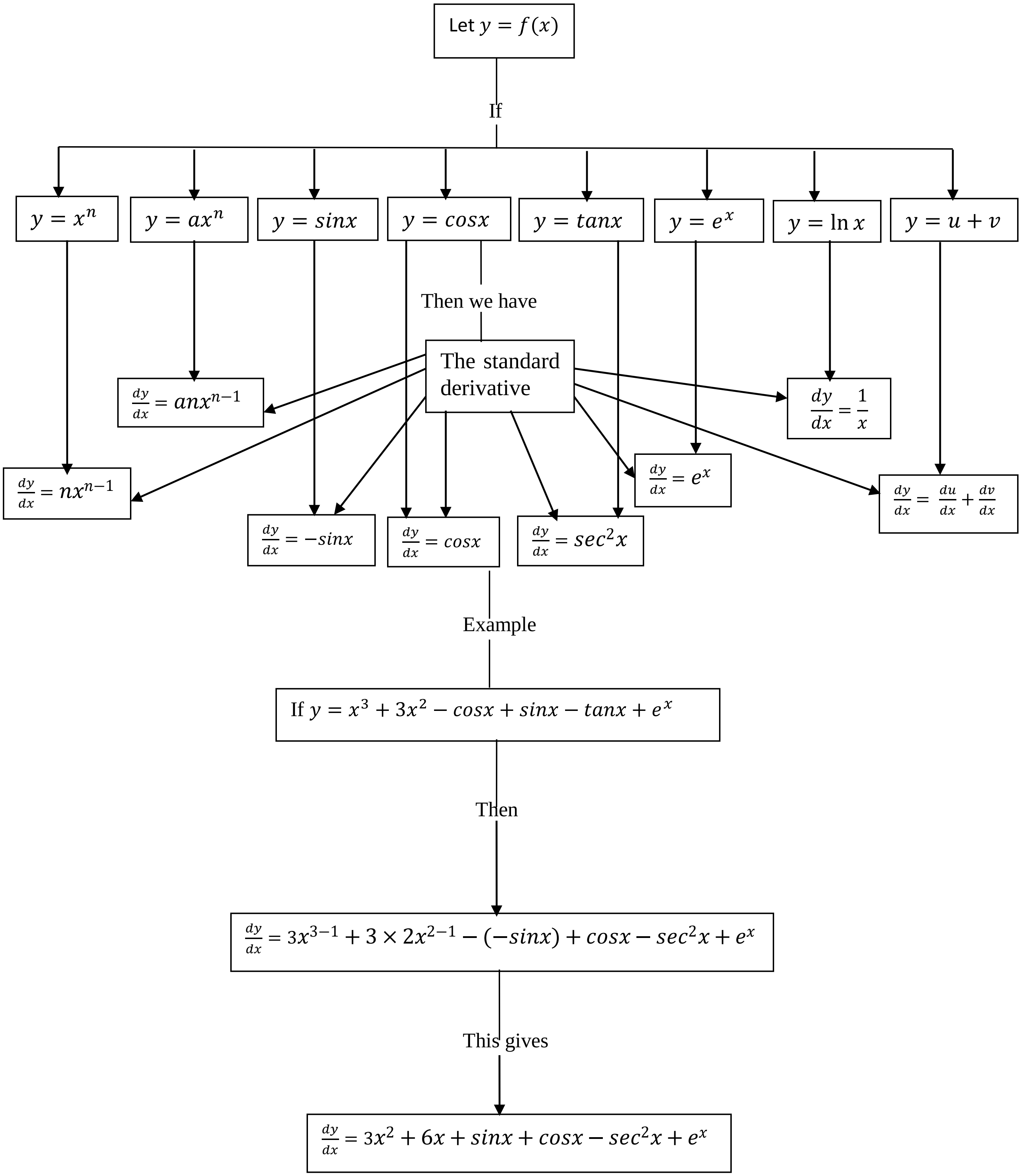
vi. If $y = e^x$, then $\frac{dy}{dx} = \frac{d(e^x)}{dx} = e^x$

vii. If $y = c$, then $\frac{dy}{dx} = \frac{d(c)}{dx} = 0$

viii. If $y = \log_e x$ or $\ln x$, then $\frac{dy}{dx} = \frac{d(\log_e x)}{dx}$ or $\frac{d(\ln x)}{dx} = \frac{1}{x}$

ix. If $y = tanx$, then $\frac{dy}{dx} = \frac{d(tanx)}{dx} = sec^2x$

THE CONCEPT MAP ON DIFFERENTIATION OF POLYNOMIALS



LESSON PLAN FOR CONCEPT MAPPING STRATEGY

(CMS GROUP)

WEEK THREE

Subject: Mathematics

Class: SS III

Duration: 80 minute

Content: Rules of Differentiation: Product and Quotient Rule

Specific Objectives: At the end of the lesson, the students should be able to:

(i) state the product rule of differentiation

(ii) find the derivatives of polynomials with respect to x using the product rule

(iii) state the quotient rule of differentiation

(iv) find the derivatives of function with respect to x by the quotient rule

(v) solve problems involving trigonometric functions using product and quotient rule

Instructional Materials: Markerboard illustrations

Instructional Techniques: Set induction, questioning, reinforcement, illustration, explanation, non verbal clues, repetition and use of examples

Entry Behaviour: The students can:

i. find the derivative of polynomials with respect to x.

Test on Entry Behaviour:

i. find the derivative of the polynomial with respect to x: $y = 2x^4 - x^3 + 3x^2$ at $x = 2$

Instructional Procedure:

Steps	**Content Development (CD)**	**Teacher's Activities**	**Students' Activities**	**Strategies**
1	Introduction	The teacher introduces the lesson by asking the students to find the derivative of the polynomial $y = 2x^4 - x^3 + 3x^2$ at $x = 2$ with respect to x.	The students solve the problems and provide the answers to the questions. Students are expected the questions thus: $\frac{dy}{dx} = \frac{d(2x^4)}{dx} - \frac{d(x^3)}{dx} + \frac{d(3x^2)}{dx}$ $\frac{dy}{dx} = 8x^3 - 3x^2 + 6x$ At $x = 2$ $\frac{dy}{dx} = 8(2)^3 - 3(2)^2 + 6(2)$ $\frac{dy}{dx} = 8 \times 8 - 3 \times 4 + 12$ $\frac{dy}{dx} = 64 - 12 + 12 = 64$	Questioning and Reinforcement

2	Differentiation by Product Rule	The teacher first explains to the students the objectives of the lesson and what is expected of them. The teacher gives each student the concept map advance organizer (see the back page of this lesson plan) and asks them to study it in 15 minutes. The teacher guides the students on how to follow the concept map with the arrows directions. After the 15 minutes, the teacher asks the students to stop and then begins to explain how to derive the product rule of differentiation Let $y = uv$, where u and v are functions of x, then $y + \Delta_y = (u + \Delta_u)(v + \Delta_v)$ $\Delta_y = (u + \Delta_u)(v + \Delta_v) - y$ $\Delta_y = (u + \Delta_u)(v + \Delta_v) - uv$ $= vu + u\Delta_v + v\Delta_u + \Delta_u\Delta_v - uv$ $= u\Delta_v + v\Delta_u + \Delta_u\Delta_v$ $\frac{\Delta_y}{\Delta_x} = \frac{u\Delta_v}{\Delta_x} + \frac{v\Delta_u}{\Delta_x} + \frac{\Delta_u\Delta_v}{\Delta_x}$	Each student studies the concept map independently. The students then listen attentively to the teacher and ask questions if any.	Explanation, questioning and non verbal clues

		As $\Delta x \to 0, \Delta u \to 0, \Delta v \to 0$ $\lim \overset{\Delta x}{\to} 0 \frac{\Delta_y}{\Delta_x} = \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} + 0$ $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ Therefore, if $y = uv$, where u and v are functions of x, then we have the product rule $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ The teacher at this point asks the students to compare this with what they have in the concept map		

3	Derivatives of Polynomials by Product Rule	The teacher teaches the students how to find the derivatives of polynomials by product rule with this example <u>Example</u> Differentiate $y = (3x + 2)(x^2 + 2)$ w.r.t x at $$x = \frac{1}{3}$$ <u>Solution</u> $$y = (3x + 2)(x^2 + 2)$$ $$u = (3x + 2), \qquad v = (x^2 + 2)$$ $$\therefore y = uv$$ $\frac{du}{dx} = 3,\ \frac{dv}{dx} = 2x$ $$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$ $$\frac{dy}{dx} = (3x + 2)(2x) + (x^2 + 2)(3)$$ It can be noticed that to differentiate product of two functions keep the first, differentiate the second plus keep the second, differentiate the first. Now, $$\frac{dy}{dx} = 6x^2 + 4x + 3x^2 + 6$$ $$= 9x^2 + 4x + 6$$	The students listen attentively and write down the example while comparing the steps with what they have in the concept maps	Use of examples and explanation

