STATIC ANALYSIS AND DESIGN OF LATERIZED CONCRETE CYLINDRICAL SHELLS FOR FARM STORAGES

NWAKONOBI, THERESA UKAMAKA (MRS.)
(PG/Ph.D/98/25931)

DEPARTMENT OF AGRICULTURAL AND BIORESOURCES ENGINEERING
FACULTY OF ENGINEERING
UNIVERSITY OF NIGERIA, NSUKKA

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CERTIFICATION

Mrs. Nwakonobi, Theresa Ukamaka, a Postgraduate student of Agricultural and Bioresources Engineering Department with Registration Number PG Ph.D 98 25931 has satisfactorily completed the requirements in course and Research work for the award of Ph.D in Agricultural Engineering. The work embodied in this thesis is original and has not been submitted in part or full for any other Diploma or Degree of this or any other University.

Engr. Prof. N.N. Osadebe
Supervisor

21/02/07
Engr. Dr. C.C. Mbajiorgu
Supervisor

Engr. Dr. G.O. Chukwuma
Head of Department

21-02-'07
Engr. Prof. J. A. Osunade
External Examiner

DEDICATION

This Ph.D work is dedicated to the Almighty God for His infinite mercy and protection on me and my family.

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Nwakonobi, Theresa Ukamaka (Mrs.)
(PG/Ph.D/98/25931)

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NOTATION

b = Breadth of section

d = Depth of section

D = Cylindrical rigidity

E_c = Static modulus of elasticity of laterized concrete

F_c = Compressive strength of Laterized concrete

F_{tb} = Tensile strength of laterized concrete

h = Thickness of shell

k = Lateral to vertical pressure ratio

L = Height of cylindrical silo

M = Bending moment

N = Hoop tension

P_w = Hydrostatic pressure

P_g = Grain pressure

Q = Shear force

R = Radius of cylindrical shell

W = Deflection

α = A constant

γ_w = Unit weight of water

γ_g = Unit weight of grain

Θ = Rotation

ϕ = Angle of internal friction of grain

σ = Stress

μ = Poisson's ratio

ABSTRACT

As the prices of building materials in the country continue to increase sharply, there is growing need to source local materials as alternative for farm building constructions. This project investigates the performance of a cylindrical shell built of laterized concrete for storage of foodgrains or water. In order to optimize the mix ratio of the constituent materials that will produce maximum strength, a mathematical model which relates the strength of laterized concrete and its component ratios was developed using Scheffe's simplex lattice approach. The optimum mix proportion obtained was used to determine the structural characteristics of laterized concrete in the laboratory. The values obtained for the material properties were applied in performing the static analysis of the cylindrical shell structure under the action of hydrostatic and foodgrain pressures using Pasternack's equations formulated on the basis of theory of shell of revolution. The reduced equation of static equilibrium was solved by the Initial-value method. A generalized solution of the cylindrical shell made of laterized concrete was obtained and used to evaluate the performance of the structure through the determination of deflection and stresses. Adequate cross-section of the cylindrical shell was determined.

The experimental data are very well fitted to the regression model, which satisfies the student-t and chi-square tests. The optimum strength value of 27N/mm^2 corresponding to an optimum mix proportion of 1:1:2:0.650 of cement, laterite, gravel and water-cement ratio respectively was predicted by the model. For the structural characteristics, the values obtained for the optimum mix ratio were higher than that obtained for the mix ratio of 1:2:4:0.791. The results of the analysis indicate that the maximum stresses developed due to hydrostatic loading is greater than the strength of the laterized concrete in reservoir thickness of 50mm, 10m in diameter and 5m in height. The wall thickness of 100mm was found adequate. The maximum stress developed due to soybean grain loading is greater than the material strength in silo thickness of 100mm, 20m in diameter and 8m in height. The wall thickness of 150mm was found adequate. Thus, the possibilities of using laterized concrete in constructing cylindrical storage structures were successfully demonstrated.

CHAPTER ONE

INTRODUCTION

1.1 Need for the Study

The post harvest handling of food grains and seeds has constituted a matter of great concern to Agricultural Engineers and other agricultural experts. Storage in agriculture has been as old as agriculture itself and had helped farmers overcome several difficulties in agricultural production. These include, among other things – availability of food produce throughout the year, stability of food prices; maintenance of seed viability, and provision of buffer stocks. In developed countries, efficient storage and processing of food have been recognized as major factors in the solution of food problems (Aboaba, 1976).

In Nigeria, about 70 % of the farming populations are of low income earning capacity. Thus, most farmers cannot afford the provision of such modern storage structures as steel or concrete silos, large warehouses etc. These farmers therefore, store their farm produce by making use of traditional storage structures. Some of these traditional storage structures have been presented by some researchers in survey studies. These include rhumbus, underground pit, plastic or metal containers, gourds, earthen pots, platform, cribs, etc. (Aboaba, 1976; Olumeko, 1996)

But the use of these structures are associated with numerous problems which, according to Olumeko (1996), are that they are not durable, not watertight, not rodent proof, susceptible to insect attack, low viability of grains, change in grain texture, appearance and flavour and of low capacity. These defects and shortcomings of the traditional structures usually result in tremendous losses estimated to be up to 30 – 40 % (Igbeka and Olumeko, 1991). To overcome these problems efforts are now being directed towards evolving low-cost, effective storage structures made of locally available materials for use by low to medium scale farmers. Laterized concrete has been identified by Lasisi and Osunade (1990) as a possible low-cost construction material. According to building experts and suppliers of building materials in the project area the cost of obtaining lateritc soil is less than sand particularly in areas where there is no sand deposit.

The justification for this research work arises as a result of tremendous losses usually incurred during storage of agricultural produce. Because of the numerous problems associated with the use of traditional storage structures and the high cost of modern storage structures, the introduction of low-cost and effective storage structures becomes very necessary. Justifications for this research study are that:

i. Low-cost storage structures be made possible for rural farmers.

ii. Provision of storage structures that can be constructed from locally available materials.

iii. Introduction of structures that will be very easy to construct as no special skills are required

iv. Minimized losses incurred from poor storage environment thereby increasing the income of rural farmers.

v. Structure for multi-purpose storage i.e., either for farm produce or water.

1.2 Structural Storage Requirements:

Every storage structure, no matter what it is made of or how it is built must satisfy the following conditions:

i. Structural stability: must be able to resist all applicable loads including dead (self-weight of structure) and imposed (stored material) loads.

ii. Environmental protection: desirable environmental condition to keep produce cool and dry as well as protect it from insects and rodents.

To meet the above requirements Olumeko (1996) discussed the following practices to be adopted:

1. The storage structure should be built of materials of adequate strength and low thermal conductivity.

2. The structure should be watertight.

3. The structure should be rodent proof in all possible ways.

4. The storage structure should be protected from direct sunlight as agricultural produce needs to be kept cool and protected from large outside temperature variations.
5. Structure, which has old produce, dust, straw and insects, should be well cleaned and disinfected before putting in fresh produce.
6. The produce (when applicable) must be well-dried (12-13% moisture content) before putting into storage.
7. Rule of cleanliness, drying and application of insecticide should be strictly followed.
8. Regular checking should be done while in storage in order to monitor and prevent insect infestation.

1.3 Background

1.3.1 Laterized concrete

Concrete is a mixture of Portland cement, fine and coarse aggregates (sand and gravel), water, and in some cases, admixtures. However, laterized concrete is concrete in which some or all the fine aggregate is from lateritic soil. Sand, which is the second largest of the concrete volume, is not always available.

According to Osunade (1997), sand is the second most costly item per unit volume of concrete produced. Thus, replacing sand with lateritic soil can reduce construction cost since the latter is abundant and readily available at construction sites.

In recent times, research efforts have been directed towards determining as well as improving some physical and engineering properties of lateritic soils (Lasisi and Osunade, 1984; Masida, 1978). Several investigators have studied the effective structural applications of lateritic soils as a component in concrete (Osunade et al., 1990; Osunade, 1993; Osunade 1994). Some of the research findings were very encouraging.

However, a lot still remains to be done on laterized concrete in order to provide adequate information for the effective utilization of this material for construction purposes.

1.3.2 Shell Structures

A shell is an object whose material is confined to the close vicinity of a curved surface, the middle surface of the shell. In most cases, the shell is made of a solid material filling the space between two parallel (or almost parallel) surfaces, called shell thickness, and middle surface is defined as the surface that halves the shell thickness everywhere.

The distinguishing characteristic of these structures is that two dimensions of the structure are much larger than the third one, i.e., the thickness. Secondly, they have great strength due to the special character of working under the action of external loads (Denttartog, 1952; Muhammed, 2000). Although the theory of shells goes back to about a hundred years and an enormous mass of publication has accumulated, it is still an evolving field. For a long time the bulk of publication relating to theoretical work dealt with solution for shells of particular geometry and under a particular type of loading.

In the recent time, attention has turned to the solvability of equation, the assessment of different shell theories, dynamics and stability under general conditions thus arousing renewed interest in the foundation of shell theory.

1.3.3 Why a shell structure?

The Science of structural engineering is at present confronted with the problem of lowering, as much as possible, the weight and the material consumption in the construction of buildings while increasing their reliability and longevity. The solution of this problem is mainly, better planning and design decision-making and the wide use of light thin-walled structures such as shells.

1.4 Objective of the Research

The objective primarily is to explore the possibility of using a cylindrical shell structure made of laterized concrete to store farm produce or water. The specific objectives are to

a. test and measure the strength properties of laterized concrete, using laterite from Nsukka.

b. develop an optimization model for mix proportioning of the components of laterized concrete.

c. analyze (based on 'b' above) the cylindrical shell structure made of laterized concrete using Pasternak's semi-moment equation, which was formulated on the basis of cylindrical theory of shell of revolution.

d. develop a general solution for design of a shell cylindrical silo built of laterized concrete.

e. evaluate the performance of the structure through the determination of deflection and stresses.

1.5 The Scope

Simplex lattice design proposed by Scheffe (1958) was used to formulate a Mathematical model which relates compressive strength of laterized concrete and its component ratios of cement, laterite, gravel and water-cement ratio. A Computer based optimization was carried out using this model to establish the mix ratio for the optimum strength.

Laboratory tests were carried out employing the optimum ratio to establish some engineering properties of laterized concrete which include flexural strength, shear modulus, young modulus, poisson's ratio and density. The values obtained were used to carry out static analysis of cylindrical shell using Pasternack's semi moment equations, which stem from classical theory of shells of revolution. The reduced equation of static equilibrium was solved using method of initial-value due to Krylov (1931). General solutions were obtained for a cylindrical shell built of laterized concrete and were used for the determination of deflection, bending moment, shear force, hoop tension and rotation. The thicknesses of the cylindrical shell to resist load due to hydrostatic and grain pressures were determined.

CHAPTER TWO

REVIEW OF LITERATURE

2.1 Importance of Food grains

Grain is the collective name for the most important farming plants that belong to the grass family. According to Bakker-Arkema (1999), the term grain is interpreted broadly and includes the cereal grains (maize, rice, wheat, sorghum, barley, oats, millet), the oil - seed grains (soybeans, groundnut, sunflower, oil-palm, caster), and the grain legumes (cowpea, field beans, peas). The nutritional values of grains and their many utilization possibilities, limited only by human imagination have made grains most important as a source of food. According to Boumans (1985), a typical grain cereal consists of 70 % carbohydrates, 10 % protein and the remainder of fats, vitamins, minerals and water. Grain legumes and oil – seed grains have high protein (30 - 40 %) and mineral salt content, and are more difficult to preserve than cereals as they are more vulnerable to certain insect attacks (Appert, 1987).

2.2 Factors Affecting the Efficiency and Performance of Storage Structure

The following factors if not properly monitored and managed do negatively affect the efficiency and performance of storage structures.

2.2.1 Physical Properties

According to Appert (1987), the inherent physical properties of agricultural produce such as food grains influence the efficiency of their storage structures. These include viz: water content, thermal conductivity and granular structure as are briefly described below.

Water Content:

The water contained in grains exists in two forms – water of composition, which is contained within the plant cells, and free water which is on the surface of the cells and part of which is absorbed superficially by those cells (Appert, 1987). It is this free moisture content that is measured and which determines grain preservation. It is noted that cereals and legumes will deteriorate if their moisture content exceeds 13 % and 15 % (wb), respectively.

Thermal Conductivity:

Food grains have a low specific heat and low thermal conductivity. Whenever there is heating in the mass of grains, the natural movement of the air between the grains is usually inadequate to evacuate the heat produced and the temperature may due to low specific heat of the environment rise to extreme high levels.

Granular Structure:

Grains in bulk behave like a fluid and can run or be sucked up with the flow depending on the form, size, and moisture content of the grains and the extent of cleanliness. Dampness or heating may cause a number of grains to stick together and coagulate as a mass, creating blockage in the outlet of the structure.

2.2.2 Environment Factors

According to Appert (1987), these factors cause deterioration of grains and seeds in storage. They are temperature, relative humidity, water and gases as are briefly described below.

Temperature:

The effect of temperature is to modify air relative humidity, since warm air absorbs much more water than cold air. The rays of the sun cause the air in a store to heat up causing decrease in its relative humidity. The warm air absorbs part of the moisture from the grains as the latter's mass does not warm up as quickly. Temperature affects the various biological organisms present in the speed and extent of their activities and physiological development. According to Appert (1987), the thermal optimum for most stored arthropods and microorganisms is around 30 °C. Below 18 °C, their development slows down or even ceases altogether and a temperature in excess of 35 °C is lethal for certain species. Much of the tropical region is characterized by climates that make it unsafe to store grains. High temperatures and relative humilities predominate over prolonged periods of time, thus increasing the potential for deterioration due to insects, birds, molds and rodents. Even in the best-constructed and best-managed facilities, storing grains in tropics is a very difficult task (De Sousa e Silva and Berbert 1999). It is therefore essential to control the temperature of the stored grains as any abnormal rise in temperature leads to a proliferation of insects.

Relative Humidity:

When the air is dry, the grain loses moisture to the ambient air; if it is damp, the grain absorbs moisture from it. These exchanges occur in either direction until a balance is achieved. The time needed for the balance to establish itself and the direction in which the exchange takes place depends on the ease with which the air can circulate through the grains. If the grain is stored in such a way that there is poor air circulation, the balance will be close to the water content of the grain. For good circulation of air around the produce the water content of the grains will depend on the relative humidity of the atmosphere. It is essential to know the water content of each batch of grains entering a store in order to determine whether the produce is sufficiently dry to keep properly or whether arrangements need to be made for drying. Table 2.1 below shows the maximum moisture content for long-term storage, as suggested by Appert (1987).

Table 2.1: Maximum Moisture Content for Long - Term Storage

Type of Grain	Maximum moisture Content % (wb)
Groundnut	7
Sorghum	12.5
Wheat	13
Maize	13
Rice	13
Paddy	14
Legumes	15
Millet	16

Source: Appert (1987)

Water:

Water, in liquid form or in the form of moisture in the grain or the ambient air, is a common cause of deterioration. The presence of water increases microorganism and insert activity, thus lowering the quality of the produce. Damp grain heating is noted to cause the grain to cake and rot as a result of fungi, yeasts and bacteria infestation. According to De Sousa e Silva and Berbert (1999), high initial moisture contents of grains at harvest and warm weather throughout the year in the tropic leads to intense levels of biological activity (grain respiration, insect and mould development), and thus require frequent supervision of grain in storage. Walls must be adequately protected from rain through accurate design and construction of the storage structures. Such walls must be insulated from the foundation to prevent water rising up from the ground by capillary action (Appert, 1987). Thus, the insulation also prevents the damaging effect of the condensed moisture on the walls of the store as a result of day and night time temperature variations.

Gases:

The respective proportions of the gases (oxygen, carbon dioxide, nitrogen) contained in the air surrounding the grains have effect in inhibiting or accelerating the vital phenomena of oxidation, fermentation etc.

2.2.3 Biological Factors

Stored grain is subjected to the deleterious effects of grain pests such as microorganism, insects and rodents (Montross et al., 1999). According to Montross et al., (1999), some writers (Christensen and Kaufman, 1969; Christensen and Meronuck, 1986) reported that the degree of pest activity is a function of the moisture content and temperature of the grain, the damage level of the grain, the foreign-material content of the grain and the type of the grain. They submitted that the lower the moisture content, the temperature, the damage level, and the foreign-material content of the grain, the longer the grain can be stored without being affected by one of the grain pests.

Microorganism:

Grains are infected by a large number of different types of bacteria, moulds and yeasts right from harvesting. This infection continues to increase during the various handling operations, which the grains undergo until consumption. The bacterial and fungal spots start to develop and cause damage as soon as they encounter favourable temperature and humidity conditions.

According to Appert (1987), the species of fungi, which develop on the surface of and inside food grains, depend on the contamination, nature of the grain, temperature, humidity and oxygen content of the air. A number of types of damage caused by fungal growth in grains are decrease in germinability, discolouration of the seed germ, heating and development of mustiness, biochemical changes, potential production of toxins and loss in dry matter as reported by some writers (Appert, 1987; Montross et al., 1999).

According to Montross et al., (1999), Pitt (1995) found that several grain moulds produce toxins, called mycotoxins. They are defined as fungal metabolites which when ingested, inhaled or absorbed through the skin cause lowered performance, sickness or death in man or animals (Pitt, 1995). If grain is properly dried field moulds do not further develop in storage (Montross et al., 1999).

Insects:

Insects are small animals, some of which compete with man for crops both before and after harvesting. They are responsible for considerable losses in storage and so, it is necessary to act against them, Several writers (Appert, 1987; Brooker et al., 1992; Pedersen, 1992) have reported damages caused by insects in various ways. The insects, whether adult or larvae, feed on the grain and eat the albumen or germ and even both of them. The attack on the endosperm, according to Appert, (1987, results in a loss of weight, a reduction in the nutrients and deterioration of their quality.

By eating the germ, the insect causes a reduction in the seed's ability to germinate. The specific gravity also decreases, lowering the market value of the product. The presence of insects in produce also causes contamination by adding excrement, empty eggs, larval molts, empty cocoons, and adult corpses all that are deposited within the grains and cannot be removed. The secretion of many silken threads cake the grains together. The metabolism of insects is accompanied by the production of heat and moisture, which consequently bring about hot spots that form with the grains. The temperature difference between these hot sports and the outside leads to a movement of air in the grain mass. Condensation occurs where the warm air meets the cool air. The water deposited results in caking together, proliferation of toxin-producing mould and fungi and rotting of the grains (Appert, 1987). Hundreds of insect species have been found in grain storage. Table 2.2 shows lists of some common grain-storage insect and their optimal growth conditions

Table 2.2: Optimal development conditions of some common insect species found in grain storages

Insect species	Temperature (°C)	Relative humidity (%)
Rice weevil	26 –31	70
Maize weevil	27 – 31	70
Lesser grain borer	32 – 34	50 – 60
Large grain borer	25 – 32	80
Grain moth	26 – 30	75
Red flour beetle	32 –35	70 -80
Rusty flour beetle	33	70 – 80
Sawtoothed grain beetle	31 –34	90

Sources: Pedersen (1992) as reported in Montross et al. (1999).

According to Montross et al., (1999), Brooker et al., (1992) suggested steps to be taken in controlling infestation of stored grain by insects as follows:

- Remove all gain debris from in and around the store before the harvest season.
- Treat the walls of store with proper chemicals at least two (2) weeks before the store is to be filled.
- Use a grain cleaner
- Grain to be stored at proper moisture content.
- Cool stored grain by aeration or chilling to below 10 ^{o}C (if possible)
- Monitor the grain temperature and moisture content and the presence of moulds and insects.
- Clean and sell or fumigate grain immediately after insects are discovered.