		At $x = \frac{1}{3}$, $\frac{dy}{dx} = 9(\frac{1}{3})^2 + 4(\frac{1}{3}) + 6$ $= 9(\frac{1}{9}) + 4(\frac{1}{3}) + 6$ $= 1 + \frac{4}{3} + 6$ $= \frac{25}{3}$		
4	Differentiation by Quotient Rule	The teacher explain how to derive the quotient rule of differentiation Let $y = \frac{u}{v}$, where u and v are functions of x, then $y + \Delta_y = \frac{u + \Delta_u}{v + \Delta_v}$ $\Delta_y = \frac{u + \Delta_u}{v + \Delta_v} - y$ $\Delta_y = \frac{u + \Delta_u}{v + \Delta_v} - \frac{u}{v}$ $\Delta_y = \frac{uv + v\Delta_u - uv - u\Delta v}{v(v + \Delta_v)}$ $\Delta_y = \frac{v\Delta_u - u\Delta v}{v(v + \Delta_v)}$ $\frac{\Delta_y}{\Delta_x} = \frac{v\Delta_u - u\Delta v}{\Delta_x v(v + \Delta_v)}$ $\frac{\Delta_y}{\Delta_x} = \frac{\frac{v\Delta_u}{\Delta_x} - \frac{u\Delta_v}{\Delta_x}}{v(v + \Delta_v)}$ As $\Delta u \to 0$ and $\Delta v \to 0$ $\lim \overset{\Delta x}{\to} 0 \frac{\Delta_y}{\Delta_x} = \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v(v + 0)}$	The students listens attentively to the teacher and ask questions if any	Explanation, repetition and questioning

		$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ Therefore, if $y = \frac{u}{v}$ where u and v are functions of x, then we have the quotient rule $$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$ The teacher asks the students compare this with the concept map		
5	Finding Derivative of Functions by Quotient Rule	The teacher explains how to find the derivative of functions using the quotient rule with the following example: <u>Example1</u> Differentiate $y = \frac{1+x^2}{1-x^2}$ w.r.t x at $x = -2$ <u>Solution</u> $$y = \frac{1+x^2}{1-x^2}$$ $u = 1 + x^2, \qquad v = 1 - x^2$ $\therefore y = \frac{u}{v}$ $\frac{du}{dx} = 2x, \ \frac{dv}{dx} = -2x$ $$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$ $\frac{dy}{dx} = \frac{(1-x^2)(2x)-(1+x^2)(-2x)}{(1-x^2)^2}$ Notice that in order to differentiate quotient of two functions of x, keep the second function differentiate the first minus keep the first	The students listen attentively to the teacher and write the example in their notes	Use of examples and expanation

		differentiate the second all over the second squared. Now, $$\frac{dy}{dx} = \frac{2x - 2x^3 + 2x + 2x^3}{(1 - x^2)^2}$$ $$= \frac{4x}{(1 - x^2)^2}$$ At $x = -2$ $$\frac{dy}{dx} = \frac{4(-2)}{(1 - (-2)^2)^2}$$ $$= \frac{-8}{(1 - 4)^2}$$ $$= \frac{-8}{(-3)^2}$$ $$= -\frac{8}{9}$$		
6	Problems Involving Trigonometry	The teachers teaches the students how to solve problems involving trigonometry with the following examples: <u>Example 1</u> If $y = x^3 sinx$ <u>Solution</u> $u = x^3, v = \sin x$ $\frac{du}{dx} = 3x^2,\ \frac{dv}{dx} = cosx$ Using the product rule $$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$	The students listen attentively and write the example in their notebook	Use of examples explanation and questioning

we have

$$\frac{dy}{dx} = x^3 cosx \ + sinx. 3x^2$$

$$\frac{dy}{dx} = x^2(xcosx \ + 3sinx)$$

<u>Example 2</u>

If

$y = \frac{sinx}{x^2+cosx}$, find $\frac{dy}{dx}$

when

$$x = 0$$

<u>Solution</u>

$$y = \frac{sinx}{x^2 + cosx}$$

The first function is

$$u = sinx$$

and the second function is

$v = x^2 + cosx$

$\frac{du}{dx} = cosx, \ \frac{dv}{dx} = 2x - sinx$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$\frac{dy}{dx} = \frac{(x^2+cosx)(cosx) - sinx(2x - sinx\ )}{(x^2+cosx)^2}$

$\frac{dy}{dx} = \frac{x^2+cos^2x-2xsinx+sin^2x\ )}{(x^2+cosx)^2}$

$\frac{dy}{dx} = \frac{x^2-2xsinx+cos^2x+sin^2x}{(x^2+cosx)^2}$

		Recall from trigonometry that $$cos^2x + sin^2x = 1$$ $$\therefore \frac{dy}{dx} = \frac{x^2 - 2xsinx + 1}{(x^2 + cosx)^2}$$ At $x = 0$, $$\frac{dy}{dx} = \frac{0^2 - 2(0)sin0 + 1}{(0^2 + cos0)^2}$$ $$\frac{dy}{dx} = \frac{0 - 0 + 1}{(0 + 1)^2}$$ $$\frac{dy}{dx} = \frac{1}{(1)^2} = \frac{1}{1} = 1$$		
7	Evaluations	The teacher evaluates the lesson by asking the students to: i. state the product rule of differentiation ii. Find the derivatives of $y = (7x^2 + 2x)(4x + 5)$ w.r.t x using the product rule iii. state the quotient rule of differentiation iv. Find the derivatives of function $y = \frac{x^2+4}{5-x^2}$ with respect to x by the quotient rule v. Solve the following problems	The students give answers to the questions. The expected are: (v)The product rule states that If $y = uv$, where u and v are functions of x, then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ (vi)$y = (7x^2 + 2x)(4x + 5)$ $u = (7x^2 + 2x), v = (4x + 5)$ $\frac{du}{dx} = 14x + 7,\ \frac{dv}{dx} = 4$	Questioning and reinforcement

		involving trigonometric functions vi. Differentiate $y = x^2 cosx$ at using product vii. Find the derivative of $y = \frac{sinx}{x^3+cosx}$ at $x = 0$ using quotient rule	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ $\frac{dy}{dx} = (7x^2 + 2x)(4) + (4x + 5)(14x + 7)$ $\frac{dy}{dx} = 56x^2 + 28x + 70x + 35 = 56x^2 + 98x + 35$ (vii)The quotient rule of differentiation of the function of the form $y = \frac{u}{v}$ where u and v are functions of x, is given by $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ (viii)a.$y = x^2 cosx$ $u = x^2, v = cos\ x$ $\frac{du}{dx} = 2x, \frac{dv}{dx} = -\sin x$ Using the product rule $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ we have $\frac{dy}{dx} = -x^2 sinx\ +$	

			$cos x.2x$ $\frac{dy}{dx} = -x^2 sinx\ +$ $2xcosx$ $\frac{dy}{dx}$ $= 2xcosx - x^2 sinx$ $\frac{dy}{dx} = x(2cosx\ +$ $xsinx)$ $y = \frac{sinx}{x^3 + cosx}$ $u = sinx$ $v = x^3 + cosx$ $\frac{du}{dx} = cosx, \qquad \frac{dv}{dx} =$ $3x^2 - sinx$ $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ $\frac{dy}{dx} =$ $\frac{x^3 + cosx(cosx) - sinx(3x^2 - s}{(x^3 + cosx)^2}$ $\frac{dy}{dx} =$ $\frac{x^3 + cos^2x - 3x^2 sinx + sin^2x}{(x^3 + cosx)^2}$ $\frac{dy}{dx} =$ $\frac{x^3 - 3x^2 sinx + cos^2x + sin^2x}{(x^3 + cosx)^2}$ Recall from trigonometry that $cos^2x + sin^2x = 1$	

			$\therefore \frac{dy}{dx} = \frac{x^3 - 3x^2 sinx + 1}{(x^3 + cosx)^2}$ At $x = 0,$ $\frac{dy}{dx} = \frac{0^3 - 3(0)^2 sin0 + 1}{(0^3 + cos0)^2}$ $\frac{dy}{dx} = \frac{0 - 0 + 1}{(0 + 1)^2}$ $\frac{dy}{dx} = \frac{1}{(1)^2} = \frac{1}{1} = 1$	

Summary: The teacher summarizes the lesson thus:

1. If $y = uv$

where u and v are functions of x, then the product rule is

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

that is: to differentiate product of two functions keep the first, differentiate the second plus keep the second, differentiate the first

2. The quotient rule of differentiation of the function of the form $y = \frac{u}{v}$ where u and v are functions of x, is given by

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

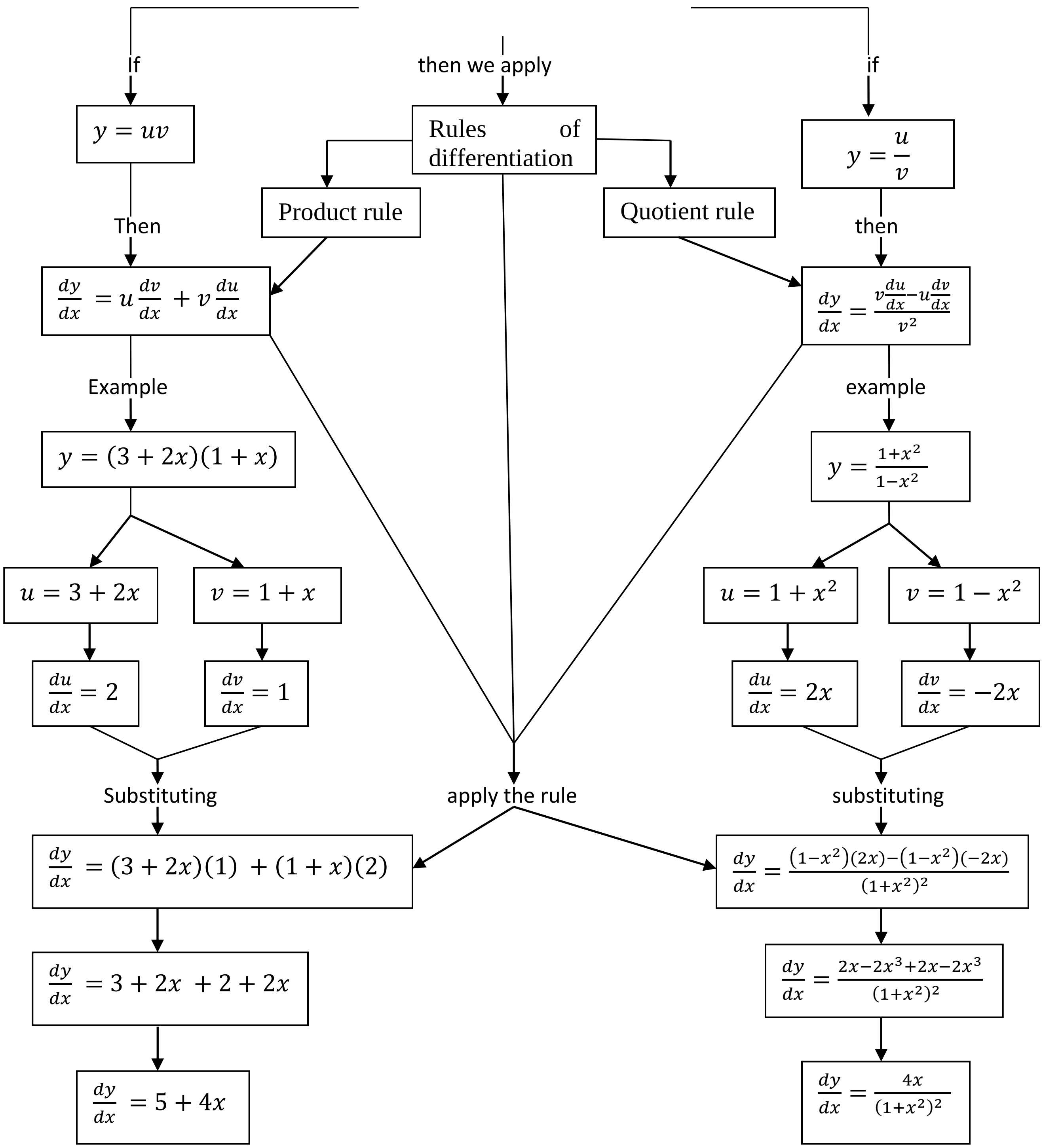
That is: put down the denominator (differentiate the numerator) – put down the numerator (differentiate the denominator) all over the denominator squared

Assignment:

1. Differentiate the following functions with respect to x:
   a. $y = (4sinx)(4x + 5)$
   b. $y = \frac{7x^2}{x + sinx}$

CONCEPT MAP ON DIFFERENTIATION BY PRODUCT AND QUOTIENT RULE

Let u and v be functions of x



LESSON PLAN FOR CONCEPT MAPPING STRATEGY

(CMS GROUP)

WEEK FOUR

Subject: Mathematics

Class: SS III

Duration: 40 minutes

Content: Differentiation by Chain Rule

Specific Objectives: At the end of the lesson, the students should be able to:

(i) identify functions of function

(ii) state the chain rule of differentiation

(iii) find the derivatives of function with respect to x by the chain rule

(iv) solve problems involving differentiation by chain rule

Instructional Materials: Markerboard illustrations

Instructional Techniques/ Stategies: Set induction, questioning, reinforcement, illustration, explanation, non verbal clues, repetition and use of examples

Entry Behaviour: The students can

i. find the derivative of a function with respect to x.

Test on Entry Behaviour:

1. Find the derivative of $y = (4 + x^3)(x^2 + 1)$ with respect to x

Instructional Procedure:

Steps	Content Development (CD)	Teacher's Activities	Students'	Strategies

			Activities	
1	Introduction	The teacher introduces the lesson by asking the students to find the derivative of $y = (4 + x^3)(x^2 + 1)$ with respect to x	The students solve the problems and provide the answers to the questions. Students are expected to answer the questions thus: $y = (4 + x^3)(x^2 + 1)$ $u = 4 + x^3$, $v = x^2 + 1$ $\therefore y = uv$ $\frac{du}{dx} = 3x^2$, $\frac{dv}{dx} = 2x$ $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ $\frac{dy}{dx} = (4 + x^3)(2x) + (x^2 + 1)(3x^2)$ $= 8x + 2x^4 + 3x^4 + 3x^2$	Questioning and Reinforcement

			$= 5x^4 + 3x^2$ $+ 8x$	
2	Function of Functions	The teacher first explains to the students the objectives of the lesson and what is expected of them. The teacher then gives each student the concept map advance organizer (see the back page of this lesson plan) and asks them to study it in 15 minutes. The teacher guides the students on how to follow the concept map with the arrows directions. After the 15 minutes, the teacher asks the students to stop and then begins to explain function of function thus: Let $y = f(u)$ and $u = h(x)$. This means that y is a function of u and u is a function of x. Then $y = f(h(x))$ is said to be a function of a function of x or a composite	Each student studies the concept independent. The students then listen attentively to the teacher while making comparions with the advance organizer.	Explanation, question and non-verbal clues

		function. Examples of functions of functions are: $y = (2x^3 + 1)^3$ $y = e^{(3x+5)}$		

3	Chain Rule for Differentiation	The teacher explains the chain for differentiation thus: Given that $y = f(u)$ and $u = h(x)$, then derivative of y w.r.t u is $\frac{dy}{du}$ The derivative of u w.r.t x is $\frac{du}{dx}$ $\therefore \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ This is called the chain rule of differentiation.	The students listen attentively to the teacher. The students compares the chain rule with what is in the concept map	Explanation, repetition and questioning
4	Differentiation of functions of functions by chain rule	The teacher explains how to differentiation of functions of functions by chain rule with the following example: <u>Example</u> Find the derivative of $y = (2x^3 + 1)^3$ w.r.t x <u>Solution</u> $y = (2x^3 + 1)^3$ $u = 2x^3 + 1$ Then $y = u^3$ $\frac{dy}{du} = 3u^2$ $\frac{du}{dx} = 6x^2$ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $\frac{dy}{dx} = 3u^2 \times 6x^2$ $= 18x^2u^2$ $= 18x^2\,(2x^3 + 1)^2$	The students listen attentively to the teacher and write down the examples in their notebooks as make reference to the concept map to discover relationships	Use of example and explanation

5	Further problems involving Chain rule	The teacher solves further problems involving differentiation by chain rule with the following examples: <u>Example</u> Differentiate the following functions with respect to x: . $y = tan(4x + 1)$ i. $y = e^{(4x^2 + 5)}$ <u>Solution</u> $y = tan(4x + 1)$ $u = 4x + 1$ Then $y = \tan u$ $\frac{dy}{du} = sec^2 u$ $\frac{du}{dx} = 4$ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $\frac{dy}{dx} = sec^2 u \times 4$ $= 4sec^2 u$ But $u = 4x + 1$ $\therefore \frac{dy}{dx} = 4sec^2(4x + 1)$. $y = e^{(4x^2 + 5)}$	The students listen attentively to the teacher and ask questions	Use of examples and explanation

		Let $u = 4x^2 + 5$, then $y = e^u$ $\frac{dy}{du} = e^u$, $\frac{du}{dx} = 8x$ $$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$ $$\frac{dy}{dx} = e^u \times 8x$$ $$\frac{dy}{dx} = 8xe^u$$ But $$u = 4x^2 + 5$$ $$\therefore \frac{dy}{dx} = 8xe^{(4x^2+5)}$$ A quick way to differentiate function of a function is to differentiate the function inside the bracket, multiply it by the derivative of the main function and then simply		

6	Evaluation	The teacher evaluates the lesson by asking the students to: Identify which of the following is are functions of function: $e^{(4x)}$, $7sinx$, $14x^3$ $(2x^3+1)^3$ (i) State the chain rule of differentiation (ii) Find the derivatives of function $cos2x^2$ with respect to x by the chain rule (iii)Solve this problem involving differentiation by chain rule: $y = e^{(sinx+5)}$	The students give answers to the questions thus: The students give answers to the questions thus: $e^{(4x)}$, $(2x^3+1)^3$ The chain rule is given by $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $y = cos2x^2$ $u = 2x^2$ Then $y = \cos u$ $\frac{dy}{du} = -sin$u, $\frac{du}{dx} = 4x$ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $\frac{dy}{dx} = sin$u $\times 4x$ $= 4xsin$u But $u = 2x^2$ $\therefore \frac{dy}{dx} = -4xsin2x^2$ $y = e^{sin(x+5)}$	Questioning

			$u = sin(x+5)$ $y = e^u$ $\frac{du}{dx} = \cos(x+5)$ $\frac{dy}{du} = e^u$ $\frac{dy}{dx}$ $= e^u \times \cos(x+5)$ $= \cos(x+5)e^u$ $\frac{dy}{dx}$ $= \cos(x$ $+ 5)e^{sin(x+5)}$	

Summary: The teacher summarizes the lesson thus:

i. If $y = f(u)$ and $u = h(x)$, then $y = f\big(h(x)\big)$ is said to be a function of a function of x Examples of function of functions are: $(2x^3 + 1)^3$, $e^{(3x+5)}$

ii. The chain rule: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ is used for the differentiation of function of a function.