Gasga (1996) suggested that the fumigation (i.e. chemical treatment) of stored grain should be implemented by professional fumigators.

Birds and Rodents:

These are responsible for losses of grains both in the field and in storage. Birds are harmful particularly in terms of the amount of grains they can consume in feeding and in contaminating the grains with their droppings, feather or various materials carried by them when building their nests (Appert, 1987). Rodents are considered a storage pest because they consume and damage grains in the field and in storage, destroy baggage and storage structures, and transmit diseases that are dangerous to humans through urine and droppings (Montross et al., 1999).

The following are recommended by Montross et al., (1999) for rodent-proofing a storage facility:

- Exterior structures should be constructed of gnaw-proof materials
- Openings should be closed or protected with doors, screens or gates
- Dead space within the interior of the storage structure should be avoided.

2.2.4 Technical Factors

The condition of grains at the time of storage affects or contributes to their deterioration. For instance, if inadequately dried, improperly cleaned or damaged in some way, the deterioration will be more. Appert (1987) reported that separation of the grain from its protective outer coating

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2.2.4 Technical Factors

The condition of grains at the time of storage affects or contributes to their deterioration. For instance, if inadequately dried, improperly cleaned or damaged in some way, the deterioration will be more. Appert (1987) reported that separation of the grain from its protective outer coating leaves it exposed to moisture, oxygen and certain insects and microorganisms, which then have easy access to wreak their destruction.

According to Montross et al., (1999) the degree of pest activity is a function of the moisture content and temperature of the grain, the damage level of the grain, the foreign – material content of the grain, the type and hybrid of the grain, and the interstitial atmosphere around the grain. Thus, the lower the moisture content, the temperature, the damage level, and the foreign – material content of the grain, the longer it can be stored without being affected by one of the grain pests.

2.3 Structures and Systems for Food grain Storage

2.3.1 Traditional Methods and Systems

Farmers utilize local, natural materials to build structures that can keep well, their produce. After harvesting, grains are threshed or shelled, sun dried and manually winnowed before being transferred to the storage structures, grains also are stored in un-threshed form after harvesting. The various traditional methods of grain storage have been reported in several survey and review studies (Hall, 1957; Aboaba, 1976; Boumans, 1985; Appert, 1987; Olumeko, 1991; Olumeko, 1996) as are discussed below:

2.3.1.1 Open Structure

Aerial Storage:

Un-shelled maize cobs and other un-threshed cereals are suspended in bunches or sheaves, using rope or plant material from the branches of trees or top of poles driven into the ground near homes, the grains gets dried in the air and the sun until it is gradually and finally consumed by the farmer and his household. This method applies only to small quantities of grains and is a common practice in the middle belt of Nigeria (Olumeko, 1996).

Storage on the ground:

This is for temporary storage and follows on immediately from harvesting and lasting only a few days. This results or happens when the farmer is either not having time to bring in what he has harvested or he wants to let it dry in this for a while if there is no prospect of rain. Considerable losses result from attack by field rodents carrying grains to burrows. This practice is common among peasant farmers irrespective of the region of the country he is.

Platforms:

Platforms are four-cornered racks made of branches covered with leaves or grass, usually fixed more than two meters from the ground on posts consisting of large branches driven into ground.

On these platforms the grain is preserved either in containers placed on top, or in bulk un-threshed, in heaps or regular layers. In the latter case, fire may be lit underneath in order to speed up drying and deter pests. Sometimes the platforms are completely covered with a detachable straw roof, which is lifted off from time to time. Sometimes the platform could be indoors by fireplace. The structure was found to be very inefficient and leads to losses due to rodents, termites, and moisture. It is prone to pilfering and fire hazard (Igbeka and Olumeko, 1991)

Cribs:

This structure is rectangular in shape with width ranging from 0.6 – 4m and ventilated sides made of bamboo, grass stalks or even wire netting or palm frond stalks or planks. The thatched or metal roof with overhang protects the whole structure from rain. The floor elevation ranges from 0.6 – 1.2m and, in some cases, the structure has either a wooden or metal door for loading and offloading the grains. An elevated floor, made of wood or branches and fixed to posts, supports the contents, keeping them away from dampness of the ground. The legs are fitted with metal anti-rat guards. With this system maize could be stored in husked or un-husked forms with capacity varying from 1 – 4t. Olumeko (1991) reported that this structure is fairly efficient depending on the materials and method of construction, but require some modifications.

According to Montross et al., (1999), Hall (1957) reported the use of cylindrical wire – cage crib for the storage of ear maize.

2.3.1.2 Enclosed Structures

According to Olumeko (1996), farmers generally store their crops in granaries, inside their houses and in underground pits. The granaries, mostly found in the Northern Zone of the country, Nigeria, are of two types-Ventilated granaries made of plant materials (Thatched rhumbu) and granaries with solid mud walls (mud rhumbu).

Domestic Indoor-storage Structures:

Several writers (Aboaba, 1976; Appert, 1987, Olumeko, 1991) reported that unthreshed cereals are commonly stored under the roof of the dwellings, hanging from the roof timber or spread out on a grid above the fire, the heat and smoke ensuring that they keep well.

Containers:

For small amount of grains, which are drawn upon frequently for daily food requirements, the grain, either threshed or un-threshed, is kept in containers such as bags, basket, jar, gourds or drums in the dewellings, granaries (on top of produce already in store) or on platforms (Olumeko, 1996). They are safe from theft. Large earthenware vessels placed underground were used in the olden times (Boumans, 1985).

Underground pit:

This structure is commonly found in the Sahel part of the Sudan Savanna Zone (Ṡokoto, Katsina, Kaduna, Borno and Gongola State) where water table is low (Olumeko, 1996). The pit, which may be either round or square in cross-section is 2-3m deep and 1.5 – 3m in diameter or square. The pit after being dug, fire may be lit to dry walls, prior to plastering them with clay. To prevent damp, the bottom of the pit is covered with straw or cereal chaff and the walls are lined with matting. After loading grains into the pit, it is covered with a polyethylene or metal sheet, and then a layer of laterite. This structure provides good protection against pests and theft. Boumans (1985), reported that ancient Greeks made use of specially dug holes for grain storage.

Thatched Rhumbu:

These are commonly found in the North-East Zone of the country, Nigeria, which include Borno, Jigawa and Adamawa States. It is one of the major traditional storage structures (Olumeko, 1996). It is cylindrical in shape, with floor made of woven grass stems or fibres and overhanging conical roof made from straw or grass. Grains are usually stored in unthreshed forms. The top acts as the inlet and outlet for grains and the capacity ranges from 1000 kg to 2000 kg. The structure is supported on low wooden structure or by stones. The wall is provided with tension rings in two or three positions using local rope materials. This structure is reported to be inefficient as it is neither moisture proof nor airtight and is prone to rodent attack and fire hazard. It has no suitable loading and unloading facilities and even no protection against theft (Olumeko, 1996).

Mud Rhumbu:

According to survey report by Olumeko (1996), this structure is another major means of grain storage. The most common type is the outdoor type. It is usually circular in cross section with diameter ranging from 80 cm to 180 cm. The capacity range is from 2000 kg to 4000 kg. The floor is made of wood and plastered with mud; while the wall is built of specially prepared mud made by adding 2 – 3 layers each construction. The top is covered with conical shaped roof made of either thatch or mud. The improved mud rhumbus is built of sandcrete base and brick mortar. The bricks are laid in sand mortar and plastered with sand mortar in all sides. In some cases there may be partition inside to enable the farmer store different types of grains at the same time. Some of the defects of this structure are found to include: inefficient loading and off-loading facilities; not moisture proof and airtight, attacked by termites, not durable and non-viability of stored produce for long period.

2.3.2 Improved/Modern Storage Structures

2.3.2.1 CSU Improved Structures:

The crop storage unit (CSU), Ibadan under the technical assistance of Common wealth fund for technical co-operation had developed various improved indoor and out door storage structures of different capacities as solution to some of the problem and shortcomings of the existing local storage structures (Olumeko, 1996).

Ventilated Crib:

According to CSU (1989), as reported by Olumeko, (1996), the Ventilated crib can serve for both drying maize on the cobs, un-threshed rice, sorghum and millet and for storing shelled and threshed grains in bags. The main features of the ventilated cribs are basically the same with local cribs except for the improved method of construction and use of better materials. They include a raised floor, four sides with one of them open for loading and offloading the grains and a roof. The inside and outside of the walls are covered with some sorts of mats, grass or even plastic to protect the bagged grain against wind driven rain.

Improved Indoors Brick Masonry Structure:

According to CSU (1992), this is a brick masonry, rectangular structure with two compartments. The total capacity is $2.7m^3$. According to the space available inside the rural house, a structure of the desired capacity with either one or more compartments can be constructed for storage of different food grains (Figs 2.1a, b and c). The bricks are laid and plastered with cement mortar (1:6) in all sides. The structure is built on a solid base 350mm high. Each compartment is provided with a steel outlet of 150 x 150mm size at 350 mm above floor level for unloading purposes. The inlet is provided at the top with size 425 x 359 mm for loading purposes with a timber as cover. Adequate loading and unloading devices are provided both at the inlet and outlet. The structure can be located in such a place in the house that the existing walls can be utilized and hence make it more economical. The most suitable location is the side or in one corner of the room. The structure eliminates the hazards encountered in the existing local indoor storage systems.

Outdoor Reinforced Brick Masonry Structure:

According to CSU (1992), the structure consists of two bricks masonry walls each 120 mm thick with a moisture barrier like polythene sheet of 600 to 800 gauge thick to make the structure moisture proof. A polythene sheet of 1000 gauge thick is used as a moisture barrier in the construction of the base to prevent subsurface moisture absorption. The outer layer is reinforced with steel and plastered with cement mortar (1:6). The structure, which is circular in shape is about 1800 mm deep. It has flat cement concrete floor at the base and reinforced cement concrete roof at the top. An inclined steel outlet is provided at the bottom and an

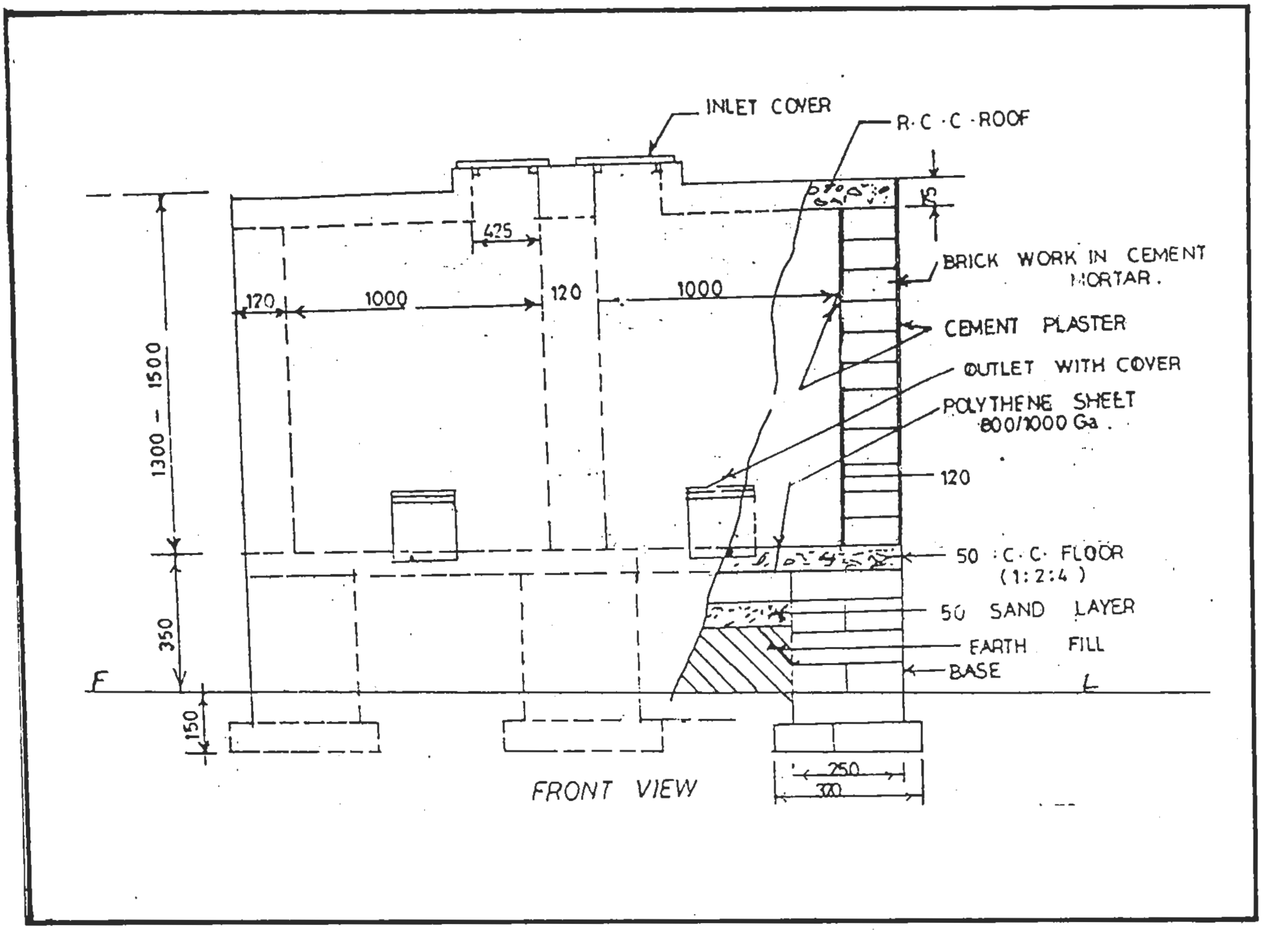


Fig. 2.1(a) Brick Masonry Structure (indoor) with Compartment (capacity : 2.0MT)

Source: CSU (1992)

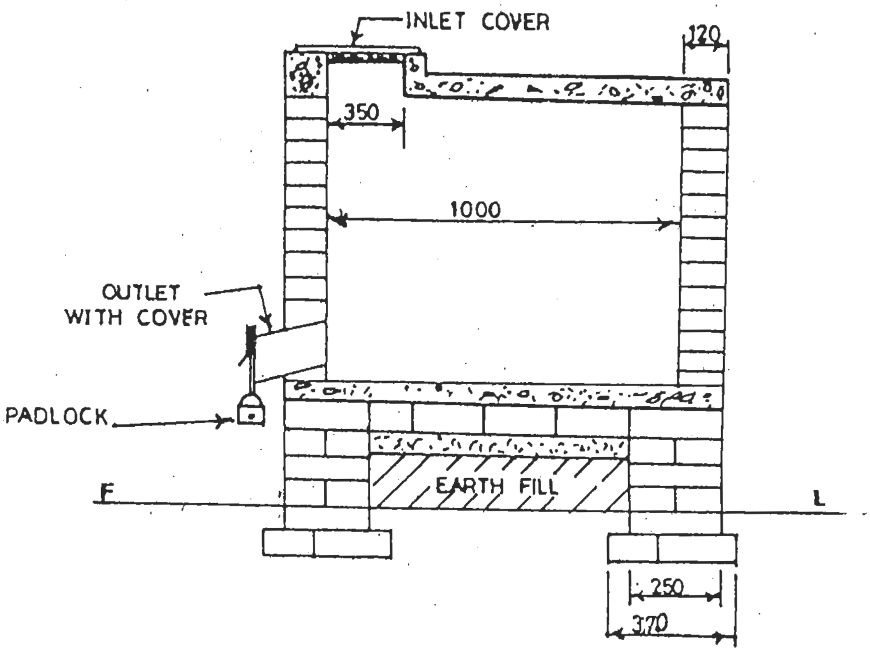


Fig. 2.1(b) Side View
Source: CSU (1992)

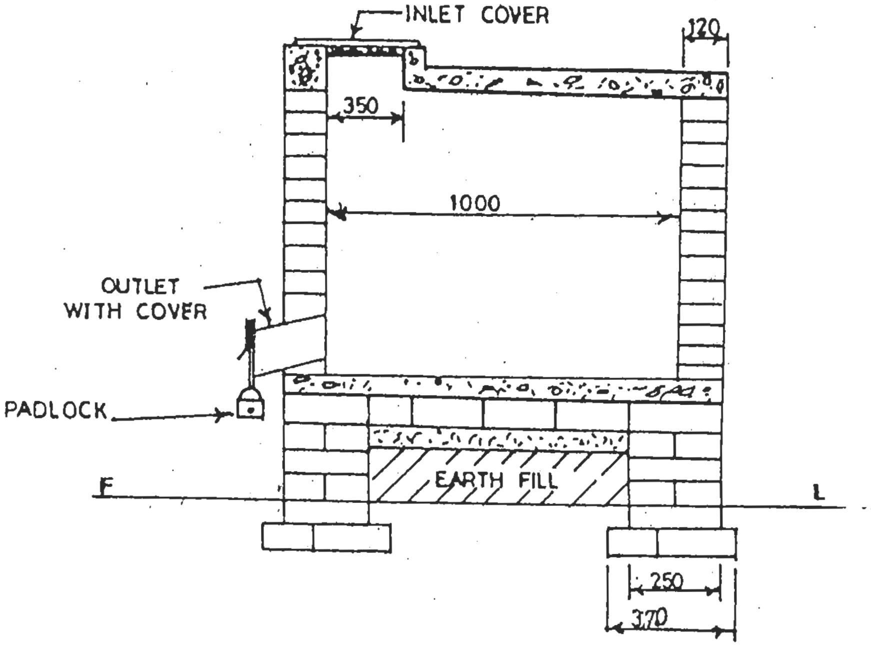


Fig. 2.1(b) Side View
Source: CSU (1992)

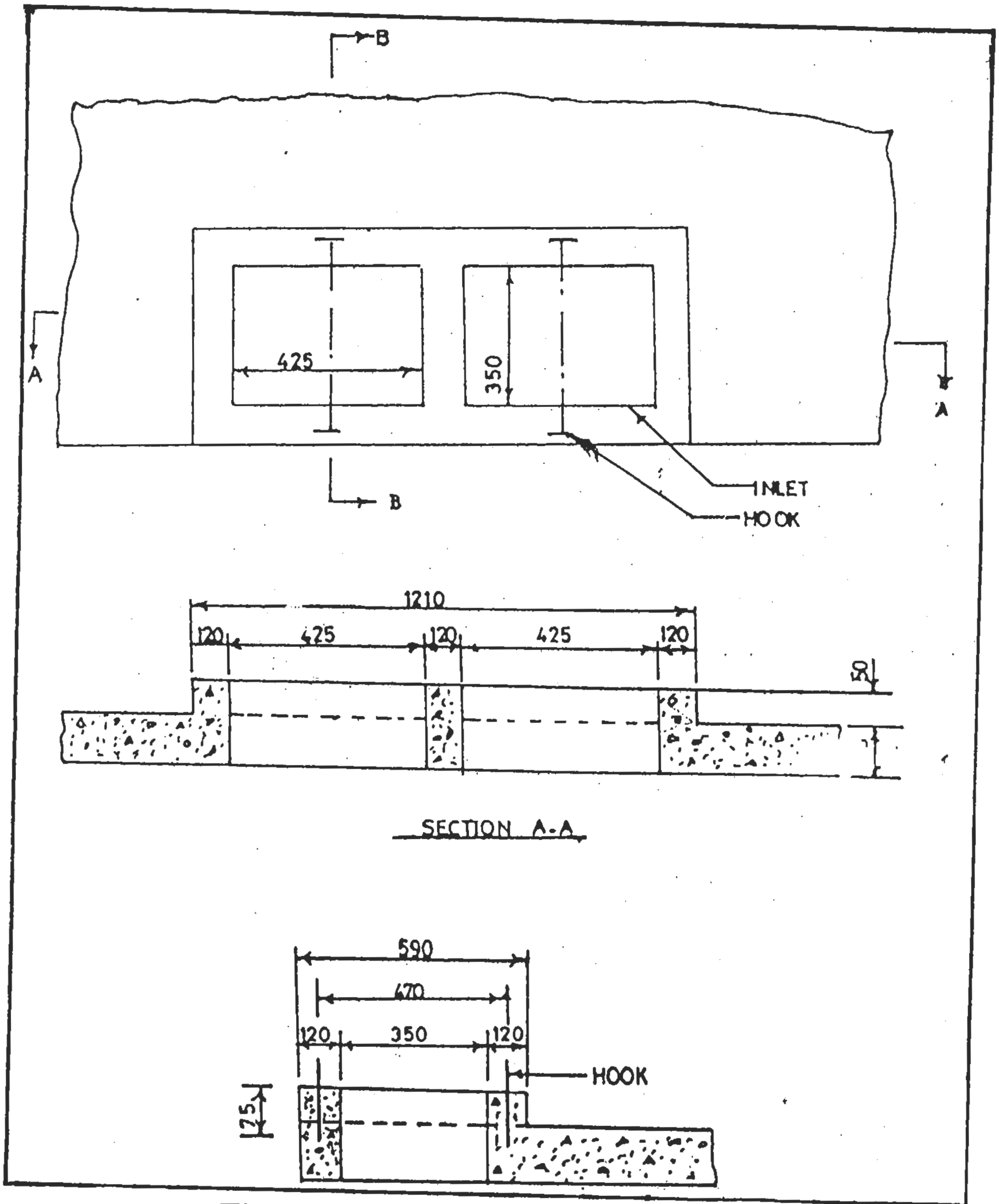


Fig. 2.1(c) sectional View
Source: CSU (1992)

inlet of 500 mm with a steel cover. The structure has a long life span and low annual maintenance cost, though the initial cost of construction is a little bit high. It provides satisfactory service. The cross-sectional view of the structure is shown in Fig 2.2.