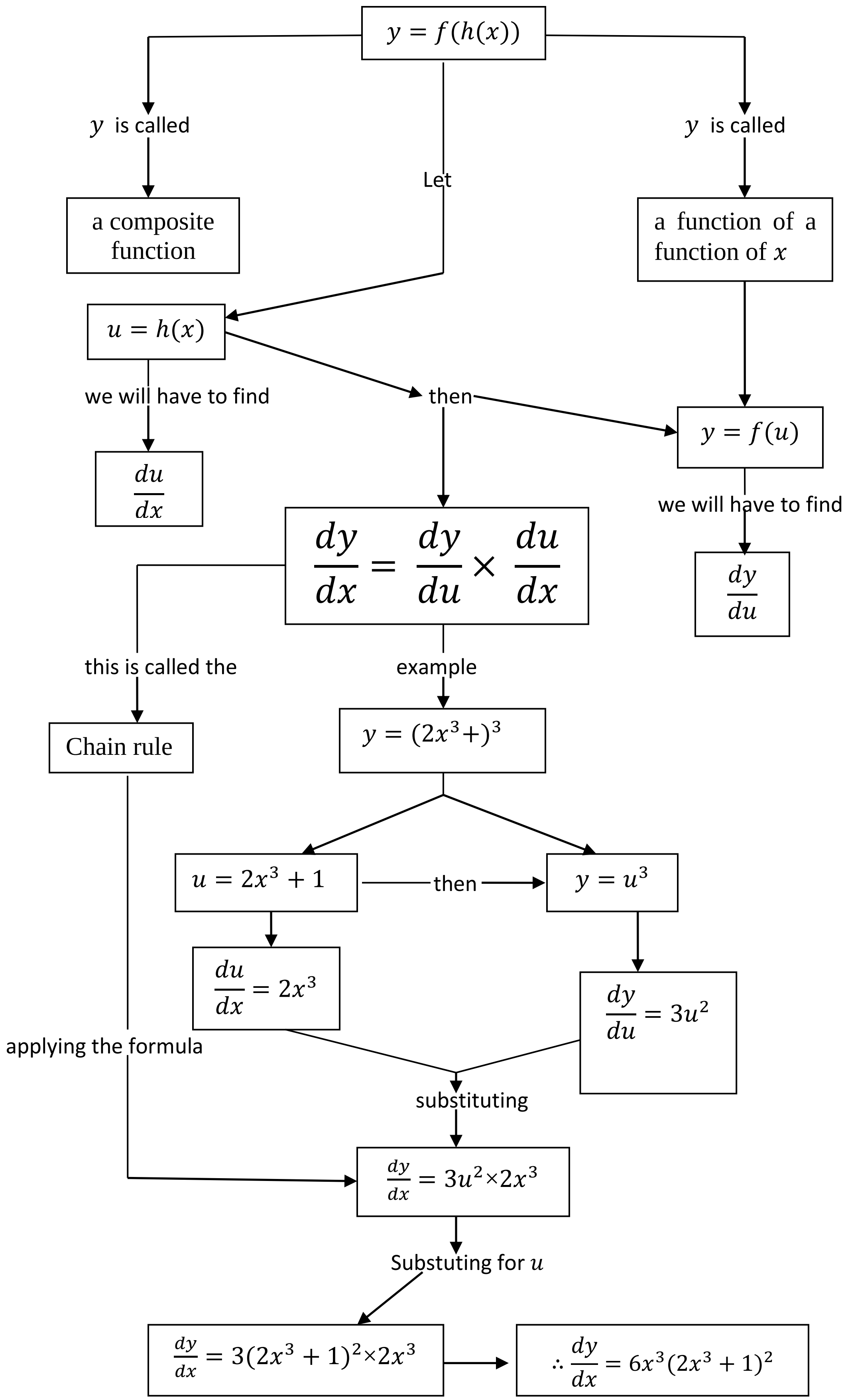
Assignments:

1. Differentiate the following with respect to x

   a. $y = ln(5x^2 + 2)$

   b. $y = e^{(3x+5)(4x-1)}$

THE CONCEPT MAP ON CHAIN RULE FOR DIFFERENTIATION



LESSON PLAN FOR CONCEPT MAPPING STRATEGY

(CMS GROUP)

WEEK FIVE

Subject: Mathematics

Class: SS III

Duration: 80 minutes

Content: Applications of Differentiation

Specific Objectives: At the end of the lesson, the students should be able to:

(i) find higher derivatives of the function $y = f(x)$

(ii) define and identify the maximum point, minimum point and point of inflexion of a curve of $y = f(x)$

(iii) solve problems involving maximum and minimum points

(iv) define velocity and acceleration

(v) apply differentiation to problems involving motion of a body

Instructional Materials: Markerboard illustrations

Instructional Techniques/ Strategies: Set induction, questioning, reinforcement, illustration, explanation, non verbal clues, repetition and use of examples

Entry Behaviour: The students can:

i. find the derivative of a function of x

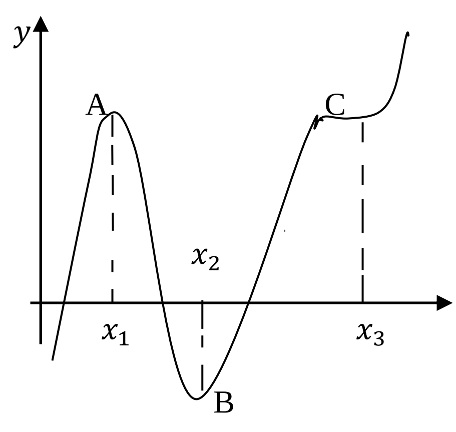
Test on Entry Behaviour:

1. find the derivative of the function $y = 4x^6$

Instructional Procedure:

Steps	Content Development (CD)	Teachers' Activities	Students' Activities	Strategies
1	Introduction	The teacher introduces the lesson by asking the students to find the derivative of the function $y = 4x^6$	The students are expected to solve the problem thus: $\frac{dy}{dx} = 4 \times 6x^{6-1}$ $\frac{dy}{dx} = 24x^5$	Questioning and Reinforcement
2	Higher Derivatives of Functions	The teacher first explains to the students the objectives of the lesson and what is expected of them. The teacher gives each student the concept map advance organizer (see the back page of this lesson plan) and asks them to study it in 15 minutes. The teacher guides the students on how to follow the concept map with the arrows directions. After the 15 minutes, the teacher asks the students to stop and then begins to explain how to find the	Each student studies the concept independently. The students then listen attentively to the teacher.	Explanation, questioning and non-verbal clues

		higher derivatives of the functions thus: Let $y = f(x)$ Then $\frac{dy}{dx}$ is the first derivative. Differentiating $\frac{dy}{dx}$ we have $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$ $\frac{d^2y}{dx^2}$ is the second derivative of y with respect to x $\frac{d^3y}{dx^3}$ is the third derivative of y with respect to x, and so on. In general, $\frac{d^ny}{dx^n}$ is nth derivative of y with respect to x. <u>Example</u> Find the first, second and the third derivatives of each of the following functions with respect to x $y = 3x^2 + 5x + 6$ $y = 2xsinx$ <u>Solution</u> $y = 3x^2 + 5x + 6$ First derivative $\frac{dy}{dx} = 6x + 5$		

		Second derivative, $\frac{d^2y}{dx^2} = 6$ $y = 2xsinx$ Applying the product rule, we have $$\frac{dy}{dx} = 2x(cosx) + sinx(2)$$ $$\frac{dy}{dx} = 2xcosx + 2sinx$$ $\frac{d^2y}{dx^2} = 2x(-sinx) + cosx(2) +$ $2cosx$ $\frac{d^2y}{dx^2} = -2xsinx + 2cosx + 2cosx$ $\frac{d^2y}{dx^2} = -2xsinx + 4cosx$		
3	Maximum and Minimum Point of a Curve	The teacher explains maximum and minimum point of a curve thus:  Consider the graph above of the function $y = f(x)$. At point A i.e. $x = x_1$, a maximum value occurs.	The students listen attentively and take note	Explanation and illustration

		Here there is a change in sign from $+ve$ to $-ve$ as we turn At point B i.e. $x = x_2$, a minimum value occurs. Here there is a change in sign from $-ve$ to $+ve$ as we turn. The maximum point A and minimum point B are called turning points. At point C i.e. $x = x_3$, the curve flattens and goes up again. We call this, point of inflexion. Points A, B and C are called stationary points. There, gradient (or slope) is zero since the tangents at these points are parallel to x-axis. That is, $\frac{dy}{dx} = 0$ at A, B and C		

4	Problems Involving Maximum and Minimum Points	The teacher explains how to solve problems involving stationary points thus: The necessary and sufficient conditions for stationary points are as follows: i. If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2}\ at\ x = a > 0$ then there is a maximum point. ii. If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0\ at\ x = a$ then there is a minimum point. iii. If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0\ at\ x = a$ then there is a point of inflexion. At this point teacher should make reference to the concept map so that the students can discover relationship between what he is teaching with what is in the advance organizer. <u>Example 1</u> Find the stationary point on the curve $y = x^3 - 5x^2 + 12x - 8$	The students listen attentively to the teacher. The students writes notes	Explanation, Repetition and use of examples