Underground Brick Masonry Structure (Out Door):

This is an underground structure of $6.6m^3$ or 5.0MT capacities for wheat, 3.65 MT capacity for sorghum constructed of brick for storage of food grains in bulks as shown in Fig. 2.3. According to CSU (1992), the structure is circular in shape having inner diameter of 2000 mm and depth 2100 mm. It is constructed below the ground level to be used for storage of different food grains. The bricks are laid and plastered with cement mortar (1:6) on all sides. The structure is constructed on solid base 350 mm high. Each compartment is provided with a steel outlet of 150 x 150 mm size at floor level for unloading purpose, and inlet of 425 x 350 mm size, and a timber inlet cover at the top for loading purpose. Adequate loading facility is provided with steel outlet. The quantity of materials and labour needed for the construction are shown on Table 2.3.

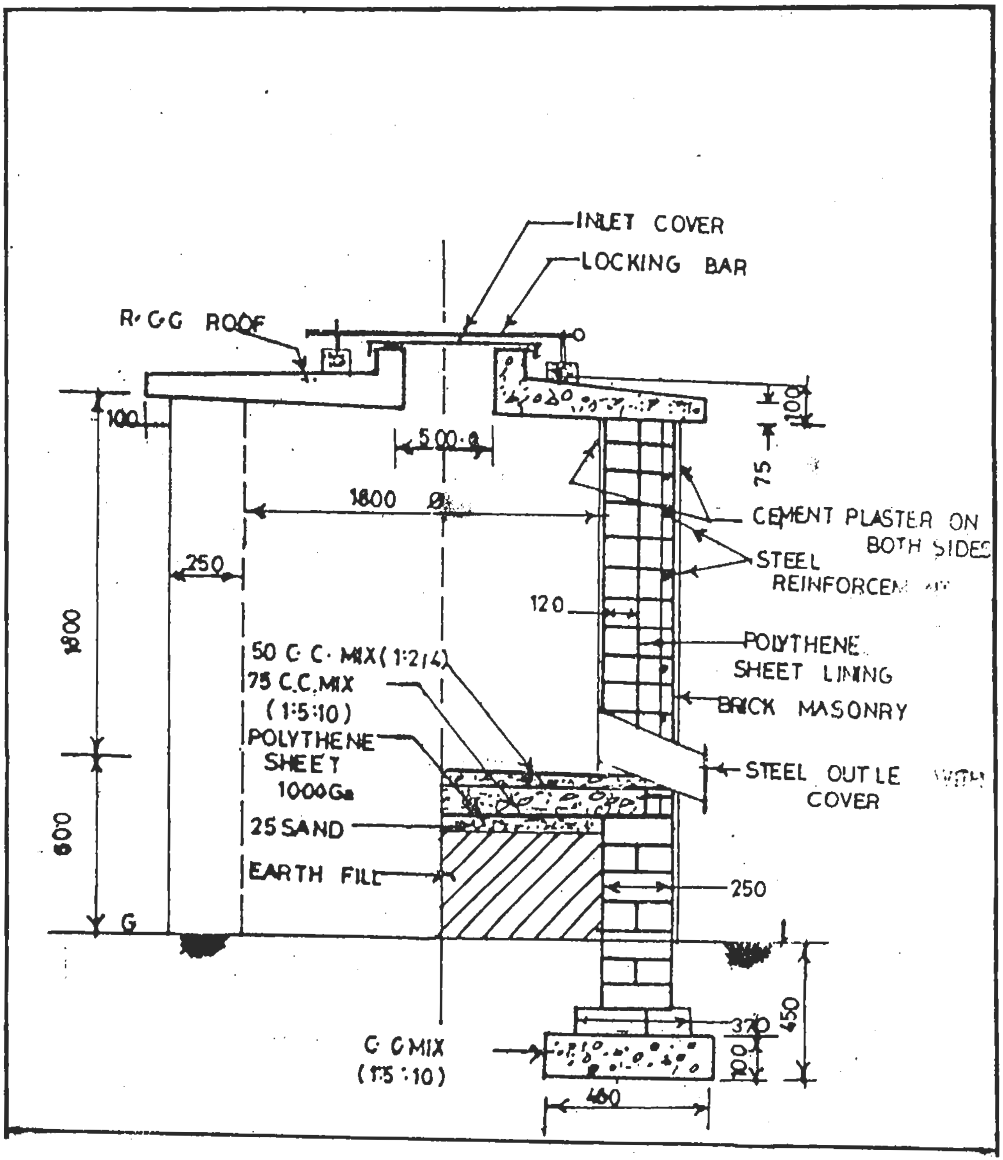


Fig. 2.2 Reinforced Brick Masonry Structure Capacity 6.1M³ or 4.6MT

Source: CSU (1992)

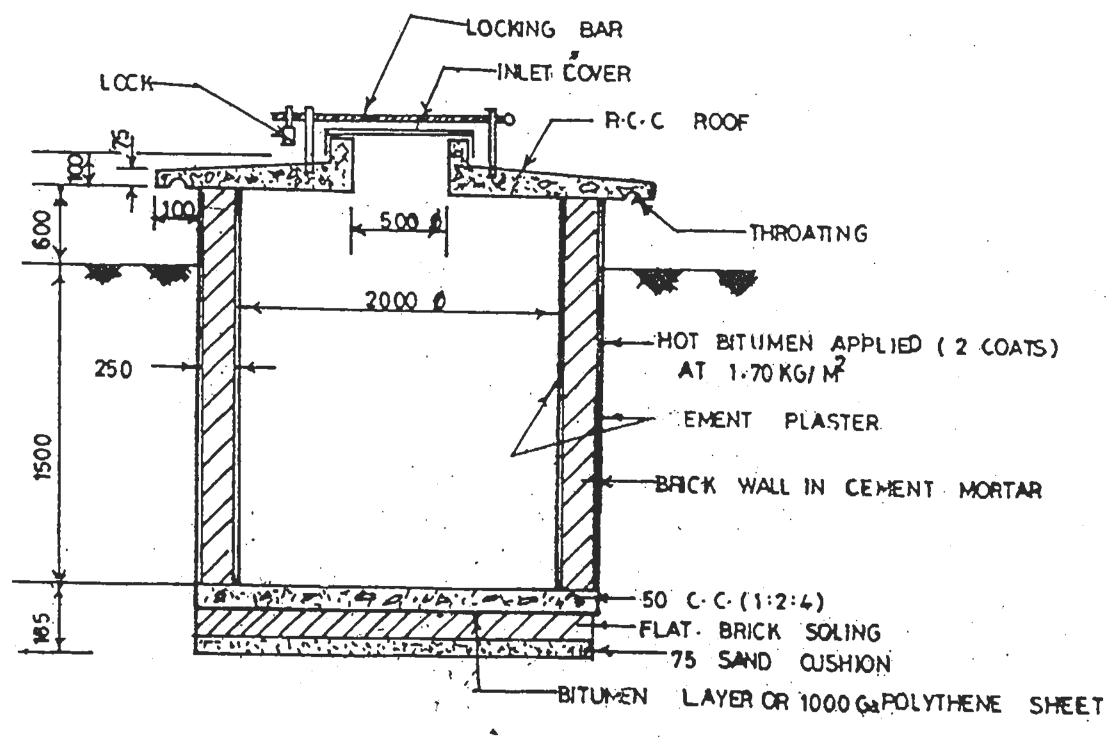


Fig. 2.3 Under ground Brick Masonry Structure Capacity 6.6.M^3 or 5.0MT

Source: CSU (1992)

Table 2.3: Material and Labour for Construction of Underground Brick Masonry Structure.

S/NO	MATERIAL	QUANTITY
1.	Cement	20 bags
2	Sand	100 cft
3	Crushed Stone 0.5″	25 cft or 4.0 cyd
4	Bricks, 250 x 120 x 60mm	1500
5	MS Bar 8mm dia.	60 m (10 length)
6	Bitumen	30 kg
7	Steel Inlet Cover	One
8	MS Hooks	Two
9	Locking Bar	One
10	Wood 1″x2″x12′ Length	10 nos
11	Wood 2″x3″x12′ Length	10 nos
	Labour	
	Brick layer	15 Man-days
	Helper	20 Man-days

Source: CSU (1992)

Underground Reinforced Cement Concrete (RCC) Storage Structure:

According to CSU (1992), this is an underground reinforced concrete, which is 4.7m3 or 35.5 MT capacity for wheat or 26 MT capacities for sorghum for storage of food grains in bulk as shown in Fig. 2.4. The structure is circular in shape having an inner diameter of 200mm and 1500 mm in depth. It is constructed below the ground level to prevent the entry of rats as well as surface or floodwater. The entire structure is constructed of reinforced concrete except the floor. The floor construction consists of two layers of concrete and a thick layer of bitumen provided underneath the concrete floor to prevent entry of subsurface moisture.

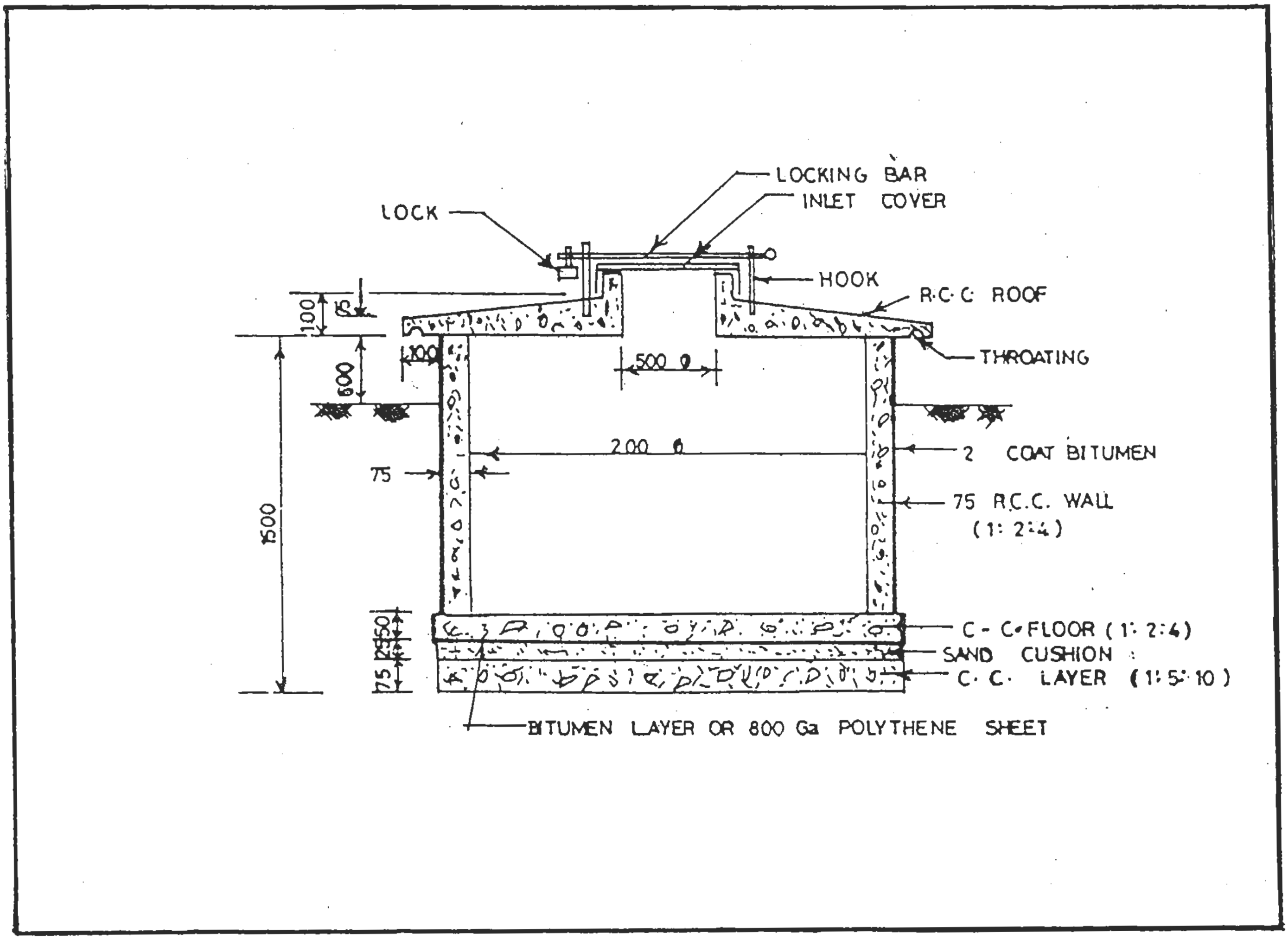


Fig. 2.4: Underground R.C.C. Structure Capacity 4.7^3 or 3.5 MT
Source: CSU (1992)

A reinforced concrete roof is constructed at the top with slope towards the outer edge to drain away rainwater. An inlet of 500 mm diameter with steel inlet cover and adequate locking arrangement is provided in the center of the roof.

Two coats of bitumen are applied on the exterior surface of the wall for moisture proofing of the structure. Besides, throating is provided all around underneath the outer edge of the roof to drain away the rainwater. This structure may prove suitable as an improvement to the existing underground pit used by farmers. Table 2.4 shows the estimated materials and labour for this construction.

Table 2.4: Material and Labour for Construction of RCC Structure

S/No.	Material	Quantity
1.	Cement	18 bags
2.	Crushed Stone	80 cft
3.	MS Bar 8mm dia.	180 m (320 length)
4.	Bitumen	25 kg
5.	Steeel inlet cover	One
6.	MS Hooks	Two
7.	Locking bar	One
8.	Wood 2"x3"x12" length	40 Nos.
9.	Singular for carpentary work to cast wall & roof	
	Labour	
	Bricklayer	12Man-Day
	Helper	20 Man-day

Source: CSU (1991)

Indoor Metal Bin:

According to CSU (1989) as reported by Olumeko (1996), metal bins of different capacities from 150 kg to 1 MT are available at crop storage unit (CSU), Ibadan. The metal bin is fabricated from 24 gauge and 16 gauge galvanized iron sheets. The major components or features are the base, body, outlet and inlet assemblies, tension ring, top plate and the bottom support. The bin opens at the top for filling and has outlet at the bottom for emptying. The bin is moisture and rat proof as well as airtight. It can be used to store various types of grains and seeds. If the bin is well managed, it has long life span with low maintenance cost. Some of the limitations of the structure include: metal sheets are more expensive than the most local available materials like mud.

Construction of bin requires skill and sheet metal for the construction must be galvanized or painted regularly to protect the metal from rusting (Olumeko, 1996).

2.3.2.1.1 Advantages of Improved Structures

According to Olumeko (1996), some of the benefits derived from the improved structures of CSU include:

1. They are rat and moisture-proof
2. There is no damage due to termites
3. They are adequately tight and therefore control of insect and pest become easy through fumigation.
4. They have efficient method of loading and offloading
5. The grains can be stored in threshed condition rather than unthreshed and thereby increasing the store capacity
6. The storage losses are comparatively less
7. There is adequate protection against theft.
8. The quality of grains can be preserved for longer period provided the moisture is within the optimum storage limit.
9. `The life of the structure is long and annual maintenance cost is low.
10. Although the initial cost of construction is high the annual cost of storage is comparatively less.

2.3.3 Modern Storage Structures

Some of the modern storage structures for grain have been discussed in detail by several writers (Boumans, 1985; Appert, 1987; Montross et al., 1999). These structures include reinforced concrete silos, corrugated iron silos as described below.

2.3.3.1 Reinforced Concrete Silos

Reinforced concrete is material widely used for building storage structures. According to Appert (1987), these silos typically consist of a cylindrical unit made of reinforced concrete, encircled with steel cables (with tensioners), to resist the lateral pressure of the grains and built on concrete slab. They are approximately 2 to 3 m high and 1.5 m in diameter. The top is hermetically sealed with a slab containing a manhole, which is used for loading.

At base, a metal chute, close with a flap, enables the silo to be emptied and piping is fitted to permit fumigation, where necessary. This type of silo is heavy so the foundation must be constructed accordingly. It is also porous and must be painted with damp-proofing agent. Silo can be built of concrete with mesh.

Montross et al., (1999) highlighted some of the benefits as well as the drawbacks of concrete grain silos. The main benefits of concrete include; long lifetime, limited maintenance, large storage volume, low thermal conductivity and limited water-condensation problems. However, the drawbacks of the concrete grain silo are the significantly higher initial cost and erection time than those of steel silos of equal storage volume. Montross et al., (1999) reported that the outside walls of concrete silos require a protective coating occasionally to prevent the damaging effect of environmental sulfurous air – pollution.

2.3.3.2 Corrugated Iron Silos

According to Boumans (1985), this type of silo is used in particular for centralized storage and there are some very large units. At rural level, it is small units of about 2m in diameter and height, which are proposed by Appert (1987). They are made of curved sheets of steel bolted together; the joints between the sections are sealed to ensure water tightness. They are covered by a metal roof. These structures are light and easy to assemble, but they have drawback of not insulating the grain from the outside (Appert, 1987). Steel bins should be galvanized, bolts and nut coated and rust-protection paint applied when rust spots appear (Montross et al., 1999).

2.3.3.3 Warehouses

According to Montross et al., (1999) horizontal sheds and warehouses are often employed in addition to bins and silos for grain storage at large grain depots. It is reported to be more costly and labour intensive to move grain into and out of this system of storage.

2.3.3.4 Metal Drums

According to Appert (1987), metal drums that have contained petroleum products if they are thoroughly cleaned, do not have any perforations and close properly make ideal containers for preserving grains, provided of course, that they are sufficiently dry, they must be located away from the sun and heat.