		And distinguish between them. <u>Solution</u> $y = x^3 - 5x^2 + 12x - 8$ $\frac{dy}{dx} = 3x^2 - 12x + 12$ At stationary point $\frac{dy}{dx} = 0$. So, $3x^2 - 12x + 12 = 0$ $x^2 - 4x + 4 = 0$ $(x-2)^2 = 0$ $x = 2$ $\frac{d^2y}{dx^2} = 6x - 12$ At $x = 2$, $\frac{d^2y}{dx^2} = 6(2) - 12 = 12 - 12 = 0$ There is a point of inflexion since $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$ at $x = 2$ <u>Example 3</u> Find the nature of the turning point of the function $\frac{x^3}{3} + x^2 - 3x + 4$ <u>Solution</u> $\frac{dy}{dx} = x^2 + 2x - 3$		

		At turning point $\frac{dy}{dx} = 0$. So, $$x^2 + 2x - 3 = 0$$ $$x^2 + 3x - x - 3 = 0$$ $$x(x + 3) - 1(x + 3) = 0$$ $$(x - 1)(x + 3) = 0$$ $x = 1,\ x = -3$ $$\frac{d^2y}{dx^2} = 2x + 2$$ At $x = 1$, $\frac{d^2y}{dx^2} = 2(1) + 2 = 2 + 2 = 4 > 0$ At $x = 1$, there is a minimum point. At $x = -3$, $\frac{d^2y}{dx^2} = 2(-3) + 2 = -6 + 2 = -4$ < 0 At $x = -3$, there is a maximum point.		

5	Definition of Velocity and Acceleration	The teacher defines velocity and acceleration thus: The motion of a particle (body) along straight line can be specified by the function $$s = f(t)$$ Where s is the displacement and t is the time. The velocity of the body is therefore the rate of change of the displacement with respect to time. It is given by $$v = \frac{ds}{dt}$$ If the body is momentarily at rest, then $$\frac{ds}{dt} = 0$$ The acceleration of the body is the rate of change of velocity with respect to time given by $$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}$$ At constant velocity, $\frac{dv}{dt} = 0$ If $\frac{dv}{dt} < 0$, we have retardation or deceleration	The students pays attention and write down notes while comparing the lesson with the concept map	Explanation and questioning

6	Application of Differentiation to Motion	The teacher explains application of differentiations to motion with the following examples: <u>Example 1</u> The motion of a particle along a straight line is described by the distance function $$s = t^3 - 9t^2 + 27t - 10$$ Find: (a) the velocity and acceleration after 2 secs. (b) the distance when the particle is momentarily at rest <u>Solution</u> $$s = t^3 - 9t^2 + 27t - 10$$ Velocity $$v = \frac{ds}{dt} = 3t^2 - 18t + 27$$ When $t = 2$ secs. $$v = 3(2)^2 - 18(2) + 27$$ $$v = 12 - 36 + 27$$ $$v = 3ms^{-1}$$ $$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 6t - 18$$	The students listen attentively to the teacher and ask questions	Explanation and use of examples

		At $t = 2$, $$a = 6 \times 2 - 18 = 12 - 18$$ $$a = -6ms^{-2}$$ which is a retardation. (b) If the body is momentarily at rest, then $$\frac{ds}{dt} = 0$$ That is, $$3t^2 - 18t + 27 = 0$$ $$t^2 - 6t + 9 = 0$$ $$t^2 - 6t + 9 = 0$$ $$t^2 - 3t - 3t + 9 = 0$$ $$t(t-3) - 3(t-3) = 0$$ $$(t-3)(t-3) = 0$$ $$\therefore t = 3\ secs$$ Hence, $$s = t^3 - 9t^2 + 27t - 10$$ $$s = t^3 - 9t^2 + 27t - 10$$ $$s = 27 - 9 \times 9 + 27(3) - 10$$ $$s = 27 - 81 + 81 - 10$$ $$s = 27 - 10$$ $$s = 17m$$		

7	Evaluation	The teacher evaluates the lesson asking the students to: i. Find third derivative of the function $y = 5x^3 - 6x^2 + 7x - 10$ w.t. x ii. Define and identify the maximum point, minimum point and point of inflexion of a curve of $y = f(x)$ iii. Solve the following problems involving maximum and minimum points: Find the nature of the turning point of the function $\frac{x^3}{3} + x^2 - 3x + 4$ iv. Define velocity and acceleration v. Apply differentiation to solve the following problems involving motion of a body: Find the velocity and acceleration of the function $s = t^2 - 3t + 4$ at $t = 5s$	The students give answers to the questions. The students are expected to provide the following answers: (i) $y = 5x^3 - 6x^2 + 7x - 10$ $\frac{dy}{dx} = 3 \times 5x^{3-1} - 2 \times 6x^{2-1} + 7 - 0$ $\frac{dy}{dx} = 15x^2 - 12x + 7$ $\frac{d^2y}{dx^2} = 30x - 12$ $\therefore \frac{d^3y}{dx^3} = 30$ (ii) If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2}$ $at\ x = a > 0$ then there is a maximum point. If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0\ at\ x = a$ then there is a minimum point. If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0\ at\ x = a$ then there is a point of inflexion. (iii) $y = \frac{x^3}{3} + x^2 - 3x + 4$	Questioning and Reinforcement

			For turning point, $$\frac{dy}{dx} = 0$$ $$\frac{dy}{dx} = x^2 + 2x - 3$$ $$= 0$$ $$x^2 + 2x - 3 = 0$$ $$x^2 + 3x - x - 3 = 0$$ $$x(x+3) - 1(x+3)$$ $$= 0$$ $$(x+3)(x-1) = 0$$ $x + 3 = 0, x - 1 = 0$ $x = -3, x = 1$ $$\frac{d^2y}{dx^2} = 2x + 2$$ $\frac{d^2y}{dx^2}$ at $x = -3$ $$\frac{d^2y}{dx^2} = 2(-3) + 2$$ $$= -6 + 2 = -4$$ (iv) The velocity of a body with the displacement $s = f(t)$ is given by $v = \frac{ds}{dt}$. The acceleration of the body is given by $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}$. (iv) $s = t^2 - 3t + 4$ at $t = 5s$ $$v = \frac{ds}{dt} = 2t - 3$$	

			At $t = 5s$ $v = 2(5) - 3$ $10 - 3 = 7m/s$ $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$ $= 2.$ At $t = 5s$ $a = 2ms^{-2}.$	

Summary: The teacher summarizes the lesson thus:

i. To find the second derivatives $\frac{d^ny}{dx^n}$ of the function $y = f(x)$ is we differentiate the function n times

ii. The stationary points have the following conditions:

a. If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2}$ $at\ x = a > 0$ then there is a maximum point.

b. If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2}$ $< 0\ at\ x = a$ then there is a minimum point.

c. If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2}$ $= 0\ at\ x = a$then there is a point of inflexion.

iii. The velocity of a body with the displacement $s = f(t)$ is given by $v = \frac{ds}{dt}$. If the body is momentarily at rest, then $\frac{ds}{dt} = 0$

iv. The acceleration of the body is then given by $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}$. At constant velocity, $\frac{dv}{dt} = 0$. If $\frac{dv}{dt} < 0$, we have retardation or deceleration.

Assignment

1. Given the distance function

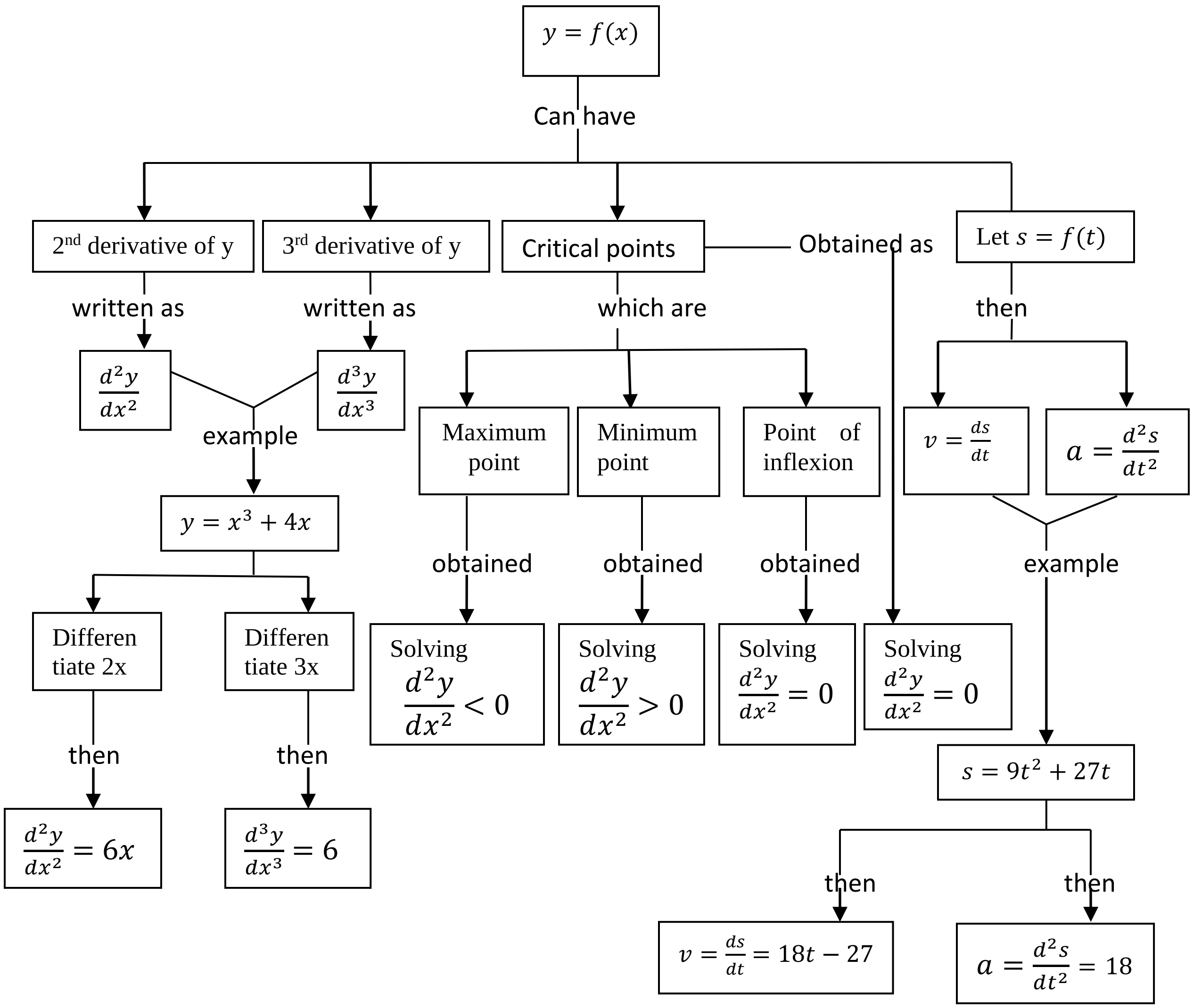
$$s = 4t^3 + 7t - 10$$

Find:

(a) the velocity and acceleration after 2 secs.

(b) the distance assuming the particle is momentarily at rest

THE CONCEPT MAP ON APPLICATION OF DIFFERENTIATION



APPENDIX C_5

THE TEST DEVELOPMENT PROCESS

CONTENT DIMENSION

S/No	TOPIC	OBJECTIVES	PERCENTAGE (%)
1	Differentiation from the First Principle	3	15
2	Differentiation of Polynomials	3	15
3	Product and Quotient Rules of Differentiation	5	25
4	Differentiation by chain Rule	4	20
5	Application of Differentiation	5	25
TOTAL		20	100

COGNITIVE DIMENSION

Topic	Remembering 35%	Understanding 10%	Applying 40%	Analyzing 15%	Evaluating -	Creating -	Total 100%
Differentiation from the First Principle	I	I	I	-	-	-	3
Differentiation of Polynomials	I	-	II	-	-	-	3
Product and Quotient Rules of Differentiation	II	-	II	I	-	-	5
Differentiation by chain Rule	I	I	I	I			4
Applications of Differentiation	II	-	II	I			5
Total	7	2	8	3	-	-	20

APPENDIX C6

TABLE OF SPECIFICATION FOR ACHIVEMENT FOR DIFFERENTIAL CALCULUS ACHIEVEMENT TEST (DCAT)

Content / **Topic**	**Remembering 35%**	**Understanding 10%**	**Applying 40%**	**Analyzing 15%**	**Evaluating -**	**Creating -**	**Total 100%**
Differentiation from the First Principle **15%**	2 (1,2)	1 (4)	2 (5,6)	1 (3)	-	-	6
Differentiation of Polynomials **15%**	2 (9,10)	2 (8,11)	1 (12)	1 (7)	-	-	6
Differentiation by Product and Quotient Rule **25%**	4 (13,14,15, 17)	1 (19)	3 (16,20,21)	2 (18,27)	-	-	10
Differentiation by Chain Rule **20%**	3 (30,28,29)	1 (22)	3 (25,26,23)	1 (24)	-	-	8
Applications Differentiation **25%**	4 (33,37,38)	1 (35)	4 (31,32,39,40)	2 (34,36)			10
Total	14	5	15	6	-	-	40

APPENDIX C7

DIFFERENTIAL CALCULUS ACHIEVEMENT TEST (DCAT)

Section A: Personal Data of Students

Name of School ___________________________________**Gender:** Male ☐ Female ☐

Instructions: Answer all questions. Circle the correct answer from the options lettered A-D

1. The process of finding the differential coefficient of a function is called________(A) integration (B) differentiation (C) polynomials (D) evaluation
2. One of these is NOT associated with differential coefficient (A) gradient (B) slope (C) derivative (D) tangent
3. The differential coefficient of $y = f(x)$ from the first principle is given by ___________ (A) $\frac{dy}{dx} = \lim \overset{\Delta x}{\rightarrow} 0 \, \frac{f(x+\Delta_x)-f(x)}{\Delta_x}$ (B) $\frac{dy}{dx} = \lim \overset{\Delta x}{\rightarrow} 0 \, \frac{f(x+\Delta_x)-f(x)}{x}$ (C) $\frac{dy}{dx} = \lim \overset{\Delta x}{\rightarrow} 0 \, \frac{f(x+\Delta_x)}{\Delta_x}$ (D) $\frac{dy}{dx} = \lim \overset{\Delta x}{\rightarrow} 0 \, \frac{f(x+2x)-f(\Delta_x)}{\Delta_x}$
4. The differential coefficient of $y = f(x)$ at $x = a$ from the first principle is given by ___________ (A) $\frac{dy}{dx} = \lim \overset{\Delta x}{\rightarrow} 0 \, \frac{f(a+2a)-f(\Delta_x)}{\Delta_x}$ (B) $\frac{dy}{dx} = \lim \overset{\Delta x}{\rightarrow} 0 \, \frac{f(a+\Delta_x)-f(a)}{a}$ (C) $\frac{dy}{dx} = \lim \overset{\Delta x}{\rightarrow} 0 \, \frac{f(a+\Delta_x)}{\Delta_x}$ (D) $\frac{dy}{dx} = \lim \overset{\Delta x}{\rightarrow} 0 \, \frac{f(a+\Delta_x)-f(a)}{\Delta_x}$
5. Find the derivative of the function $y = 3x^2$ from the first principle (A) $2x^3$ (B) $6x$ (C) $6x^2$ (D) $3x$
6. Determine the derivative of $y = \frac{1}{x}$ from the first principle (A) x^2 (B) x^{-1} (C) $\frac{-1}{x^2}$ (D) $1 - x$
7. Given that $y = x^n$, derive the formula for the differentiation of y with respect to x (A) $\frac{dy}{dx} = nx^n$ (B) $\frac{dy}{dx} = x^{n-1}$ (C) $\frac{dy}{dx} = nx^{n-1}$ (D) $\frac{dy}{dx} = (n-1)x^n$

8. Differentiate $y = ax^n$ with respect to x A) $\frac{dy}{dx} = ax^n$ (B) $\frac{dy}{dx} = anx^{n-1}$ (C) $\frac{dy}{dx} = nx^{a-1}$ (D) $\frac{dy}{dx} = (n-1)x^{n-a}$

9. State the standard derivative for the function $y = tanx$ (A) $secx$ (B) $cosx$ (C) $sinx$ (D) sec^2x

10. The differential coefficient of e^x is______________ (A) xe^x (B) e^{x-1} (C) e^x (D) $2e^x$

11. Find the derivative of $y = x^4 - 3x^3 + x + \frac{1}{x}$ at $x = -1$ (A) -13 (B) 5 (C) -3 (D) 25

12. Determine the derivative of $y = 6x^2 + 4x - 3sinx$ at $x = 0$ (A) 0 (B) 1 (C) 2 (D) 3

13. Given that $y = uv$ where u and v are functions of x then $\frac{dy}{dx}$ is equal to___________ (A) $u\frac{dv}{dx} + v\frac{du}{dx}$ (B) $\frac{dv}{dx} + \frac{du}{dx}$ (C) $u\frac{dv}{dx}$ (D) $u\frac{dv}{dx} - v\frac{du}{dx}$

14. The derivative $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ represents ____________rule of differentiation (A) chain (B) addition (C) quotient (D) product

15. Which of these rules of differentiation is best used to differentiate the product of two functions of x? (A) product (B) addition (C) quotient (D) chain

16. Differentiate $(3x+1)(3x-2)$ with respect to x (A) $18x - 3$ (B) $18x$ (C) $9x - 3$ (D) 9

17. If $y = \frac{u}{v}$ then $\frac{dy}{dx} =$ ___________ (A) $u\frac{dv}{dx} + v\frac{du}{dx}$ (B) $\frac{dv}{dx} + \frac{du}{dx}$ (C) $u\frac{dv}{dx} - v\frac{du}{dx}$ (D) $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

18. Find $\frac{dy}{dx}$ if $y = \frac{a(x)}{b(x)}$ (A) $a\frac{db}{dx} + b\frac{da}{dx}$ (B) $\frac{dy}{dx} = \frac{b\frac{du}{dx} - a\frac{dv}{dx}}{b^2}$ (C) $a\frac{db}{dx} - b\frac{da}{dx}$ (D) $\frac{db}{dx} + \frac{da}{dx}$