By filling to the top, oxygen is excluded from the mass, and this prevents insects from developing. Also, the content are inaccessible to rodents.

2.3.3.5 Plastic Sacks

This is used for preserving legume seeds by bagging them up in polyethylene or polypropylene sacks, which are hermetically sealed immediately after dosing with Fumigant (Appert, 1987). To avoid heating, these sacks are white and opaque. The sacks usually measure about 50 x 110 cm and 0.3 mm thick. They hold 40 kg of seed. The sacks are four-fifths filled and pre-dosed capsule containing 18gm of carbon tetrachloride is broken over the content prior to resealing the sack by twisting the neck and tying it tightly with a knot. These sacks are piled up on platforms covered with grass to avoid tearing them on anything rough on the support, and are sheltered from the sun's rays by means of leaves. The platforms are equipped with metal anti-rat guards (Appert, 1987).

2.4 Thin Shell structures and their Applications

There are many interesting aspects of the use of shells in engineering, but according to Calladine (1983), only one stands out as being of paramount importance and it is the structural aspect. Shell constructions began in the 1920s and have emerged as a major long-span concrete structure. Thin parabolic shell vaults stiffened with ribs have been built with spans up to about 90m. More complex forms of concrete shells have been made, including hyperbolic, paraboloids, or saddle shapes, and intersecting parabolic vaults less than 1.25 cm thick (Britannica, 2004).

2.4.1 Application and Uses of Shell Structure

It is desirable to give a glimpse of the wide range of applications of shell structures in engineering practice. Several investigators have assembled a list of applications from a historical point of view, and the way in which the introduction of shell as a structural form has made an important contribution to the development of several branches of engineering (Sechler, 1974; Calladine, 1983; Britannica, 2004).

- Architecture and Building

The development of Masonry domes and Vaults in middle Ages made possible construction of more spacious building; and in recent times development of reinforced concrete has stimulated interest in the use of thin shells for roofing purposes.

- Power and Chemical Engineering

The development of steam power during the Industrial Revolution depended to some extent on construction of suitable boilers. These shells were constructed from plates suitably formed and joined by riveting. In recent times the use of welding in pressure vessel construction has led to much more efficient design. Pressure – vessels and associated pipe work are key components in thermal nuclear power plant, and in all branches of the chemical and petroleum industries.

- Structural Engineering

An important problem in the early development of steel for structural purposes was to design compression members against buckling. A striking advance was the use of tabular members in the construction of the Forth railway bridge.

- Vehicle Body Structures

The construction of vehicle bodies in the early days of road transport involved a system of structural ribs and non-structural paneling or sheeting.

The modern form of vehicle construction, in which the skin plays an important structural part, followed the introduction of sheet-metal components preformed into thin doubly curved shells by large power presses, and firmly connected to each other by welds along the boundaries. The use of the curved skin of vehicles as a load bearing members has similarly revolutionized the construction of railway carriages and aircraft.

- Boat Construction

The introduction of fiberglass and similar plastic materials has revolutionized the construction of small and medium sized boats, since the skin of the hull can be used as a strong, stiff, structural shell.

2.4.2 Shell Structure as Storage Vessel

The irregularity of the yield of the seasonal harvests, and also the unequal distribution of food grain throughout the country and the world at large, have resulted in an unequal consumption of it. This has led to building up of stocks to ensure a more uniform distribution and consumption of commodities. Stockpiling is done preferably in thin-walled silos for food grains, which does away with the burdensome use of sacks and reduces labour cost. This also applies to certain food products like sugar, flours, and animal feeding stuffs or industrial products like plastic granules, various powders as a result of increasing industrialization.

The introduction of combine harvesters brought with it the need for bulk storage of food grains on the farms (Beard, 1982). Water tank or reservoir is needed for the efficient operation of farmstead particularly of livestock enterprise.

Thin-walled shells of various shapes and sizes have therefore become adoptable for farm storages, as weight and material consumption in its construction are reduced. Dinnie and Beard (1970) reported the continuous usage and the satisfactory performance of circular and rectangular grain silos built of reinforced brickwork. A 455-m^3 circular water tank constructed of high strength bricks was also reported by Johnson (1982). Various kinds of economical silos for the storage of grains are developed by the use of thin steel shell. Cooling towers for thermal stations are built using thin reinforced-concrete shells. The simplest of all shell geometry is the cylindrical shell and is therefore widely employed in various structural applications.

2.5 Materials for Construction of Thin-shell Structures

Thin-walled structures such as silos and tanks are commonly used for farm storages. Several writers (Reimbert, 1976; Whitaker, 1979; Beard, 1982; Johnson, 1982; MWPS, 1983; Shelton and Harper, 2003) have discussed materials and method of construction of silos and tanks.

2.5.1 Wood

For storing of agricultural products such as food grains, woody materials continuė to be preferred by certain farmers. The reasons being that this material is universally available, strong and has high thermal insulating value. Processing of wood can be done with very simple hand tools such as axe, saws and hammers, or on an industrial scale with sophisticated machinery. The principal industrial processes to obtain lumber are extraction (logging), sawing, drying and surfacing (planing or sanding). Pressure impregnation is used to treat wood with such chemicals as preservatives or fire retardants. Production of wood-based materials usually involves glueing and pressing as in the manufacture of plywood, particleboard and glued laminated timber. In addition, wood has natural warmth and beauty that are a significant part of its overall character and at times may even be the determining factors in its selection for a particular use. Some writers gave several reasons why timber has remained a primary construction materials for thousands of years and these are that; timber is light, strong, easily worked, durable, withstands impact, not a fire hazard and attractive (Mijinyawa and Dahunsi, 1997; Salau, 1998). According to Dolby (1999) timber has maintained its position very well in agriculture and fits quite satisfactorily in the rural environment.

Plywood has been recorded as an excellent material from which construction of farm storages is carried out (Whitaker, 1979). Reimbert (1976), noted that the walls of timber silos, unless suitably reinforced, are unable to withstand the heavy stresses generated by the mass of ensiled grains. Timber silos are usually only 5 to 6 m in height.

Timber silos set up in warehouses of large area and low height are able to compete very well with the silos of reinforced concrete or steel. The limiting capacity capable of being stored in timber silos is about 1,500 to 2,000 tonnes. Wooden silo is subject to insect attack if not properly treated and is highly inflammable.

The walls of timber silos can be constructed in several ways. The four principal ways according to Reimbert (1976) are:

- Walls constructed of planks
- Walls constructed of parquet strips with reinforcing beams.
- Walls constructed of built-up wooden panels such as plywood
- Corrugated walls.

Wooden silo of corrugated pre-fabricated panels is shown in plate. 2.1.

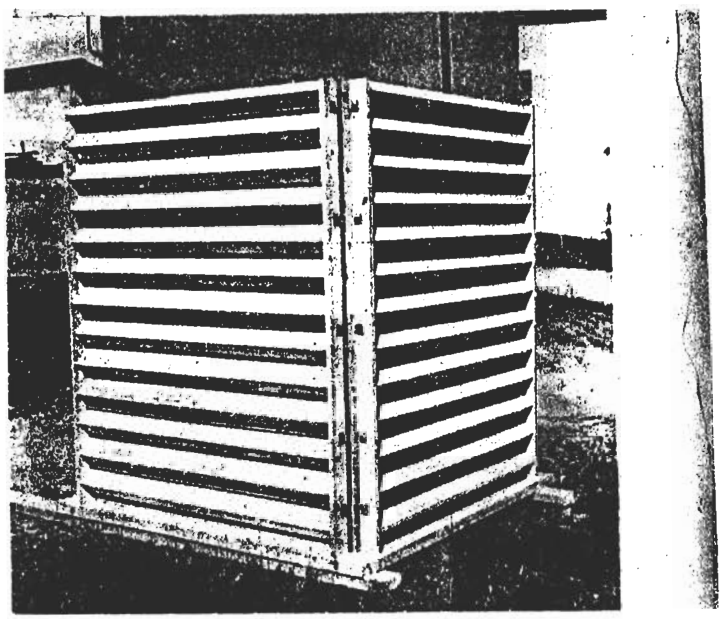


Plate 2.1: Wooden silo of corrugated pre- fabricated panels
Source: Reimbert (1976)

2.5.2 Mud

Mud is considered to be a very popular material because of its abundance and virtually free. According to Mohammed (2003) mud is used in the construction of almost all structural components (foundation, floors, walls etc.) particularly when it is mixed with clay and reinforced with dry grass to increase its binding ability. According to survey report by Mohammed (2003) the major problems identified with the use of mud was its apparent weakness, cracks easily, susceptibility to erosion due to high rainfall and low aestheticity.

2.5.3 Metal

Metal is most frequently used in forms of iron and steel. This has the advantages of not being damaged by such factors as insect attack, fungi, weathering and temperature changes. Thus the material has greater durability and strength. The disadvantage of using metal is that it is liable to corrosion, which is more severe in moist conditions, and it is very expensive and heavy (Carles, 1983). The commonest forms of this material are corrugated iron or steel sheet for roofing and walls (Bartali, 1999). These iron sheets need to be protected against corrosion using galvanization, paint, or lacquer. The main types found are steel galvanized by hot immersion, galvanized steel reacquired in oven, painted galvanized steel, and painted steel (Bartali, 1999), According to

Merritt (1976) as reported in Bartali (1999); to select paint for corrosion protection, it is necessary to begin with the function of the structure, its environment, maintenance practices, and appearance requirements. Steel silos which may be of octagonal, hexagonal or square type cells are all constructed on the same principle which usually consists of placing side by side and assembling together panels of prefabricated folded sheeting as shown in Fig. 2.5. The thickness of the sheeting may vary from 2 to 4 mm (Reimbert, 1976). The cylindrical cells are of type quite common and are constructed of steel sheet bent to a curve in the workshop and erected on site by welding (or by bolting in the case of cells which are sheltered). The helicoidal steel silos are built to contain between concentric cylinders as in Fig. 2.6, and the annular space between them constitutes the capacity for storage. This space is divided by helicoidal radial partitions forming cells.

2.5.4 Concrete

Concrete is a mixture of Portland cement, water, aggregates (coarse and fine), and some cases, admixtures. The cement and water form a paste that harden (hydrates) and bonds the aggregates together. Concrete quality is directly related to the materials used, and way that it is placed, finished and cured. According to Shelton and Harper (2003) concrete is a versatile construction material, adaptable to a wide variety of agricultural

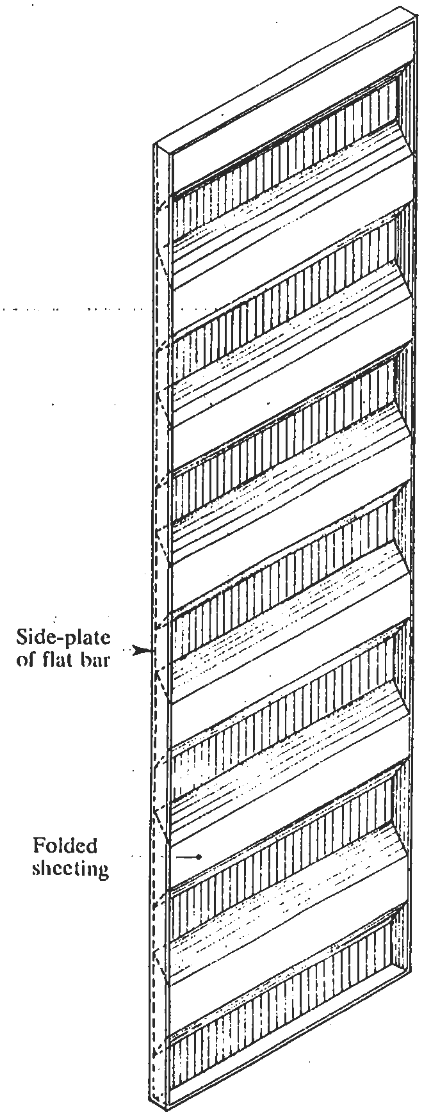


Fig 2.5: Pre-fabricated Panels
Source: Reimbert (1976)

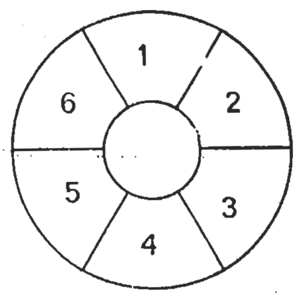


Fig 2.6: Helicoidal Steel silo
Source: Reimbert (1976)

and residential uses. With proper material and techniques, it can withstand many acids, silage, milk, manure, fertilizers, water, fire, and abrasion (MWPS, 1983). Concrete can be finished to produce surfaces ranging from glass-smooth to coarsely textured, and it can be coloured with pigment or painted.

Concrete has substantial strength in compression, but is weak in tension. Most structural uses, such as beams, slats, slabs, columns, manure tank lids and retaining walls involve reinforced concrete, which depends on concrete's strength in compression and steel's strength in tension.

Reinforced concrete is therefore among the most costly materials for farm structures. It is however, the most common material used for construction of silos and tanks. In continued efforts to introduce concrete of better quality, several investigators (Stroeven and Bui, 1997; Nehdi and Ladanchuk, 2004; Naaman et al., 2005; Montes et al., 2005) have studied various ways through which this could be achieved. The improvement on the quality of concrete is usually in terms of strength, workability, lightweight, shrinkage crack reducing, permeability reducing and/or economy. Some results are quite encouraging.

According to Reimberts (1976) the walls of reinforced concrete silos can be constructed either by assembling prefabricated units or by casting the concrete between shuttering or sliding coffering which is raised as the casting of the concrete proceeds. Silos of pre-stressed reinforced concrete have been reported. These silos are pre-fabricated and their wall are built up from cylindrical hoops superimposed and joined together by vertical pre-stressing cables, passing into holes provided for same in the pre-fabricated units (Haeger and Safarian, 1967). Construction of 200 m high natural draught cooling tower shell at Niederaussem has been reported Harte et al., 2001). This tower is said to be the largest and thinnest concrete building in the world.

2.5.5 Concrete Masonry

Walls constructed with stones, bricks, tile, cinder blocks, or concrete blocks bonded together with cement mortar are described as masonry construction. This type of construction is popular because it is durable, fire resistant, low in maintenance, and attractive in appearance. It is not affected by high humidity, termites, or most agricultural products and waste (Whitaker, 1979). However, because it is more porous and more subject to cracking than concrete, it is difficult to make it watertight. Concrete blocks are difficult to insulate (Bartali, 1999). It is obvious that brick and blockwork are strong in compression and weak in tension. Therefore, like concrete, they can be reinforced to carry tensile stresses or prestressed to eliminate them. Thus, reinforcing and prestressing widens the application, improves competitiveness and increases their potential. An early method of construction of tall silos of prestressed concrete blockwork is described by Mallagh (1982). The bins, circular in form, were built of 112 mm nominal block work and the circumferential wires stressed by pulling them together in a vertical direction alternatively upwards and downwards by a simple lever tool. Soft – wire ties then tied these wires at their displaced points together. Beard (1982) described two types of reinforced brickwork silos, one circular and deep and the other rectangular and shallow built by a small but competent or knowledge. The satisfactory performance of the silos demonstrated the effectiveness of relatively simple design requirements and construction techniques required for reinforced brickwork. The concrete masonry still remains expensive, as material is not available at every location.

2.5.6 Plastic

Plastics are synthetic materials produced by the chemical industry. They contain basic organic elements – carbon, oxygen, hydrogen and nitrogen and will flow under heat and/or pressure. American Society for Testing Materials (ASTM) defines plastics as "Materials that contain as an essential ingredient an organic substance of large Molecular Weight, are solid in their finished state, and at some stage in their manufacture or processing in finished articles can be shaped by flow" (Carles, 1983).

There are two types of plastics – thermo-plastics and thermosets. Thermoplastics soften when heated and harden when cooled. They act like metal or wax and they can be reshaped many times. Polyethylene and PVC are thermoplastics. Thermosets are similar to concrete because once they set, they cannot be made to flow again. Melamine is a typical thermoset. There are about forty different commercially produced plastic families and within the forty families. There are hundreds of plastic types (Carles, 1983). A plastic may be soft, hard, clear, heat-resistant or heavier than iron, depending on the type. Plastics are generally more resistant to environmental attacks than steel and wood. According to Carles (1983), the great range of properties available in plastics means that this basic material can meet almost any requirements, but high cost limits its wide use. Reimberts (1976) describes the moulding of silo shells using plastic material (Fig. 2.7). The material forming the wall is, in general, polyster resin reinforced with glass fibre. This is unaffected by water, sun and light. Its maintenance is nil. It is moulded in two shells by projecting it on a steel matrix. This silo is light and it can be transported and put in position in a single piece, after sticking the two shells together in the workshop by means of a glass fibre tissue. The use of silos of plastic

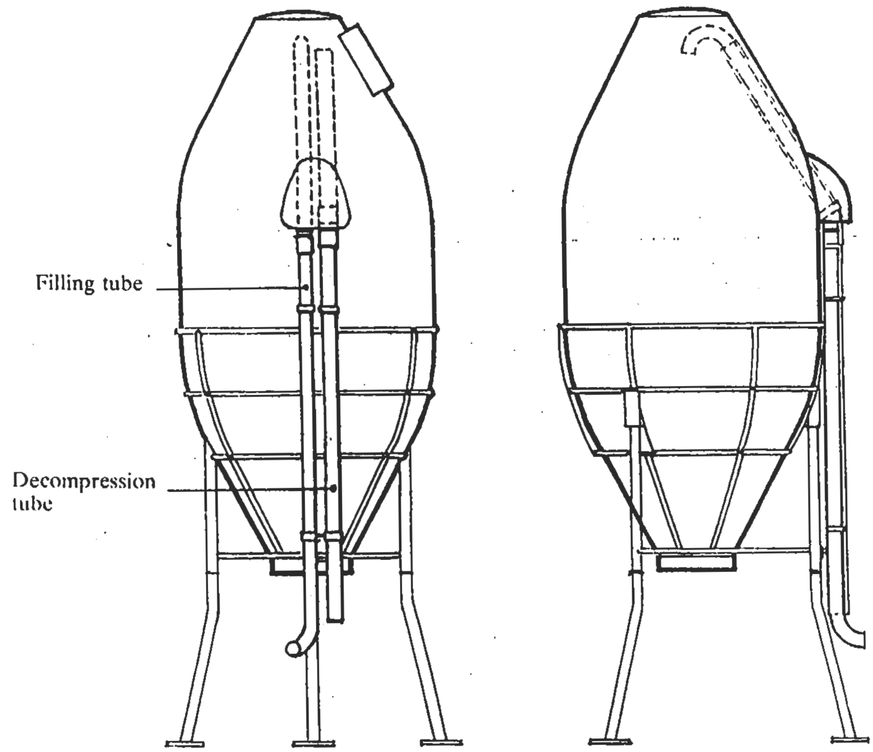


Fig. 2.7: Silo of plastic material .
Source: Reimbert (1976)

material to store water is common. Nevertheless, they are used for storages of relatively small capacities.

2.6 Methods of Analysis of shell Structures

A known problem in the study of shells is how to determine whether the analysis based on a particular theorem is adequate, satisfactory or successful. Because the analysis of a shell structure is very tedious and cumbersome, theories have been developed to simplify the design of shell structures. Some of the shell theories include; Membrane theory, Variational theory, Bending or Moment theory and Half (or Semi) moment theory.