19. The derivative of $y = tanx$ can best be found by ____________ (A) quotient rule (B) product rule (C) chain rule (D) subtraction rule

20. Differentiate $y = \frac{2-x^2}{2+x^2}$ w.r.t x (A) $\frac{8x}{(2-x^2)^2}$ (B) $\frac{4x}{(2-x^2)^2}$ (C) $\frac{-8x}{(2+x^2)^2}$ (D) $\frac{8x}{(x^2)^2}$

21. Find the derivative of $y = cotx$ w.r.t x (A) sec^2x (B) cos^2x (C) $-cosec^2x$ (D) sin^2x

22. Which of these is not a function of function? (A) $y = (5x + 2)(4x + 2)$ (B) $y = e^{5\mathrm{x}}$ (C) $y = \sin(2x + 1)$ (D) $y = (2 + x^2)^2$

23. The derivative of the function $y = cos5x$ with respect to x can best be solved by _____________ (A) quotient rule (B) product rule (C) chain rule (D) subtraction rule

24. If $y = f(u)$ and $u = f(x)$ then $\frac{dy}{dx}$ is given by___________(A) $u\frac{dv}{dx} + v\frac{du}{dx}$ (B) $\frac{dy}{du} \times \frac{du}{dx}$ (C) $u\frac{dv}{dx} - v\frac{du}{dx}$ (D) $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

25. If $y = \sin(3x - 1)$, find $\frac{dy}{dx}$ at $x = \frac{1}{3}$ (A) 0 (B) 1 (C) 2 (D) 3

26. Let $y = \ln(5x^2 + 2)$ then $\frac{dy}{dx}$ is__________ (A) $5x \ln(5x^2)$ (B) $\ln(5x^2)$ (C) $10x\ \ln(5x^2 + 2)$ (D) $\frac{10x}{5x^2+2}$

27. Find the derivative of $y = e^{(x+3)(x+2)}$ at $x = -2$ (A) 3 (B) 1 (C) 2 (D) 5

28. Which of these is NOT a rule for differentiation? (A) quotient rule (B) product rule (C) chain rule (D) subtraction rule

29. Which of these is NOT correct? (A) $u\frac{dv}{dx} + v\frac{du}{dx}$ (B) $\frac{dy}{du} \times \frac{du}{dx}$ (C) $u\frac{dv}{dx} - v\frac{du}{dx}$ (D) $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

30. Differentiation is applied in problems involving the following areas except (A) dynamics (B) slope of a curve (C) Economics (D) Law

31. Find the third derivative of the function $y = 6x^2 + 7x - 6$ (A) 0 (B) $12x + 7$ (C) $12x - 6$ (D) $7x - 6$

32. Find the second derivative of the function $y = xe^x$ (A) e^x (B) $2e^x$ (C) $e^x(x + 2)$ (D) $2xe^x$

33. One of these is NOT a stationary point (A) maximum point (B) minimum point (C) yielding point (D) point of inflexion

34. If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$ at $x = a$ then we have _____________ (A) maximum point (B) minimum point (C) yielding point (D) point of inflexion

35. At what value of x does the function $f(x) = x^3 - 3x^2 - 2x$ have a point of inflexion? (A)3 (B) 1 (C) 4 (D) 8

36. The necessary condition for $y = f(x)$ to have a maximum point is that_____________ (A) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$ (B) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$ (C) $\frac{dy}{dx} = \frac{d^2y}{dx^2}$(D) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$

37. Let $s = f(t)$ be a distant function where s is the displacement and t is the time. Then the velocity and acceleration are defined as __________ and _____ respectively (A) $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ (B) $\frac{ds}{dt}$ and $\frac{d^2t}{dx^2}$ (C) $\frac{ds}{dt}$ and $\frac{d^2s}{dt^2}$ (D) $\frac{dt}{ds}$ and $\frac{d^2t}{ds^2}$

38. If a particle with the distance function $s = f(t)$ travelling with a velocity is momentarily at rest, then__________ (A) $\frac{d^2t}{dx^2} = 0$ (B) $\frac{ds}{dt} = 0$ (C) $\frac{ds}{dt} = \frac{d^2s}{dt^2}$ (D) $\frac{dt}{ds} - \frac{d^2t}{ds^2}$

The motion of a particle along a straight line is described by the distance function

$$s = \frac{t^3}{3} - 2t^2 - 3t$$

Use the information above to answer questions 39 and 40.

39. What is the velocity of the particle? (A) $t^2 - 4t - 3$ (B) $4t - 3$ (C) $t^2 - 4t$(D) $9t - 12$

40. Find the acceleration of the body at $t = 3\ secs.$ (A) $5ms^{-2}$ (B) $3ms^{-2}$ (C) $2ms^{-2}$ (D) $1ms^{-2}$

APPENDIX C$_8$

MARKING SCHEME FOR DIFFERENTIAL CALCULUS ACHIEVEMENT TEST

Item Number	Correction Options
1	B
2	D
3	A
4	B
5	B
6	C
7	C
8	B
9	D
10	C
11	A
12	D
13	A
14	D
15	A
16	A
17	D
18	B
19	A
20	C
21	C
22	A
23	C
24	B
25	A
26	D
27	D
28	D
29	C
30	D
31	A
32	C
33	C
34	B
35	B
36	D
37	B
38	B
39	A
40	C

APPENDIX C

APPENDIX D_1

KUDDER-RICHARDSON, K-R20 RELIABILITY

Question No.	P	Q	P_i	Q_i	$P_i\ Q_i$
1	46	04	0.92	0.08	0.0736
2	42	08	0.84	0.16	0.1344
3	28	22	0.56	0.44	0.2464
4	21	29	0.42	0.58	0.2436
5	38	12	0.76	0.24	0.1824
6	15	35	0.30	0.70	0.2100
7	34	16	0.68	0.32	0.2176
8	28	22	0.56	0.44	0.2464
9	22	28	0.44	0.56	0.2464
10	12	38	0.24	0.76	0.1824
11	10	40	0.20	0.80	0.1600
12	10	40	0.20	0.80	0.1600
13	32	18	0.64	0.36	0.2304
14	15	35	0.30	0.70	0.2100
15	24	26	0.48	0.52	0.2496
16	19	31	0.38	0.62	0.2356
17	32	18	0.64	0.36	0.2304
18	29	21	0.58	0.42	0.2436
19	20	30	0.40	0.60	0.2400
20	15	35	0.30	0.70	0.2100
21	28	22	0.56	0.44	0.2464
22	09	41	0.18	0.82	0.1476
23	14	36	0.28	0.72	0.2016
24	27	23	0.54	0.46	0.2484
25	11	39	0.22	0.78	0.1716
26	08	42	0.16	0.84	0.1344
27	26	24	0.52	0.48	0.2496
28	41	09	0.82	0.18	0.1476
29	13	37	0.26	0.74	0.1924
30	41	09	0.82	0.18	0.1476
31	16	34	0.32	0.68	0.2176

32	04	46	0.08	0.92	0.0736
33	37	13	0.74	0.26	0.1924
34	10	40	0.20	0.80	0.1600
35	25	25	0.50	0.50	0.2500
36	15	35	0.30	0.70	0.2100
37	32	18	0.64	0.36	0.2304
38	28	22	0.56	0.44	0.2464
39	23	27	0.46	0.54	0.2484
40	17	33	0.34	0.66	0.2244
				Total	**8.0932**

Kudder-Richardson, K-R20 = $\left[\frac{n}{n-1}\right]\left[1-\frac{\sum piqi}{Si^2}\right]$

Where

$n =$ number of items

$Pi =$ proportion of students that pass question i

$Qi =$ proportion of students that fail question i

$Si^2 =$ Variance for the test

$$n = \mathbf{40}$$

$$\sum PiQi = \mathbf{8.0932}$$

$$Si^2 = 39.3824$$

Kudder-Richardson, $KR_{20} = \left[\frac{n}{n-1}\right]\left[1-\frac{\sum PiQi}{Si^2}\right] = \left[\frac{40}{40-1}\right]\left[1-\frac{8.0932}{39.38932}\right]$

$$= \left(\frac{40}{39}\right)(1-0.2055)$$

$$= 1.02564 \times 0.7945$$

$$= 0.8149$$

$$= \mathbf{0.81}$$

APPENDIX D2

Cronbach Alpha Reliability

```
[DataSet1] C:\Users\user\Desktop\New folder\Original DCIS 20 Items.sav
```

Case Processing Summary

		N	%
Cases	Valid	30	100.0
	Excluded[a]	0	.0
	Total	30	100.0

a. Listwise deletion based on all variables in the procedure.

Scale: ALL VARIABLES

Reliability Statistics

Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items
.916	.916	20

APPENDIX E

CONSTRUCT VALIDITY OF DIFFERENTIAL CALCULUS INTEREST SCALE (DCIS)
The 35-Item Interest Scale

SECTION A: Demographic Data of the Respondents

Instruction: Please, fill in the gap your personal data.

Gender: Male ☐ Female ☐

Name of School: ………………………………...

This instrument is developed to help you express your feelings towards differentiation (differential calculus). Please, read each statement carefully and tick against the point that best expresses your feelings or the extent to which you like the topic. I promise to treat all the information you will provide with utmost confidentiality.