Membrane theory: According to Zingoni et al., (2000) membrane theory is applicable to shells with finite bending rigidity but in which the moments that are developed are so small as to be negligible. In this case, the problem of stress analysis is greatly simplified, since resultants moment and resultant shearing forces varnish. Thus, the only unknowns are the three quantities N_x,N_y and $N_{xy} = N_{yx}$ and which can be determined from the conditions of equilibrium of an element of a shell. The N_x and N_y represent normal forces per unit length acting on the transverse and longitudinal sections respectively. N_{xy} is the membrane shearing forces acting tangentially on the plane sections. The problem becomes statically determinate if all the force acting on shell are known.

According to Timoshenko and Woinowsky-Krieger (1959) the forces obtained in this manner are sometimes called membrane forces, and the theory of shell based on the omission of bending stresses is called membrane theory. However, there are some limitations associated with the application of membrane theory in the analysis of shell structures. This theory according to Timoshenko and Woinowsky-Krieger (1959) fails to represent the true stresses in those portions of the a shell close to the edges, since the edge conditions usually cannot be completely satisfied by considering only membrane stresses. A similar condition in which the membrane theory is inadequate is found in cylindrical pressure vessels at the joints between the cylindrical portion and the ends of the vessel. At these joint the membrane stresses are usually accompanied by local bending stresses which are distributed symmetrically with respect to the axis of the cylinder (Timoshenko and Woinowsky-Krieger, 1959). Zingoni et al., (2000) also noted similar conditions under which membrane theory is not valid which include condition at certain locations in the interior of the shell. There exist at these locations transverse shearing force and moments to satisfy the condition of continuity. Also are conditions at the supports of the shell. Any component of the forces reaction at the support in the direction normal to the shell's middle surface would induce transverse shear in the shell, which the, membrane theory does not admit.

Variational Theory:

This theory is deficient as "bending terms" are not adequately accounted for due to approximations in the strain energy function (Muhammad, 2000).

Bending Theory:

This theory can be used in obtaining reduced (finite) set of equations, which involve no more than stress resultants and stress couples for the description of stress distribution, together with the appropriate boundary conditions. Bending or moment theory is the most accurate of all the theories but the analysis is very cumbersome as the analyst are involved in the generally very difficult task of searching for the actual particular solution corresponding to a given surface loading (Zingoni et al., 2000). But in this theory, transverse shearing force and bending terms are accounted for in the stress analysis of shell structures.

Half Moment Theory:

This theory is known as Pasternack's theory. This theory tried to simplify the moment theory. In this case, only vertical force is neglected. The half moment theory is accurate and the analysis is also simple. This theory was therefore adopted in this present study based on its above-mentioned strength.

Solving Equations of Static Equilibrium of Shell Structures:

Several researchers using various methods solve the reduced equation of static equilibrium of circular cylindrical shell structure. Ren and kuan- Chen (2001) tried to develop series solutions to the differential equations for a complete cylindrical shell under various point loads. Instead of expanding the solution series in both axial and circumferential directions, the solution obtained was only expanded in circumferential coordinates and each term in the series represent an exact solution to the set of basic equations. An exact three-dimensional elasticity solution is obtained for an infinitely long, thick transversely isotropic circular cylindrical shell panel, simply supported along the longitudinal edges and subjected to a radial patch load (Chandrashekhera and Rao, 1996). The boundary value problem is reduced to Bessel's differential equation using a set of three displacement functions. In another study, Chandrashekhera and Rao (1996) presented an approximate three-dimensional elastic solution for an infinitely cross-ply laminated circular cylindrical shell panel with simply supported boundary conditions, subjected to an arbitrary discontinuous transverse loading. The solution is based on the principal assumption that the ratio of the thickness of the lamina to its middle surface radius is negligible compared to unity. It is also shown that for very shallow panels the definition of a thin shell be based on the ratio of thickness to chord width rather than the ratio of thickness to mean radius as the case with Love's first approximation. Muhammad (2000) carried out analysis of thin cylindrical shell subjected to local and continuous axi-symmetric loads using Pasternack's equations formulated on the basis of theory of shell of revolutions. The reduced differential equation was solved using classical and Initial-value methods. The results obtained in each of the method are the same indicating the applicability of either of the two methods in solving equation of static equilibrium of cylindrical shell. Initial value method has been applied in the analysis of circular cylindrical shell under hydrostatic pressure and ring forces (Amadou, 2002).

The verification of the finite element analysis technique was carried out by Song et al., (2002). Failure analysis of reinforced concrete slabs loads were performed and numerical results were compared with experimental data. In addition, reinforced concrete dome structures designed with reinforcement ratios were analyzed to check the applicability of the technique. Their results showed that the technique could be applied effectively to failure analysis of various types of reinforced concrete shell structures. Shi and Ohtsu (2001) had applied Linear programming to the limit analysis of conical-shaped steel tanks under hydrostatic pressure. The comparison of this with the experimental and numerical results available in literature indicates that the linear programming approach is quite high, with respect to the limit loads and failure modes of the conical shell structures (Shi and Ohtsu, 2001). The linear programming also eliminated the overestimation of stiffness for thin plates of shells resulting from inadequate constitutive equations often encountered in the non-linear finite-element analysis for thin-walled panels or shells. For better prediction of the actual buckling load of a shell structure, a new methodology involving the proper combination of eigen-value buckling analysis and geometric nonlinear analysis has been reported (Bagchi and Paramasivam, 1996). This method is computationally cheaper than nonlinear buckling analysis and more reliable than linear buckling analysis (Bagchi and Paramasivam, 1996).

2.6.1 Imperfection as it affects Buckling of Silos and Tanks

The strength of thin-walled cylindrical shell structures is highly dependent on the nature and magnitude of imperfections. Circumferential imperfections have been reported to have an especially detrimental effect on the buckling resistance of these shells under axial load (Pircher and Bridge, 2001). Due to the manufacturing techniques commonly used during the erection of silos and tanks, particularly of steel type, specific imperfections are introduced into these structures, among them circumferential weld-induced imperfections between strakes of steel plates. Pircher and Bridge (2001) studied several factors that influence the buckling of silos and tanks using the finite-element method. They and susceptibility to volume changes, which describe the physical behaviour of laterite, are functions of the particle size distribution and the amount of fines that pass through the 75µm sieve. Generally, when water is added to a lateritic soil mass, films of observed water cover the soil particles. The thickness of the water covering the particles increases with increase in water moisture content of the soil. Over a certain range of moisture, the particles slide over each other. The resistance of the soil to this shear action depends on the presence of various particles sizes in the soil (Ahire, 2001)

2.7.1 Engineering Problems of Lateritic Soils

The behaviour of soil mass depends on the amount of water present in its system, the chemical and mineral composition, sizes and shape of the soil particles.

Lateritic soils vary in their characteristic from one location to other. However, general geotechnical analysis by a scientist (Gidigasu, 1976) have indicated that laterite, irrespective of their locations or modes of formation do have some engineering problems, among them are outlined as follows:

- Poor permeability and sub-drain
- High compressibility
- Shear failure

- Easy settlement
- Cohesion and held-water
- Susceptibility to volume changes
- Crushing and weathering

Lateritic soils have high clay contents, which make them, behave partially like cohesive soils. The compatibility of the laterite soils depends on the internal friction of the soil particles, the degree of interlocking between the particles and the amount of fines to act as binders.

2.7.2 The Application of Laterite in Building Construction

Laterite has featured in building construction works for a very long time in Nigeria. The majority of houses in rural areas in use today were built of lateritic soils. The practice varies depending on the location and related local problems as well as physical properties of the lateritic soils. Studies have been made to determine the usefulness of lateritic soils in the building and allied industries. The properties of lateritic soils that will influence its rate and ease of working include its degree of finess, density, particle shape, stickiness, chemical stability and chemical composition (Brand and Hangenoi, 1969; Newill and Dowling, 1964). Stabilization with cement, lime, bitumen, etc. have been found by Ola (1985) to be effective means of improving engineering properties of lateritic soils for road construction and low cost housing. In house construction, the two most important methods are the rammed earth and the masonry units approaches. In the case of rammed earth construction, the soil is rammed into shape in-situ and large shrinkage occurs during the drying out period. This initial drying shrinkage has been found to be usually larger than any subsequent moisture movement.

The use of mould, which may be wooden or metal type of known dimension, has been introduced to standardize the size. The soil is "rammed" by hand and the units are allowed to cure before being used. Lasisi and Osunade (1984) employed this method to study the strength characteristics of cubes made from unstabilized lateritic soils. The results indicate that the finer the grain sizes of lateritic soil the higher the compressive strength. It was reported also in the same study that the formation probably constitutes one of the factors determining strength, as well as the source from which the laterite soil are obtained.

Masonry units (burnt or unburnt) have been in use since the time of the colonial administration in Nigeria. These units were solid bricks, fired and more than not made from lateritic soils and any other manageable earth materials.

2.8 Mix Design for Concrete Materials

The problem is the formulation of an optimal model for laterized concrete mix proportioning that has advantages over other alternatives methods. This model must possess the assets of economy and ease of reformulation as well as an inherent cognizance of the effect of the variation in the source of ingredients of concrete on the strength of concrete produced. Before considering different approaches to mix design, it is necessary to differentiate between the various purposes for which a design may be required as one or more of the following:

- Approximate design
- Safe design for instant use
- Accurate design for safety and economy
- Comparisons of performance and economy of different materials or mixes.

The best method to suit any of these purposes may be found as one or as a combination of the following:

- Laboratory trial mix (es)
- Full-scale trail mix (es)
- Previous production data bank
- 'Ready-to-use' standard data
- Simplified methods of design
- 'Comprehensive' method of design

The aspects of material of construction (concrete or laterized concrete) to be covered by the design may include a selection of the

According to (Murdock et al., 1991), for mix design purposes, relationships between strength and water-cement ratio or cement content are usually based on average strength, so that information is required on how to estimate the average strength to be targeted for any given specified strength ($f_{cv,}$).

Statistical theory and concreting practice have combine to confirm that the proportion of results expected to lie below the characteristic strength is related to the margin when expressed as a multiple, K, of the standard deviation as shown in Table 2.5. Thus, the minimum margin is 1.64 x standard deviation when the specified, maximum percentage is 5 percent. Compliance requirements for strength are usually set at a level to discourage the use of low values for K. In the UK, practice values of K of less than 2 would not normally be adopted. Similarly, in USA, although K can be as low as 1.28, a value of 1.64 or higher would be recommended (Murdock et al., 1991).

The QSRMC (1989) defines the Target mean strength (T) as follows:

$T \geq f_{cv} + KSy$ for values of T = 27 N/mm^2 and above

or $T \geq \dfrac{f_{cv}}{1+\dfrac{KS_y}{27}}$ for values or T below 27 N/mm^2

Table 2.5: Maximum Percentage of result below the Characteristic Strength level and the Margin

Maximum Percentage of result below the Characteristic Strength level ($f$cv)	Minimum value of K
1.28	1.28
1.64	1.64
2.00	2.00
2.33	2.33
2.58	2.58
3.00	3.00

Source: Murdock et al., (1991)

where, K is the factor
S_y is standard deviation
f_{cv} is the characteristic strength

These formulae assume standard deviation increases uniformly with strength level until a mean of 27 N/mm^2 is reached and is constant above that level. The minimum design margin, M (K_{sy}), is not normally permitted to be less than 7 N/mm^2 or K less than 2. When test rates are low, the lowest permitted values for M and K rise to 14 N/mm^2 and 3 respectively (QSRMC, 1989).

2.8.2 Methods of Mix Design

Simplified mix design – The Department of Environment (DOE) method: According to DOE (1988), this provides for designs using

Portland cement to BS 12 and also includes modifications allowing for the use of air entrainment. In this method, a comprehensive set of graphs is provided enabling the proportion of fine to total aggregate to be estimated, which is dependent on maximum aggregate size, free w/c, workability and fine aggregate grading (percent passing 600μm).

Trial Mix

There are a number of factors to keep in mind when preparing for, and making, trial mixes.

- All materials should be as representative as possible of those to be used in the construction.
- Preferably materials should be used in the same condition as those to be used in the construction, i.e. usually wet, in the case of aggregates.
- Trial mixes will lose workability due to evaporation (and absorption, if the aggregates are dried) at a much faster rate than large batches use in construction.
- Interpolation between the results of two trial batches is preferable to extrapolation from a single trial batch.
- Results from a trail batch are only representative of the qualities of the material used in the batch.

It is necessary to consider making allowances for normal variations in larger-scale concrete production. It is important to treat the estimated water content from the 'paper' mix design only as a guide to the amount needed for the required workability.

When trial mixes are made it is essential to judge whether the cohesion is adequate for the intended purpose. If the mix has a tendency to segregate, the mix should be redesigned with a higher proportion of fine aggregate or, if this does not solve the problem, by use of a finer sand, a plasticizer, an increased cement content or a change of cementitious material. Slight over-cohesion is essential to take account of variations, which may occur, in full-scale production but if the mix is excessively cohesive, the proportion of fine aggregate should be reduced.

Quantities of materials per cubic metre can be calculated from the mix proportions used in trial mixes and the plastic density determined as follows:

Cement content, (C) = (C) (kg) x scale factor

Where the scale factor is $\dfrac{\text{(PD)}}{\text{Total trial mix weight (kg)}}$

(C) is the trial mix weight of cement and

(PD) is the plastic density.

Similarly, for the other materials (i.e. fine and coarse aggregate).

Ready-To-Use Mix Design:

BS 5328 caters for safe designs for instant use for site mixing by the provision of standard mixes (see Table 2.6)

Table 2.6: Mix Proportions for Standard Mixes

Standard Mix	Nominal Maximum Size of aggregate (mm)	40 20
	Slump (mm)	75 125 75
ST1	<u>Constitution</u> Cement Total aggregate	(kg) 180 200 210 230 (kg) 2010 1950 1940
ST2	Cement Total aggregate	(kg) 210 230 240 260 (kg) 1980 1920 1920 1860
ST3	Cement Total aggregate	(kg) 200 260 270 300 (kg) 1950 1900 1890 1820
ST4	Cement Total aggregate	(kg) 280 300 300 330 (kg) 1920 1860 1860 1860
ST5	Cement Total aggregate	(kg) 320 340 340 370 (kg) 1890 1830 1830 1770

Source: BS 5328 as reported by Murdock et al., (1991)

Comprehensive Mix Design:

The British Ready Mixed Concrete Association has developed BRMCA method of concrete mix design over many years. It has been adopted throughout the industry and has been used in the production of several hundred million cubic metres of concrete (Dewar, 1986). The major stages in the method are as shown in Fig. 2.8.

A key feature of the method is that in stage II, mixes are designed for optimum performance in the plastic state to ensure that the concrete is suitable for transport, handling, compacting and finishing. Only when the mixes have been designed to achieve these properties should the performance of the mixes be determined in stage III. The analysis and recording of trial mix and strength test data in stage IV are based on cement content as the primary mix pas well as between cement content and the weight of each constituent in the mix.

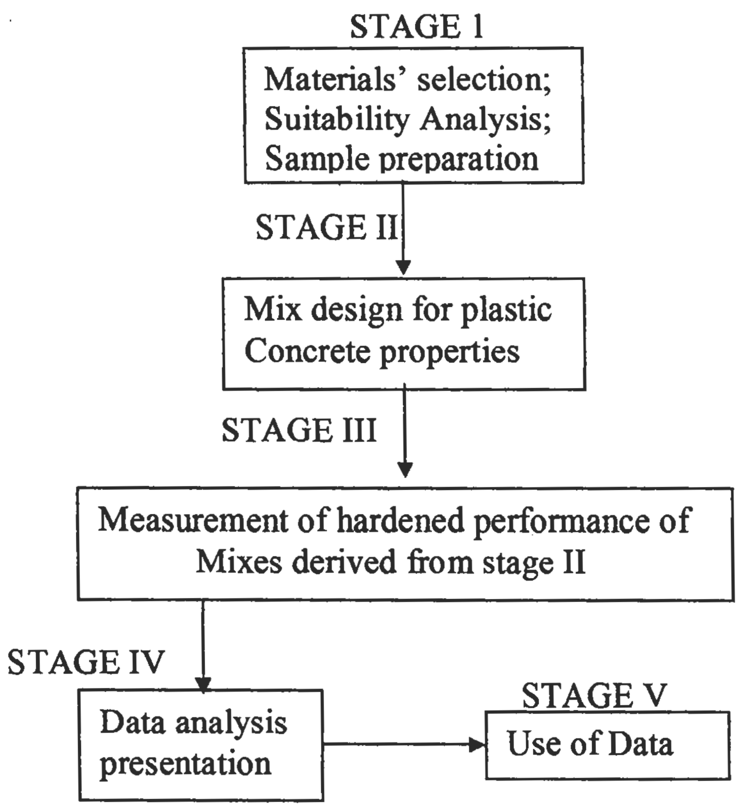


Fig. 2.8 Major Stages of BRMCA method of Mix Design
Source: Murdock et al., (1991)

Conversion from Proportions by Weight to Proportions by Volume:

It is always preferable for aggregates and cement to be batched by weight, even when the quantities are small. Portable weigh-batching equipment is usually available for hire.

However, in the few instances where this is not possible and, only then for low quality applications, the cement may be batched by whole (usually 50 kg) bags and the aggregates by volume. The conversion from weight proportions to volume proportions involves knowledge of the weight per m^3 of the fine and coarse aggregates. The values for the aggregates may be obtained by experiment. Once the proportions by weight have been established, equivalent proportions of aggregate by volume may be calculated as given in the following example by Murdock et al (1991):

(1) Weight of material: Kg	Cement 50	fine aggregate 100	Coarse aggregate 170
(2) By volume dry to 50 kg cement	50	$\frac{100}{1600} = 0.063\,m^3$	$\frac{170}{1440} = 0.063\,m^3$
(3) Add for bulking		+20% = $0.013\ m^3$	
(4) Proportion of damp aggregate to 50 kg	50	$0.076\ m^3$	$0.118\ m^3$

Weights per m^3 of fine and coarse aggregates are taken as 1600 and 1440 kg/m^3 respectively.

Purpose-made containers, of a suitable size for handling when full, can be prepared to enable the volumes of each aggregate to be gauged accurately.

Scheffe's Simplex-Lattice Design

The most common simplex lattice designs are proposed by Scheffe (1958) and Zedginidze (1971). These designs provide a uniform scatter of points over the (q – 1)-simplex. The points form a (q,n)-lattice on the simplex where 'q' is the number of mixture components, 'n' is the degree of polynomial. This method has been employed in formulation of an optimal model for concrete mix proportioning that has advantages over other alternative methods. Nwakonobi et al., (1998) adopted Scheffe's simplex design employing second order polynomial for quaternary mixtures to formulate a mathematical model for mix proportioning of Clay-Ricehusk-Cement mixture. The optimal model developed is a predictor of strength, given any mix proportions of the constituent materials and alternatively, strengths related to various mixes can be predicted using the model.

CHAPTER THREE

ANALYSIS OF CYLINDRICAL SHELL BY PASTERNACK'S THEORY

3.1 Derivation of Differential Equation of Equilibrium of Cylindrical Shell

The simplest of all shell geometry is that of the circular cylindrical shell shown in Fig. 3.1 below.

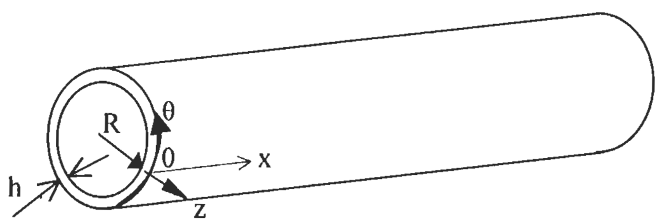


Fig. 3.1: Circular Cylindrical Shell

3.1.1 Assumptions and their Implications

In shell theory, all the assumptions used in plate theory also apply and are given as follows:

1. Points of the shell lying initially on a normal to the middle plane of the shell;

   (a) remain on the normal to the middle plane after deformation;

   (b) undergo at most a translation and rotation with respect to the original co-ordinate system.