SECTION B: Differential Calculus Interest Scale (DCIS)

Instruction: Please, tick against the point that best expresses your feelings

SA = Strongly Agree

A = Agree

D = Disagree

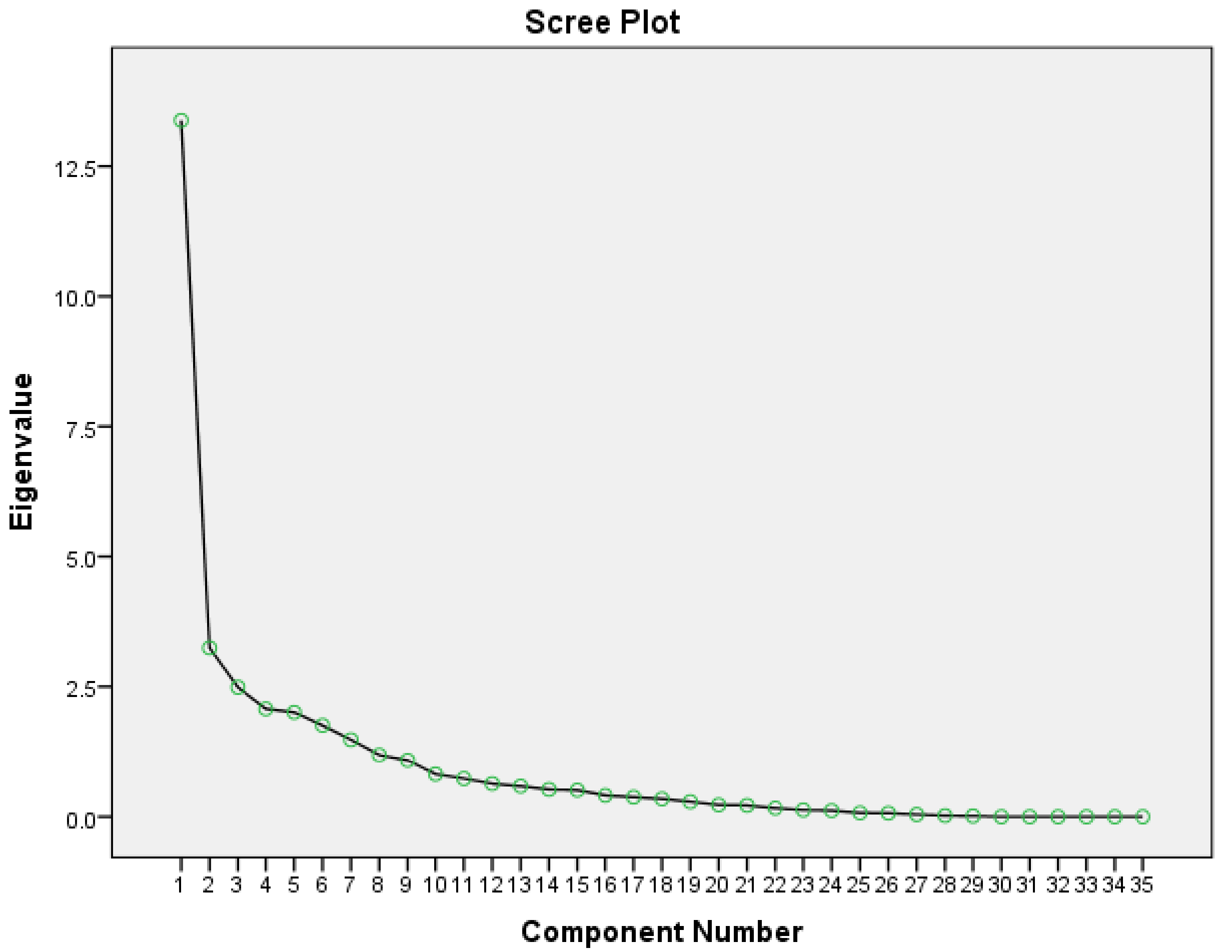
SD = Strongly Disagree

S/No	Item Statement	SA	A	D	SD
1	I like learning differential calculus through group discussion				
2	The real life applications of differential calculus make it even more interesting to me				
3	I am interested to know courses where differential calculus are applied in the university				
4	I get excited whenever it is time to learn differential calculus.				
5	I try on my own to derive rules of differentiation				
6	Differential calculus is a very interesting topic				
7	I ask questions during differential calculus lessons				
8	I attend extra classes to learn what I did not understand during differential calculus lessons				
9	I do not like missing lessons on differential calculus				
10	I enjoy differential calculus lessons more than other topics in mathematics				
11	I would want differential calculus to be removed from the mathematics syllabus				

12	Differential calculus is one of my best topics in mathematics				
13	I ask a lot of questions to discover reasons during differential calculus lessons				
14	Using rules of differentiation to solve problems is very interesting				
15	I often discuss differential calculus problems with my mates during free periods				
16	I solve more difficult tasks on differential calculus with my mates				
17	I study differential calculus ahead, in preparation for the next day’s lesson				
18	I look for more challenging tasks on differential calculus				
19	I try to solve assignments on differential calculus by myself				
20	I feel happy whenever an assignment is given on differential calculus				
21	I always feel like going out of the class during lessons on differential calculus				
22	I do not like differential calculus lessons because they are boring				
23	I do not like differential calculus because it is very difficult to understand				
24	I would like to attempt questions on differential calculus in external examinations such as WAEC and NECO				
25	I enjoy differential calculus lessons when the teacher allows us to figure out solutions to problems by ourselves				
26	I doze off during differential calculus classes				
27	Differential calculus has helped me to develop more likeness for mathematics				
28	I am easily distracted during lessons on differential calculus				
29	I feel uncomfortable whenever it is time to learn differential calculus				
30	I always feel that the teacher should continue teaching us differential calculus beyond the period allocated to it				
31	I do not like differential calculus because of my teacher’s method of teaching it				
32	I like differential calculus lessons because they are real and practical				
33	I take time to analyze a question in order to figure out the best rules of differentiation to apply				
34	I make efforts to discover why I fail a particular question on differential calculus and then take my correction				
35	Differential calculus lessons have made me to develop interest in studying related courses in the university				

Factor Analysis

[DataSet1] C:\Users\user\Desktop\Coded DCIS.sav



Rotated Component Matrix[a]

	Component			
	1	2	3	4
ITEM1	.500	-.673	.118	-.094
ITEM2	-.180	-.113	-.809	.046
ITEM3	.312	.670	.384	-.213
ITEM4	.406	-.110	-.067	-.607
ITEM5	.033	.220	.166	.781
ITEM6	.219	.385	.291	.163
ITEM7	.186	-.174	.023	.466
ITEM8	-.037	.086	.283	-.604
ITEM9	-.104	-.604	.367	.055
ITEM10	.023	.366	.162	.185

ITEM11	-.354	.263	.232	-.433
ITEM12	.219	.385	.291	.163
ITEM13	.186	-.174	.023	.466
ITEM14	-.037	.086	.283	-.604
ITEM15	.023	.366	.162	.185
ITEM16	.312	.670	.384	-.213
ITEM17	.406	-.110	-.067	-.607
ITEM18	.033	.220	.166	.781
ITEM19	-.163	.750	.088	-.038
ITEM20	-.104	-.604	.367	.055
ITEM21	.020	-.014	-.149	.716
ITEM22	-.337	.259	-.324	-.337
ITEM23	.204	-.647	.274	-.195
ITEM24	.435	.021	-.576	.136
ITEM25	.312	.670	.384	-.213
ITEM26	.406	-.110	-.067	-.607
ITEM27	.020	-.014	-.149	.716
ITEM28	.239	.328	-.839	-.002
ITEM29	-.864	.361	-.047	.104
ITEM30	.500	-.673	.118	-.094
ITEM31	-.180	-.113	-.809	.046
ITEM32	.312	.670	.384	-.213
ITEM33	.406	-.110	-.067	-.607
ITEM34	.033	.220	.166	.781
ITEM35	-.163	.750	.088	-.038

Extraction Method: Principal Component Analysis.

Rotation Method: Varimax with Kaiser Normalization.

a. Rotation converged in 9 iterations.

Component Transformation Matrix

Component	1	2	3	4
1	.807	.412	.411	.103
2	-.346	.195	.263	.879
3	-.091	.790	-.606	-.030
4	.471	-.410	-.628	.464

Extraction Method: Principal Component Analysis.

Rotation Method: Varimax with Kaiser Normalization.

APPENDIX F

STATISTICS OF STUDENTS' ACHIEVEMENT IN MATHEMATICS

YEAR	Total No. Who Sat	No. of Students that Obtained Credit & Above (A1 - C6)	% of Students with Credit & Above (A1-C6)	No. of Students with (D7- F9)	% of Students with (D7- F9)
2010	1,351,557	453,447	33.55	898,110	66.45
2011	1,540,250	587,630	38.93	952,620	61.07
2012	1,675,224	819,390	49.00	852,834	51.00
2013	1,543,683	555,726	36.00	987,957	64.00
2014	1,692,435	529,732	31.30	1,162,703	68.70
2015	1,593,442	544,638	34.18	1,048,804	65.82
2016	1,544,234	597,310	38.68	946,924	61.32

Source: Test Development Division, West African Examination Council (WAEC) Lagos, Nigeria.