2. A shell resists lateral and in-plane loads by bending, transverse shear stresses, and in-plane action.
3. A linear element through the thickness does not elongate or contract and remain straight upon load application.

In addition, there is another condition known as Love's first approximation which is consistent with the neglect of the transverse shear deformation and is written as:

4. $\frac{h}{R} \ll 1$, where h is the thickness of the shell and R its radius of curvature.

Assumption (3) requires that the strain in Z-direction is zero; $\epsilon_z = 0$. It also implies that for any Z, the shear strains in XZ plane (γ_{xz}) and YZ plane (γ_{yz}) at any specific location (X,Y) on the middle surface are constant.

Assumption (1a) specifies that $\gamma_{yz} = \gamma_{xz} = 0$. Assumption (2) means that $\sigma_z = 0$ in the stress-strain relations.

3.1.2 Forces and Moments acting on the Shell

Consider an element cut out from the cylindrical shell by two adjacent axial sections and two adjacent transverse sections. Fig. 3.2 shows the corresponding element with the middle surface of the shell.

In Fig.3.2a, N_x and N_θ represent normal forces per unit length acting on the transverse and longitudinal sections respectively, while Q_x and Q_θ are shear forces per unit length acting transversely (out of plane) on the two sections. $N_{x\theta}$ and $N_{\theta x}$ are membrane-shearing forces on the two sections acting tangentially (in-plane).

Fig. 3.2b shows the moments per unit length. M_x is the longitudinal moment acting on the transverse section. The transverse moment M_θ acts on the longitudinal section. $M_{x\theta}$ and $M_{\theta x}$ are the twisting moments acting along the longitudinal and transverse sections respectively.

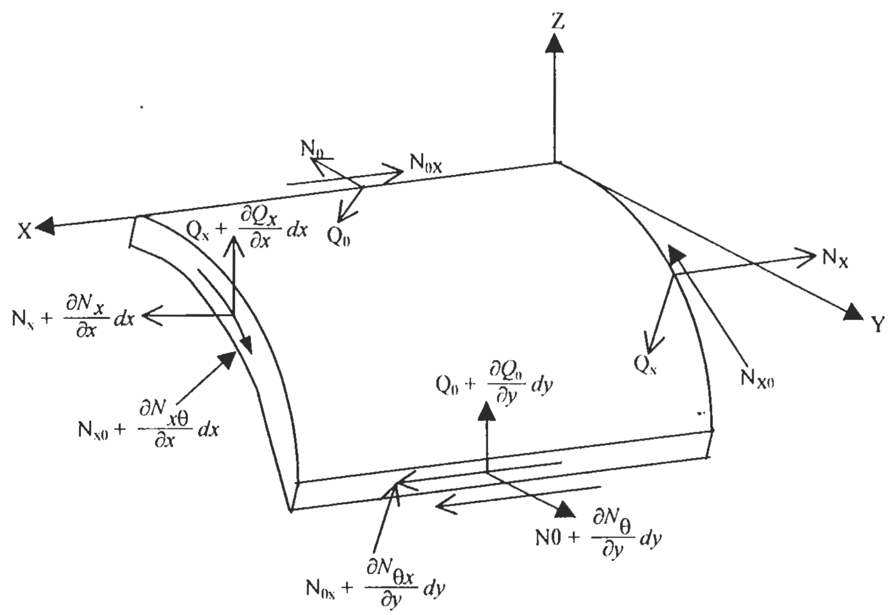


Fig. 3.2a: Direct and Shear Forces in an Element of a Shell

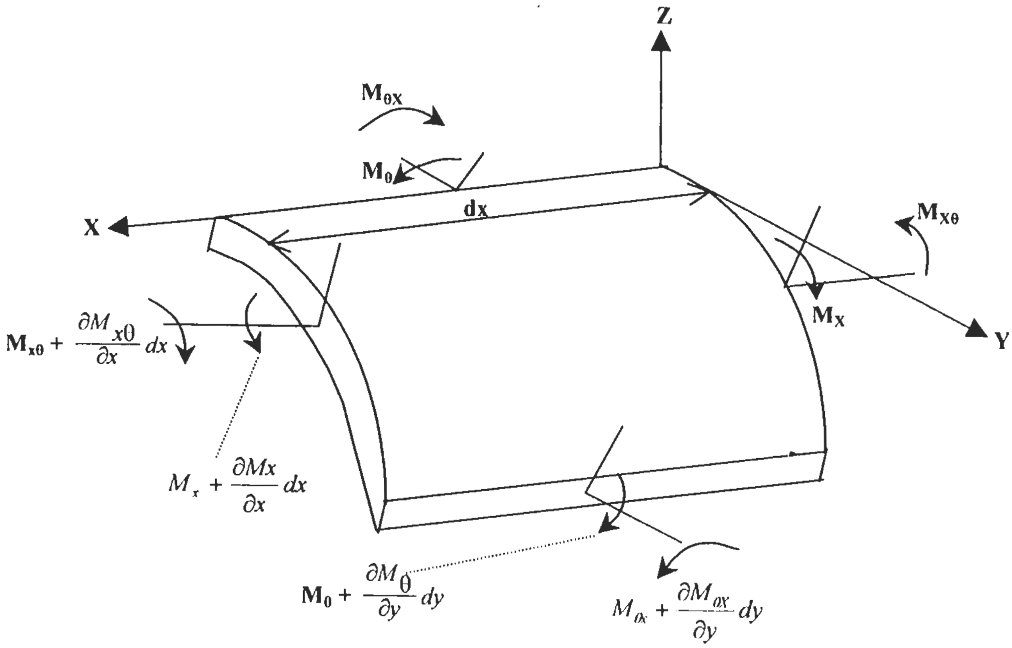


Fig. 3.2b:Bending and twisting Moments in an Element of a shell

3.1.3 Stress Equilibrium Equations

We take the x,y,z-axes at a given point od the middle surface to be the direction to the axs of the cylinder, the tangent to the circumference, and the normal to the middle surface of the shell respectively. Let us assume a transverse distributed load whose components in x, y, and z directions are X, Y, and Z respectively, R is the radius o f the cylinder dx and dy are the sizes of the element in x and y directions

Projecting all forces in x-direction, we obtain:

$$\left[N_x + \frac{\partial N_x}{\partial x} dx\right].R.d\theta - N_x.R.d\theta + \left[N_{\theta x} + \frac{\partial N_{\theta x}}{\partial y} dy\right].dx$$

$$-N_{\theta}xdx + X.dx.R.d\theta = 0$$

Expanding the above expression and noting that $dy = R\tan d\theta \approx Rd\theta$, we get

$$\frac{\partial N_x}{\partial x}.dx\,.\mathrm{R.d\theta} + \frac{\partial N_{\theta x}}{\partial y}\mathrm{R.d\theta\ dx} + \mathrm{X.dx.R.d\theta} = 0$$

Divide all through by dx.R.dθ to get:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{\theta x}}{\partial y} + \mathrm{X} = 0 \quad\text{.........................(3.1)}$$

Projecting all forces in y-direction to obtain:

$$\left[N_\theta + \frac{\partial N_\theta}{\partial y}dy\right]\mathrm{dx.cos\ d\theta}\ - \mathrm{N_\theta dx} + \left[Q_\theta + \frac{\partial Q_\theta}{\partial y}dy\right]\mathrm{.dx.sin\ d\theta} +$$

$$\left[N_{x\theta} + \frac{\partial N_{x\theta}}{\partial x}dx\right]\mathrm{.R.d\theta} - \mathrm{N_{x\theta}.R.d\theta} + \mathrm{Y.R.d\theta.dx} = 0$$

Expanding the above expression and noting that, for small angles, tan dθ ≈ sin dθ ≈ dθ and cos dθ ≈ 1, we get:

$$\frac{\partial N_\theta}{\partial y}\mathrm{.R.d\theta.dx} + \frac{\partial Q_\theta}{\partial y}\mathrm{.R.d\theta.dx.d\theta} + \mathrm{Q_\theta.dx.d\theta} + \frac{\partial N_{x\theta}}{\partial x}\mathrm{.R.\ dx.d\theta} +$$

$$\mathrm{Y.R.d\theta.dx} = 0$$

But, $\frac{\partial Q_\theta}{\partial y}\mathrm{.R.d\theta.dx.d\theta} \approx 0$

After further simplification by dividing all through by R.dθ.dx, we obtain:

$$\frac{\partial N_\theta}{\partial y} + \frac{Q_\theta}{R} + \frac{\partial N_{x\theta}}{\partial x} + \mathrm{Y} = 0 \quad\text{......................(3.2)}$$

Projecting all forces along Z-axis to get:

$$\mathrm{Z.R.d\theta.dx} + \left[Q_x + \frac{\partial Q_x}{\partial x}dx\right]\mathrm{.\ R.d\theta.cos.}\frac{d\theta}{2} - \mathrm{Q_x.R.d\theta\ cos.}\frac{d\theta}{2}$$

$$+\left[Q_\theta+\frac{\partial Q_\theta}{\partial y}dy\right].dx.\cos d\theta - Q_\theta.dx - \left[N_\theta+\frac{\partial N_\theta}{\partial y}dy\right].dx\sin d\theta = 0$$

Noting that tan $d\theta \approx$ sin $d\theta$ and cos $d\theta \approx \cos.\frac{d\theta}{2} \approx 1$ for small angle $d\theta$

and neglecting third order terms, to obtain:

$$Z+\frac{\partial Q_x}{\partial x}+\frac{\partial Q_\theta}{\partial y}-\frac{N_\theta}{R}=0 \qquad (3.3)$$

Considering the moment equilibrium with respect to **x-axis results in:**

$$\left[M_\theta+\frac{\partial M_\theta}{\partial y}dy\right]dx - M_\theta dx + \left[M_{x\theta}+\frac{\partial M_{x\theta}}{\partial x}dx\right].R.d\theta$$

$$- M_{x\theta}.R.d\theta - \left[Q_\theta+\frac{\partial Q_\theta}{\partial y}dy\right].dx.R.d\theta = 0$$

After further simplification gives:

$$\frac{\partial M_\theta}{\partial y}+\frac{\partial M_{x\theta}}{\partial x} - Q_\theta = 0 \qquad (3.4)$$

Considering the moment equilibrium with respect to the y-axis, to get:

$$\left[M_x+\frac{\partial M_x}{\partial x}dx\right].R.d\theta - M_x.R.d\theta + \left[M_{\theta x}+\frac{\partial M_{\theta x}}{\partial y}dy\right].dx$$

$$- M_{\theta x}.dx - \left[Q_x+\frac{\partial Q_x}{\partial x}dx\right].R.d\theta.dx = 0$$

Expanding the above expression and neglecting third order terms to get:

$$\frac{\partial M_x}{\partial x}+\frac{\partial M_{\theta x}}{\partial y} - Q_x = 0 \qquad (3.5)$$

In semi-moment theory as applied to cylindrical shell, twisting moments are usually neglected. We can thus write:

$$M_{\theta x} = M_{x\theta} = 0 \qquad (3.6)$$

Based on the law of reciprocity of shearing stresses, we have:

$$N_{\theta x} = N_{x\theta} = S \qquad (3.7)$$

Substitution of equation (3.6) into equations (3.4) and (3.5) gives

$$\frac{\partial M_\theta}{\partial y} = Q_\theta \qquad (3.8)$$

$$\frac{\partial M_x}{\partial x} = Q_x \qquad (3.9)$$

Sustitution of equations (3.7) , (3.8), and (3.9) into equations (3.1), (3.2), and (3.3) to obtain:

$$\frac{\partial N_x}{\partial x} + \frac{\partial S}{\partial y} + X = 0 \qquad (3.10)$$

$$\frac{\partial M_\theta}{R\partial y} + \frac{\partial N_\theta}{\partial y} + \frac{\partial S}{\partial x} + Y = 0 \qquad (3.11)$$

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_\theta}{\partial y^2} + \frac{N_\theta}{R} + Z = 0 \qquad (3.12)$$

Equations (3.10) through (3.12) represent the equations of static equilibrium of a cylindrical shell under distributed forces of components X, Y, and Z in x, y and z directions respectively. It can be seen that there are five unknowns for only three equations rendering the systems statically indeterminate.

In order to solve the system, we need the following stress-strain relations and elasticity equations to facilitate the determination of the unknowns namely, N_x, N_0, S, M_0, M_x:

$$N_x = \frac{Eh}{1-\mu^2}(\varepsilon_x + \mu\varepsilon_0) \qquad (3.13)$$

$$N_0 = \frac{Eh}{1-\mu^2}(\varepsilon_0 + \mu\varepsilon_x) \qquad (3.14)$$

$$S = Gh\gamma_{x0} \qquad (3.15)$$

$$M_x = -D(X_x - \mu X_0) \qquad (3.16)$$

$$M_0 = D(X_0 - \mu X_x) \qquad (3.17)$$

where:

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad (3.18)$$

$$\varepsilon_0 = \frac{\partial v}{\partial y} + \frac{w}{R} \qquad (3.19)$$

$$X_x = \frac{\partial^2 w}{\partial x^2} \qquad (3.20)$$

$$X_0 = \frac{\partial}{\partial y}\left(\frac{v}{R} - \frac{\partial w}{\partial y}\right) \qquad (3.21)$$

$$\gamma_{x0} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \qquad (3.22)$$

$$G = \frac{E}{2(1-\mu)} \qquad (3.23)$$

u = displacement component along the x-axis

v = displacement component along the y-axis

w = displacement component along the z-axis

h = thickness of the shell

D = $\frac{Eh^3}{12(1-\mu^2)}$ is the flexural rigidity of the shell

μ = Poisson's ratio

E = Young Modulus of the material used.

3.1.4 Forces and Moments – Displacements Equations

Substituting equations (3.18) through (3.23) into equations (3.13) through (3.17) gives the stress-resultants and stress-couples in terms of the displacements u, v, and w

$$N_x = \frac{Eh}{1-\mu^2}\left[\frac{\partial u}{\partial x} + \mu\left(\frac{\partial v}{\partial y} + \frac{w}{R}\right)\right] \quad (3.24)$$

$$N_\theta = \frac{Eh}{1-\mu^2}\left[\frac{\partial v}{\partial y} + \frac{w}{R} + \mu\frac{\partial u}{\partial x}\right] \quad (3.25)$$

$$S = \frac{Eh}{2(1-\mu)}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right] \quad (3.26)$$

$$M_x = -D\left[\frac{\partial^2 w}{\partial x^2} - \mu\frac{\partial}{\partial y}\left(\frac{v}{R} - \frac{\partial w}{\partial y}\right)\right] \quad (3.27)$$

$$M_\theta = D\left[\frac{\partial}{\partial y}\left(\frac{v}{R} - \frac{\partial w}{\partial y}\right) - \mu\frac{\partial^2 w}{\partial x^2}\right] \quad (3.28)$$

Substitution of above expressions into equations (3.10), (3.11), and (3.12) gives

$$\frac{\partial^2 u}{\partial x^2}+\frac{1-\mu}{2}.\frac{\partial^2 u}{\partial y^2}+\frac{1+\mu}{2}.\frac{\partial^2 v}{\partial x \partial y}+$$

$$\frac{u}{R}.\frac{\partial w}{\partial x}+\frac{1-\mu^2}{Eh}.X=0 \quad (3.29)$$

$$\frac{1+\mu}{2}.\frac{\partial^2 u}{\partial x \partial y}+\frac{1-\mu}{2}.\frac{\partial^2 v}{\partial x^2}+.\frac{\partial^2 v}{\partial y^2}+\frac{\partial}{\partial y}\left(\frac{w}{R}\right)+\frac{h^2}{12R}.$$

$$\left[\frac{\partial^2}{\partial y^2}\left(\frac{v}{R}\right)-\frac{\partial^3 w}{\partial y^3}-\mu\frac{\partial^3 w}{\partial x^2 \partial y}\right]+\frac{1-\mu^2}{Eh}.Y=0 \quad (3.30)$$

$$\frac{\partial^4 w}{\partial x^4}+\frac{\partial}{\partial y}\left[\frac{u}{R}.\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2}{\partial y^2}\left(\frac{v}{R}\right)-\frac{\partial^4 w}{\partial y^4}\right]-2\mu.\frac{\partial^4 w}{\partial x^2 \partial y^2}$$

$$-\frac{12}{Rh^2}\left[\frac{\partial v}{\partial y}+\frac{w}{R}+\mu\frac{\partial u}{\partial x}\right]+\frac{12(1-\mu^2)}{Eh^3}.Z=0 \quad (3.31)$$

From the Pasternack's theory on cylindrical shell, the following assumptions are made:

(1) The middle plane of the shell is inextensible in y-direction such that

$v = 0$

(2) It is assumed that stresses generated by hydrostatic or granular forces do not depend on y but only on the length x.

(3) The normal force N_x acting on the transverse section of the shell is neglected.

Consequently all derivatives with respect to y are set to be zero.

It should be noted that, because forces, moments and displacements are function of only x, total derivatives are used from now in the different equations.

In case of hydrostatic pressure, the external forces acting on the cylindrical shell are such that X = Y = 0 and Z = γx; where γ stands for the weight per unit volume of water.

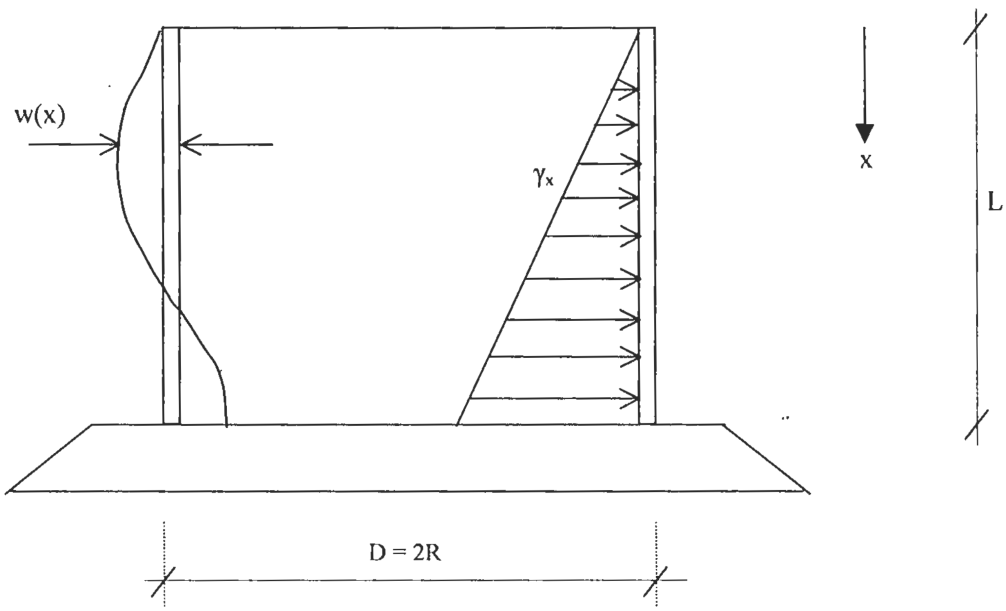


Fig. 3.3: Section of Circular Cylindrical Shell

Applying the above assumptions to equations (3.24) through (3.28) to get:

$$N_x = \frac{Eh}{1-\mu^2}\left[\frac{du}{dx} + \mu.\frac{w}{R}\right] = 0 \Rightarrow \frac{du}{dx} = u' = -\mu\frac{w}{R} \qquad (3.32)$$

$$N_\theta = \frac{Eh}{1-\mu^2}\left[\frac{w}{R} + \mu.\frac{du}{dx}\right] = \frac{Eh}{1-\mu^2}\left[\frac{w}{R} - \mu^2.\frac{w}{R}\right] \Rightarrow N_\theta = \frac{Ehw}{R} \qquad (3.33)$$

$$S = \frac{Eh}{2(1+\mu)}[0+0] \Rightarrow S = 0 \qquad (3.34)$$

$$M_x = M = -Dw'' \quad \text{...(3.35)}$$

$$M_\theta = -\mu Dw'' \Rightarrow M_\theta = \mu M \quad \text{..................................(3.36)}$$

where, $w'' = \dfrac{d^2w}{dx^2}$

3.1.5 Differential Equation of Equilibrium

Making use of the previous results, it can be seen that equations (3.29) and (3.30) are satisfied.

Substitute these same results into equation (3.31) to get:

$$\frac{d^4w}{dx^4} + 4\alpha^4 w = \frac{\gamma x}{D}$$

which can be written as:

$$W^{iv} + 4\alpha^4 W = \frac{\gamma}{D}x \quad \text{.................................(3.37)}$$

where:

$$W^{iv} = \frac{d^4w}{dx^4}$$

$$\alpha^4 = \frac{3(1-\mu^2)}{R^2h^2}$$

$$D = \frac{Eh^3}{12(1-\mu^2)}$$

Equation (3.37) is due to Pasternack and is only applicable to cylindrical shell subject to axi-symmetric loading ie. the forces and moments do not vary along the circumferential section. It is the differential equation for cylindrical shell.

3.2 General Solution of Static Equilibrium

3.2.1 A Case of Circular Cylindrical Shell subject to Hydrostatic Pressure

Consider the following vertical circular cylinder under hydrostatic forces

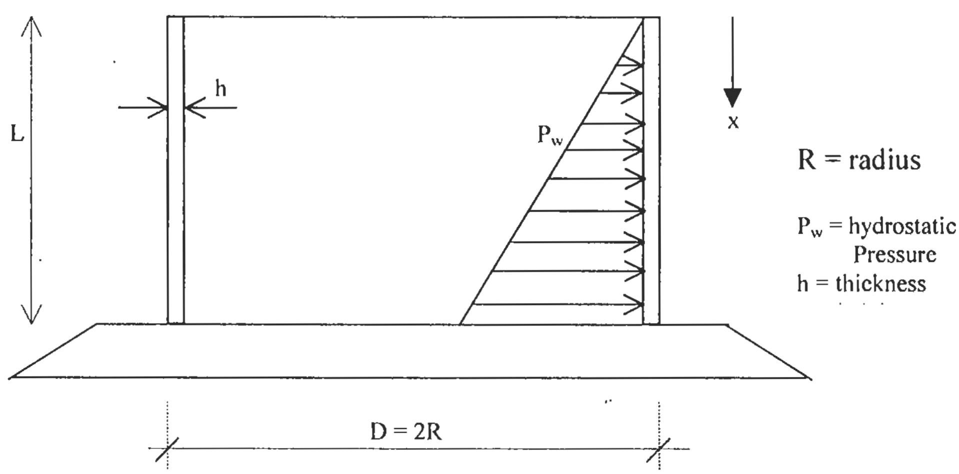


Fig. 3.4: Hydrostatic Pressure Distribution

Recalling that:

$P_w = \gamma.x$, where γ is the weight per unit volume of water

Also recall equation (3.37)

$$W^{iv} + 4\alpha^4 W = \frac{\gamma}{D}x \quad \ldots\ldots(3.38)$$

where

$$\alpha^4 = \frac{3(1-\mu^2)}{R^2h^2} \text{ and } D = \frac{Eh^3}{12(1-\mu^2)}$$

3.2.1.1 Classical Solution

3.2.1.1.1 The Homogeneous Solution

The homogeneous equation of static equilibrium is given as:

$$W^{iv} + 4\alpha^4 W = 0 \qquad (3.39)$$

Assume the solution to be in the form of:

$$W(x) = e^{\lambda x} \qquad (3.40)$$

where λ is constant.

Substitution of equation (3.40) into (3.39) gives:

$$\lambda^4 + 4\alpha^4 = 0 \qquad (3.41)$$

$$\text{Let } r = \lambda^2 \qquad (3.42)$$

$$\text{Thus: } r^2 = -4\alpha^4 \qquad (3.43)$$

which implies that;

$$r = \pm 2\alpha^2 i \qquad (3.44)$$

solving equation (3.44) as required by equation (3.42) to get four values of λ:

$$\lambda_1 = \sqrt{2\alpha^2 i} \qquad (3.45a)$$

$$\lambda_2 = -\sqrt{2\alpha^2 i} \qquad (3.45b)$$

$$\lambda_3 = \sqrt{-2\alpha^2 i} \qquad (3.45c)$$

$$\lambda_4 = -\sqrt{-2\alpha^2 i} \qquad (3.45d)$$

Given a complex number a + ib, let us solve for its root u + iv

$$\sqrt{a + ib} = u + iv \qquad (3.46)$$

Squaring equation (3.46) to obtain:

$$a + ib = (u^2 - v^2) + 2uvi \qquad (3.47)$$

Comparing the right hand and left hand complex number (ie. equating term):

$$\left.\begin{array}{l} u^2 - v^2 = a \\ 2uv = b \end{array}\right\} \qquad (3.48)$$

Solving equation (3.48) yields:

$$u = \pm\left[\frac{1}{2}\sqrt{a^2 + b^2} + \frac{a}{2}\right]^{\frac{1}{2}} \qquad (3.49)$$

$$v = \pm\left[\frac{1}{2}\sqrt{a^2 + b^2} + \frac{a}{2}\right]^{\frac{1}{2}} \qquad (3.50)$$

Equations (3.49) and (3.50) together with equation (3.45a) through (3.45d) yield:

$$\lambda_1 = \alpha + i\alpha \qquad (3.51a)$$

$$\lambda_2 = \alpha - i\alpha \qquad (3.51b)$$

$$\lambda_3 = -\alpha - i\alpha \qquad (3.51c)$$

$$\lambda_4 = -\alpha + i\alpha \qquad (3.51d)$$

Substitution of the above into equation (3.40) yields four independent solutions:

$$W_1 = e^{(\alpha+i\alpha)x} = e^{\alpha x} * e^{i\alpha x} \quad \text{.........................(3.52a)}$$

$$W_2 = e^{(\alpha-i\alpha)x} = e^{\alpha x} * e^{-i\alpha x} \quad \text{.........................(3.52b)}$$

$$W_3 = e^{(-\alpha-i\alpha)x} = e^{-\alpha x} * e^{-i\alpha x} \quad \text{.....................(3.52c)}$$

$$W_4 = e^{(-\alpha+i\alpha)x} = e^{-\alpha x} * e^{i\alpha x} \quad \text{.....................(3.52d)}$$

This gives rise to the homogeneous solution as follows:

$$\begin{aligned} W_h(x) = {} & C_1 e^{\alpha x} * e^{i\alpha x} + C_2 e^{\alpha x} * e^{-i\alpha x} + C_3 e^{-\alpha x} * e^{-i\alpha x} \\ & + C_4 e^{-\alpha x} * e^{i\alpha x} \end{aligned} \quad \text{........(3.53)}$$

where, C_1, C_2, C_3, and C_4 are constants.

The following relationship exist in mathematics:

$$e^{\alpha x} = \cosh \alpha x + \sinh \alpha x \quad \text{...........................(3.54a)}$$

$$e^{-\alpha x} = \cosh \alpha x - \sinh \alpha x \quad \text{...........................(3.54b)}$$

$$e^{i\alpha x} = \cos \alpha x + i \sin \alpha x \quad \text{..............................(3.54c)}$$

$$e^{-i\alpha x} = \cos \alpha x - i \sin \alpha x \quad \text{..............................(3.54d)}$$

Equation (3.53) can be put as:

$$\begin{aligned} W_h(x) = {} & C_1(\cosh \alpha x + \sinh \alpha x)(\cos \alpha x + i \sin \alpha x) + \\ & C_2(\cosh \alpha x + \sinh \alpha x)(\cos \alpha x - i \sin \alpha x) + \\ & C_3(\cosh \alpha x - \sinh \alpha x)(\cos \alpha x - i \sin \alpha x) + \\ & C_4(\cosh \alpha x - \sinh \alpha x)(\cos \alpha x + i \sin \alpha x) \end{aligned}$$

Expanding the above expression gives:

$$W_h(x) = A_1 \cosh \alpha x \cos \alpha x + A_2 \cosh \alpha x \sin \alpha x + A_3 \sinh \alpha x \cos \alpha x + A_4 \sinh \alpha x \sin \alpha x \quad \text{.......................(3.55)}$$

where:

$A_1 = C_1 + C_2 + C_3 + C_4$

$A_2 = (C_1 - C_2 - C_3 + C_4)i$

$A_3 = C_1 + C_2 - C_3 - C_4$

$A_4 = (C_1 - C_2 + C_3 - C_4)i$

3.2.1.1.2 The Particular Solution

Recall the static equation of equilibrium (eqn. 3.38)

$$W^{iv} + 4\alpha^4 W = \frac{\gamma}{D}x \quad \text{..............................(3.56)}$$

The particular solution is assumed to be of the form:

$$W_p(x) = ax + b \quad \text{..............................(3.57)}$$

It follows that :

$$W_p^{n}(x) = 0 \quad \text{.......................................(3.58)}$$

Substitution of equation (3.57) and (3.58) into equation (3.56) to get:

$$0 + 4\alpha^4(ax + b) = \frac{\gamma}{D}x$$

$$4\alpha^4 ax + 4\alpha^4 b = \frac{\gamma}{D}x$$

A comparison of coefficients in the above expression yields:

$b = 0$

$$a = \frac{\gamma}{4\alpha^4 D}$$

Therefore the particular solution will be:

$$W_p(x) = \frac{\gamma}{4\alpha^4 D} x \quad \text{.............................(3.59)}$$

3.2.1.1.3 The General Solution

The general solution is obtained by summing the particular and homogeneous solutions, which gives:

$$\begin{aligned} W(x) = {} & A_1 \cosh\alpha x \cos\alpha x + A_2 \cosh\alpha x \sin\alpha x + \\ & A_3 \sinh\alpha x \cos\alpha x + A_4 \sinh\alpha x \sin\alpha x + \frac{\gamma}{4\alpha^4 D} x \end{aligned} \quad \text{...........(3.60)}$$

Expression for rotation θ(x), moment M(x), shear force Q(x) and normal force N(x) are generated in the following way:

$$\begin{aligned} \theta(x) = \frac{dw}{dx} = {} & \alpha(A_1 + A_4)\sinh\alpha x \cos\alpha x + \\ & \alpha(A_4 - A_1)\cosh\alpha x \sinh\alpha x + \\ & \alpha(A_2 - A_3)\sinh\alpha x \sin\alpha x + \\ & \alpha(A_2 + A_3)\cosh\alpha x \cos\alpha x + \frac{\gamma}{4D\alpha^4} \end{aligned} \quad \text{..................(3.61)}$$

$$M(x) = -D\frac{d^2w}{dx^2} = \frac{Eh^3\alpha^2}{6(1-\mu^2)}A_1\sinh\alpha x\sin\alpha x -$$
$$\frac{Eh^3\alpha^2}{6(1-\mu^2)}A_2\sinh\alpha x\cos\alpha x +$$
$$\frac{Eh^3\alpha^2}{6(1-\mu^2)}A_3\cosh\alpha x\sin\alpha x - \qquad \text{......................(3.62)}$$
$$\frac{Eh^3\alpha^2}{6(1-\mu^2)}A_4\cosh\alpha x\cos\alpha x$$

$$Q(x) = \frac{dM}{dx} = \frac{Eh^3\alpha^3}{6(1-\mu^2)}(A_1 + A_4)\cosh\alpha x\sin\alpha x +$$
$$\frac{Eh^3\alpha^3}{6(1-\mu^2)}(A_1 - A_4)\sinh\alpha x\cos\alpha x + \qquad \text{......(3.63)}$$
$$\frac{Eh^3\alpha^3}{6(1-\mu^2)}(A_3 - A_2)\cosh\alpha x\cos\alpha x +$$
$$\frac{Eh^3\alpha^3}{6(1-\mu^2)}(A_2 + A_3)\sinh\alpha x\sin\alpha x$$

$$N(x) = \frac{Eh}{R}w(x) = \frac{Eh}{R}A_1\cosh\alpha x\cos\alpha x + \frac{Eh}{R}A_2\cosh\alpha x\sin\alpha x +$$
$$\frac{Eh}{R}A_3\sinh\alpha x\cos\alpha x + \frac{Eh}{R}A_4\sinh\alpha x\sin\alpha x + \gamma R x$$

$$\text{.....................................(3.64)}$$

3.2.1.2 Application of Initial value method in defining the constant of Integration

$$\text{Let } Z = \alpha x \qquad (3.65)$$

Differentiating the above with respect to x gives:

$$\frac{dZ}{dx} = \alpha \qquad (3.66)$$

Generally,

$$\frac{d^n w}{dx^n} = \frac{d^n}{dz^n}\left(\frac{dz}{dx}\right)^n = \frac{\alpha^n d^n w}{dz^n} \qquad (3.67)$$

Substitution of equations (3.66) and (3.67) into equation (3.38) will

provide:

$$\frac{\alpha^4 d^4 w}{dz^4} + 4\alpha^4 w(z) = \frac{\gamma z}{D\alpha}$$

Dividing all through by α^4 gives:

$$\frac{d^4 w}{dz^4} + 4w(z) = \frac{\gamma z}{D\alpha^4}\frac{z}{\alpha} \qquad (3.68)$$

Substitute for D and α^4 as defined in equation (3.38), to get:

$$\frac{d^4 w}{dz^4} + 4w(z) = \frac{4\gamma R^2}{Eh\alpha} z \qquad (3.69)$$

3.2.1.2.1 The Initial Value Homogeneous Solution

The homogeneous solution is given by:

$$\frac{d^4 w}{dz^4} + 4w(z) = 0 \qquad (3.70)$$

The solution is found using the results obtained in section (3.2.1.1.1) by taking $\alpha = 1$ and $x = z$:

$$W_h(z) \quad C_1 e^{z} * e^{i^z} + C_2 e^{z} * e^{-i^z} + C_3 e^{-z} * e^{-i^z} + C_4 e^{-z} * e^{i^z} \quad \ldots\ldots\ldots\ldots(3.71)$$

where: C_1, C_2, C_3, C_4 are arbitrary constants owing to:

ez = cosh z + sinh z

e-z = cosh z - sinh z

eiz = cos z + isin z

e-iz = cos z - isin z

Equation (3.71) can now be put as:

$$W_h(z) = \sum_{k=1}^{4} C_k Y_k(z) \ldots\ldots\ldots\ldots(3.72)$$

where:

$$Y_k(z) = a_k \cosh z \cos z + b_k \cosh z \sin z + c_k \sinh z \cos z + d_k \sinh z \sin z \quad \ldots\ldots\ldots\ldots(3.73)$$

The constants a_k, b_k, c_k, and d_k ($k = 1, 2, 3, 4$), are 16 in number and are obtained by differentiating equation (3.73) with respect to z till the fourth order and making use of the supplementary initial conditions which are given as:

$$Y_1(0) = 1 \qquad Y_1^{i}(0) = 0 \qquad Y_1^{ii}(0) = 0 \qquad Y_1^{iii}(0) = 0$$

$$Y_2(0) = 0 \qquad Y_2^{i}(0) = 1 \qquad Y_2^{ii}(0) = 0 \qquad Y_2^{iii}(0) = 0$$

$$Y_3(0)=0 \qquad Y_3^i(0)=0 \qquad Y_3^{ii}(0)=1 \qquad Y_3^{iii}(0)=0$$

$$Y_4(0)=0 \qquad Y_4^i(0)=0 \qquad Y_4^{ii}(0)=0 \qquad Y_4^{iii}(0)=1$$

Differentiation of equation (3.73) four times gives:

$$\begin{aligned} Y_k^i &= a_k(\sinh z\cos z - \cosh z\sin z) + b_k(\sinh z\sin z + \cosh z\cos z) + \\ &\quad c_k(\cosh z\cos z - \sinh z\sin z) + d_k(\cosh z\sin z + \sinh z\cos z) \end{aligned} \qquad \text{..............................(3.74)}$$

$$\begin{aligned} Y_k^{ii} &= -2a_k(\sinh z\sin z) + 2b_k(\sinh z\cos z) \\ &\quad - 2c_k(\cosh z\sin z) + d_k(\cosh z\cos z) \end{aligned} \qquad \text{..................(3.75)}$$

$$\begin{aligned} Y_k^{iii} &= -2a_k(\cosh z\sinh z + \sinh z\cos z) + \\ &\quad 2b_k(\cosh z\cos z - \sinh z\sin z) - \\ &\quad 2c_k(\sinh z\sin z + \cosh z\cos z) + \\ &\quad 2d_k(\sinh z\cos z - \cosh z\sin z) \end{aligned} \qquad \text{.......................(3.76)}$$

$$\begin{aligned} Y_k^{iv} &= -4a_k(\cosh z\cos z) - 4b_k(\cosh z\sin z) \\ &\quad - 4c_k(\sinh z\cos z) - d_k(\sinh z\sin z) \end{aligned} \qquad \text{..................(3.77)}$$

Applying the supplementary initial conditions to equations (3.73) through (3.76) to get the arbitrary constants as follows:

K = 1

$\therefore Y_1(0) = a_1 \cosh z \cos z + b_1 \cosh z \sin z + c_1 \sinh z \cos z + d_1 \sinh z \sin z$

Substituitng z = 0 into above equation to get:

$Y_1(0) = a_1$

but $Y_1(0) = 1; \Rightarrow a_1 = 1$

$$Y_1^i(0) = a_1(\sinh z \cos z - \cosh z \sin z) + b_1(\sinh z \sin z + \cosh z \cos z) +$$
$$c_1(\cosh z \cos z - \sinh z \sin z) + d_1(\cosh z \sin z + \sinh z \cos z)$$

Substituting z = 0 into above equation to get:

$$Y_1^i(0) = a_1(0) + b_1(1) + c_1(1) + d_1(0)$$

$$= b_1 + c_1 = 0 \text{ since } Y_1^i(0) = 0$$

Substituting z = 0 into equation (5.75)

$$Y_1^{ii}(0) = -2a_1(0) + 2b_1(0) - 2c_1(0) + 2d_1(1)$$

but $Y_1^{ii}(0) = 0; \Rightarrow 2d_1 = 0; \therefore d_1 = 0$

Substituting z = 0 into equation (5.76)

$$Y_1^{iii}(0) = -2a_1(0) + 2b_1(1) - 2c_1(1) + 2d_1(0)$$

$$Y_1^{iii}(0) = 2b_1 - 2c_1$$

but $Y_1^{iii}(0) = 0; \Rightarrow 2b_1 - 2c_1 = 0$

$$\therefore b_1 - c_1 = 0; b_1 = c_1 = 0$$

$K = 2$

$$\left.\begin{aligned} &a_2 = 0 \\ &b_2 + c_2 = 1 \\ &2b_2 - 2c_2 = 0 \\ &d_2 = 0 \end{aligned}\right\} \quad a_2 = d_2 = 0;\ b_2 = c_2 = \tfrac{1}{2}$$

$K = 3$

$$\left.\begin{aligned} &a_3 = 0 \\ &b_3 + c_3 = 0 \\ &2d_3 = 1 \\ &2b_3 - 2c_3 = 0 \end{aligned}\right\} \quad \Rightarrow a_3 = b_3 = c_3 = 0;\ d_3 = \tfrac{1}{2}$$

$K = 4$

$$\left.\begin{aligned} &a_4 = 0 \\ &b_4 + c_4 = 0;\ 2d_4 = 0 \\ &2b_4 - 2c_4 = 1 \end{aligned}\right\} \quad a_4 = d_4 = 0;\ b_4 = -c_4 = \tfrac{1}{4}$$

Summarily:				
	$a_1 = 1$	$a_2 = 0$	$a_3 = 0$	$a_4 = 0$
	$b_1 = 0$	$b_2 = \frac{1}{2}$	$b_3 = 0$	$b_4 = \frac{1}{4}$
	$c_1 = 0$	$c_2 = \frac{1}{2}$	$c_3 = 0$	$c_4 = -\frac{1}{4}$
	$d_1 = 0$	$d_2 = 0$	$d_3 = \frac{1}{2}$	$d_4 = 0$

Substituting these constants into equation (3.73) to get:

$$Y_1(z) = \cosh z \cos z \ldots\ldots\ldots\ldots \quad (3.78)$$

$$Y_2(z) = \tfrac{1}{2}(\cosh z \sin z + \sinh z \cos z) \ldots\ldots\ldots\ldots (3.79)$$

$$Y_3(z) = \tfrac{1}{2}(\sinh z \sin z) \ldots\ldots\ldots\ldots (3.80)$$

$$Y_4(z) = \tfrac{1}{4}(\cosh z \sin z - \sinh z \cos z) \ldots\ldots\ldots\ldots (3.81)$$

It can be seen that Y_k, k = 1, 2, 3, 4 are four independent expressions. The homogeneous solution will be written as:

$$W_h(z) = C_1Y_1(z) + C_2Y_2(z) + C_3Y_3(z) + C_4Y_4(z) \ldots\ldots\ldots (3.82)$$

the arbitrary constants C_1, C_2, C_3, and C_4 are determined using the initial conditions. The derivations of the functions Y_k can be evaluated by using the computed constants a_k, b_k, c_k and d_k in equation (3.74) through (3.77):

$$Y_1^{i}(z) = \sinh z \cos z - \cosh z \sin z = -4Y_4(z) \ldots\ldots\ldots\ldots (3.83a)$$

$$Y_1^{ii}(z) = -2(\sinh z \sin z) = -4Y_3(z) \ldots\ldots\ldots\ldots (3.83b)$$

$$Y_1^{iii}(z) = -2(\cosh z \sin z + \sinh z \cos z) = -4Y_2(z) \ldots\ldots\ldots\ldots (3.83c)$$

$$Y_1^{iv}(z) = -4(\cosh z \cos z) = -4Y_1(z) \ldots\ldots\ldots\ldots (3.83d)$$

$$Y_2^{i}(z) = (\cosh z \cos z) = Y_1(z) \ldots\ldots\ldots\ldots (3.84a)$$

$$Y_2^{ii}(z) = (\sinh z \cos z - \cosh z \sin z) = -4Y_4(z) \ldots\ldots\ldots\ldots (3.84b)$$

$$Y_2^{iii}(z) = -2(\sinh z \sin z) = -4Y_3(z) \ldots\ldots\ldots\ldots (3.84c)$$

$$Y_2^{iv}(z) = -2(\cosh z \sin z + \sinh z \cos z) = -4Y_2(z) \quad \text{.............(3.84d)}$$

$$Y_3^{i}(z) = \frac{1}{2}(\cosh z \sin z + \sinh z \cos z) = Y_2(z) \quad \text{..................(3.85a)}$$

$$Y_3^{ii}(z) = \cosh z \cos z = Y_1(z) \quad \text{......................................(3.85b)}$$

$$Y_3^{iii}(z) = \sinh z \cos z - \cosh z \sin z = -4Y_4(z) \quad \text{..................(3.85c)}$$

$$Y_3^{iv}(z) = -2(\sinh z \sin z) = -4Y_3(z) \quad \text{................................(3.85d)}$$

$$Y_4^{i}(z) = \frac{1}{2}(\sinh z \sin z) = Y_3(z) \quad \text{....................................(3.86a)}$$

$$Y_4^{ii}(z) = \frac{1}{2}(\cosh z \sin z + \sinh z \cos z) = Y_2(z) \quad \text{..................(3.86b)}$$

$$Y_4^{iii}(z) = \cosh z \cos z = Y_1(z) \quad \text{......................................(3.86c)}$$

$$Y_4^{iv}(z) = \sinh z \cos z - \cosh z \sin z = -4Y_4(z) \quad \text{..................(3.86d)}$$

the above relationship are put in a tabular form as follows:

Table 3.1: Derivatives of Y_k

Y_k	Y^i	Y^{ii}	Y^{iii}	Y^{iv}
Y_1	$-4Y_4$	$-4Y_3$	$-4Y_2$	$-4Y_1$
Y_2	Y_1	$-4Y_4$	$-4Y_3$	$-4Y_2$
Y_3	Y_2	Y_1	$-4Y_4$	$-4Y_3$
Y_4	Y_3	Y_2	Y_1	$-4Y_4$

The relationships between the deflection W(z) and the rotation θ(z), moment M(z), shear force Q(z) and hoop tension N(z) are derived in the following manner:

Recall equation (3.72)

$$W_h(z) = \sum_{k=1}^{4} C_k Y_k(z)$$

using it together in the following expression

$$\frac{d^n w}{dx^n} = \frac{\alpha^n d^n w}{dz^n} \text{ to obtain:}$$

$$\theta_h(z) = \frac{dw_h}{dx} = \alpha w_h'(z) = \alpha \sum_{k=1}^{4} C_k Y_k^{\prime}(z)$$

$$\Rightarrow \theta_h(z) = \alpha C_1 Y_4(z) + \alpha C_2 Y_1(z) + \alpha C_3 Y_2(z) + \alpha C_4 Y_3(z) \ldots\ldots (3.87)$$

$$M_h(z) = -\frac{Dd^2 w_h}{dx^2} = -Dd^2 w_h''(z) = -\frac{Eh}{4\alpha^2 R^2}\left[\sum_{k=1}^{4} C_k Y_k''(z)\right]$$

$$\Rightarrow M_h(z) = \frac{Eh}{4\alpha^2 R^2}\left[4C_1 Y_3(z) + 4C_2 Y_4(z) - C_3 Y_1(z) - C_4 Y_2(z)\right]$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.88)$$

$$Q_h(z) = \frac{dM}{dx} - \frac{Dd^3 w_h}{dx^3} = -D\alpha^3 w_h'''(z)$$

$$= \frac{-Eh}{4\alpha R^2} w_h''' = -\frac{Eh}{4\alpha R^2}\left[\sum_{k=1}^{4} C_k Y_k'''(z)\right]$$

$$\Rightarrow Q_h(z) = \frac{Eh}{4\alpha R^2}\left[4C_1Y_2(z) + 4C_2Y_3(z) + C_3Y_4(z) - C_4Y_1(z)\right] \ldots\ldots (3.89)$$

$$N_h(z) = \frac{Eh}{R}w_h(z) = \frac{Eh}{R}\left[\sum_{k=1}^{4} C_k Y_k\right]$$

$$\Rightarrow N_h(z) = \frac{Eh}{R}\left[C_1Y_1(z) + C_2Y_2(z) + 4C_3Y_3(z) + C_4Y_4(z)\right] \ldots\ldots\ldots (3.90)$$

Taking W_o, θ_o, M_o and Q_o as initial value of $W_h(z)$, $\theta_h(z)$, $M_h(z)$ and $Q_h(z)$ respectively. the arbitrary constants C_k are found by making use of the initial supplementary conditions given in the previous section (3.2.1.2.1)

$$\theta_0 = \alpha\left[-4C_1Y_4(0) + C_2Y_1(0) + C_3Y_2(0) + C_4Y_3(0)\right]$$

$$\Rightarrow C_2 = \frac{\theta_0}{\alpha} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.91)$$

$$M_0 = \frac{Eh}{4\alpha^2R^2}\left[4C_1Y_3(0) + 4C_2Y_4(0) - C_3Y_1(0) - C_4Y_2(0)\right]$$

$$\Rightarrow C_3 = -\frac{4\alpha^2R^2}{Eh}M_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.92)$$

$$Q_0 = \frac{Eh}{4\alpha R^2}\left[4C_1Y_2(0) + 4C_2Y_3(0) + 4C_3Y_4(0) - C_4Y_1(0)\right]$$

$$\Rightarrow C_4 = -\frac{4\alpha R^2}{Eh}Q_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.93)$$

$$W_0 = \left[C_1Y_1(0) + C_2Y_2(0) + C_3Y_3(0) + C_4Y_4(0)\right]$$

$$\Rightarrow C_1 = W_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.94)$$

Substituting for C_1, C_2, C_3, and C_4 into equations (3.72) and (3.87) through (3.90) to obtain:

$$W_h(z) = Y_1(z)W_0 + \frac{Y_2(z)}{\alpha}\theta_0 - \frac{4\alpha^2 R^2}{Eh} Y_3(z)M_0 - \frac{4\alpha R^2}{Eh} Y_4(z).Q_0 \qquad \cdots\cdots\cdots(3.95)$$

$$\theta_h(z) = -4\alpha Y_4(z)W_0 + Y_1(z)\theta_0 - \frac{4\alpha^3 R^2}{Eh} Y_2(z)M_0 - \frac{4\alpha^2 R^2}{Eh} Y_3(z).Q_0 \qquad \cdots\cdots\cdots(3.96)$$

$$M_h(z) = \frac{Eh}{\alpha^2 R^2} Y_3(z)W_0 + \frac{Eh}{\alpha^3 R^2} Y_4(z)\theta_0 + Y_1(z)M_0 + \frac{Y_2}{\alpha} Q_0 \qquad \cdots\cdots\cdots(3.97)$$

$$Q_h(z) = \frac{Eh}{\alpha R^2} Y_2(z)W_0 + \frac{Eh}{\alpha^2 R^2} Y_3(z)\theta_0 - 4\alpha Y_4(z)M_0 + Y_1(z)Q_0 \qquad \cdots\cdots\cdots(3.98)$$

$$N_h(z) = \frac{Eh}{R} Y_1(z)W_0 + \frac{Eh}{\alpha R} Y_2(z)\theta_0 - 4\alpha^2 R Y_3(z)M_0 - 4\alpha R Y_4(z).Q_0 \qquad \cdots\cdots\cdots(3.99)$$

3.2.1.2.2 The Initial Value Particular solution

Consider the figure below:

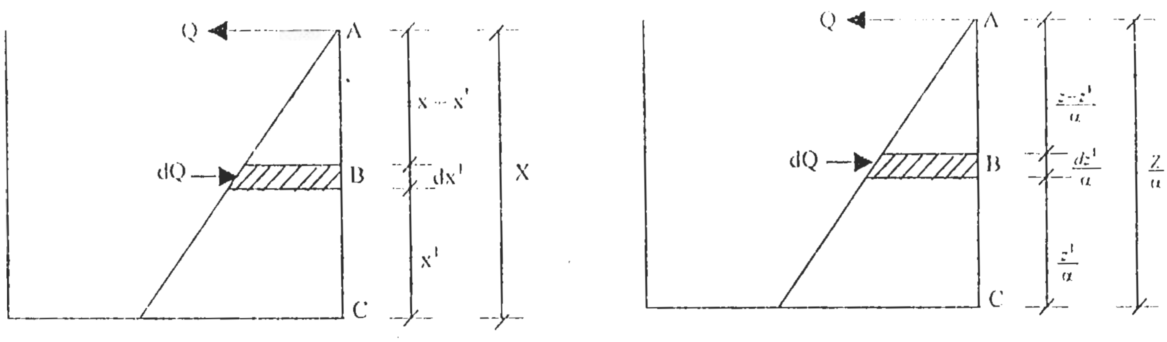


Fig. 3.5a

Fig. 3.5b

Fig. 3.5: Elemental Force Acting on Side of Cylindrical Shell

To find the particular solution we proceed to an origin transformation. The origin that was previously at A, is shifted to B, thereby introducing a new variable X.

Let $z' = \alpha x' \Rightarrow dx' = \frac{dz'}{\alpha}$

dxl is the width of the element.

The distributed load at the new origin B is given by:

$$q(z') = \frac{z - z'}{\alpha}\gamma$$

Therefore the elemental force can be expressed as:

$$dQ = q(z').\frac{dz'}{\alpha} = \frac{\gamma(z - z')}{\alpha^2}dz' \quad \text{.......................(3.100)}$$

In the process of getting the particular solution, one just needs to consider the homogeneous solution in such a way that only parameters similar to dQ are used. But only Q_0 is similar to dQ with an opposite sign. Therefore the coefficients of Q_0 in the homogeneous solution are used with the appropriate sign to formulate the particular solutions:

$$dW_p = \frac{4\alpha R^2}{Eh} Y_4(z').\frac{\gamma(z-z')}{\alpha^2} dz'$$

$$\Rightarrow dW_p = \frac{4\gamma R^2}{Eh\alpha}(z-z')Y_4(z') \quad\text{..................(3.101)}$$

$$d\theta_p = \frac{4\alpha^2 R^2}{Eh} Y_3(z').\frac{\gamma(z-z')}{\alpha^2} dz'$$

$$\Rightarrow d\theta_p = \frac{4\gamma R^2}{Eh}(z-z')Y_3(z') \quad\text{..................(3.102)}$$

$$dM_p = \frac{-Y_2(z')}{\alpha}.\frac{\gamma(z-z')}{\alpha^2} dz'$$

$$\Rightarrow dM_p = \frac{-\gamma}{\alpha^3}(z-z')Y_2(z')dz' \quad\text{................(3.103)}$$

$$dQ_p = -Y_1(z').\frac{\gamma(z-z')}{\alpha^2} dz'$$

$$\Rightarrow dQ_p = \frac{-\gamma}{\alpha^2}(z-z')Y_1(z')dz' \quad\text{................(3.104)}$$

$$dN_p = 4\alpha R Y_4(z').\frac{\gamma(z-z')}{\alpha^2} dz'$$

$$\Rightarrow dN_p = \frac{4\gamma R}{\alpha}(z - z')Y_4(z')dz' \ldots\ldots\ldots\ldots\ldots(3.105)$$

Integrating the above expressions gives the actual particular solutions:

$$W_p = \frac{4\gamma R^2}{Eh\alpha}\left[Z\int_0^z Y_4(z')dz' - \int_0^z Z' Y_4(z')dz'\right]$$

Consider the term $\int_0^z Z' Y_4(z')dz'$, which will be solved using integration by parts.

Given that $\int V du = UV - \int U dv$,

Let $v = z' \Rightarrow dv = dz'$

$$du = Y_4(z') \Rightarrow U = -\tfrac{1}{4}Y_1(z')$$

$$\therefore \int_0^z Z' Y_4(z')dz' = \left[-\tfrac{1}{4}z' Y_1(z') + \tfrac{1}{4}\int Y_1(z')dz'\right]_0^z$$

$$= \left[-\tfrac{1}{4}Y_1(z')z' + \tfrac{1}{4}Y_2(z')\right]_0^z$$

$$W_p = \frac{4\gamma R^2}{Eh\alpha}\left[-\tfrac{1}{4}zY_1(z') + \tfrac{1}{4}z' Y_1(z') - \tfrac{1}{4}Y_2(z')\right]_0^z$$

$$\Rightarrow W_p = \frac{\gamma R^2}{Eh\alpha}[Z - Y_2(z)] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.106)$$

$$\theta p = \frac{4R^2\gamma}{Eh}\left[Z\int_0^z Y_3(z')dz' - \int_0^z Z' Y_3(z')dz'\right]$$

Integrating by parts to get:

$$\int_0^z Z'Y_3(z')dz' = \left[Z'Y_4(z') - \int Y_4(z')dz'\right]_0^z$$

$$= \left[z'Y_4(z') + \tfrac{1}{4}Y_1(z')\right]_0^z$$

$$\theta_p = \frac{4\gamma R^2}{Eh}\left[zY_4(z') - z'Y_4(z') - \tfrac{1}{4}Y_1(z')\right]_0^z$$

$$\Rightarrow \theta_p = \frac{\gamma R^2}{Eh}\left[1 - Y_1(z)\right] \quad \text{.....................................(3.107)}$$

$$Mp = \frac{-\gamma}{\alpha^3}\left[Z\int_0^z Y_2(z')dz' - \int_0^z Z'Y_2(z')dz'\right]$$

Integrating by parts to get:

$$\int_0^z Z'Y_2(z')dz' = \left[z'Y_3(z') - \int Y_3(z')dz'\right]_0^z$$

$$= \left[z'Y_3(z') - Y_4(z')\right]_0^z$$

$$M_p = -\frac{\gamma}{\alpha^3}\left[zY_3(z') - z'Y_3(z') + Y_4(z')\right]_0^z$$

$$\Rightarrow M_p = -\frac{\gamma}{\alpha^3}Y_4(z) \quad \text{.........................} \quad (3.108)$$

$$Qp = \frac{-\gamma}{\alpha^2}\left[Z\int_0^z Y_1(z')dz' - \int_0^z Z'Y_1(z')dz'\right]$$

Integrating by parts:

$$\int_0^z z' Y_1(z')dz' = \left[z' Y_2(z') - \int Y_2(z')dz'\right]_0^z$$

$$= \left[z' Y_2(z') - Y_3(z')\right]_0^z$$

$$Q_p = -\frac{\gamma}{\alpha^2}\left[z Y_2(z') - z' Y_2(z') + Y_3(z')\right]_0^z$$

$$\Rightarrow Q_p = -\frac{\gamma}{\alpha^2} Y_3(z) \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.109)$$

$$N_p = \frac{Eh}{R} W_p = \frac{Eh}{R}\frac{\gamma R^2}{Eh\alpha}\left[z - Y_2(z)\right]$$

$$\Rightarrow N_p = \frac{\gamma R}{\alpha}\left[z - Y_2(z)\right] \ldots\ldots\ldots\ldots\ldots \quad (3.110)$$

5.2.1.2.3 The Initial Value General Solution

It is obtained by summing homogeneous and particular solutions:

$$W(z) = Y_1(z) W_0 + \left[\frac{Y_2(z)}{\alpha}\theta_0\right] - \left[\frac{4\alpha^2 R^2}{Eh} Y_3(z)\right] M_0 - \left[\frac{4\alpha R^2}{Eh} Y_4(z)\right] Q_0 + \frac{\gamma R^2}{Eh\alpha}\left[z - Y_2(z)\right]$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.111)$$

$$\theta(z) = -4\alpha Y_4(z)W_0 + Y_1(z)\theta_0 - \left[\frac{4\alpha^3 R^2}{Eh} Y_2(z)\right]M_0$$

$$-\left[\frac{4\alpha^2 R^2}{Eh} Y_3(z)\right]Q_0 + \frac{R^2\gamma}{Eh}[1 - Y_1(z)]$$

..........................(3.112)

$$M(z) = \left[\frac{Eh}{\alpha^2 R^2} Y_3(z)\right]W_0 + \left[\frac{Eh}{\alpha^3 R^2} Y_4(z)\right]\theta_0 + Y_1(z)M_0 + \frac{Y_2(z)}{\alpha}Q_0 - \frac{\gamma}{\alpha^3}Y_4(z)$$

.........................(3.113)

$$Q(z) = \left[\frac{Eh}{\alpha R^2} Y_2(z)\right]W_0 + \left[\frac{Eh}{\alpha^2 R^2} Y_3(z)\right]\theta_0 - 4\alpha Y_4(z)M_0 + Y_1(z)Q_0 - \frac{\gamma}{\alpha^2}Y_3(z)$$

..........................(3.114)

$$N(z) = \left[\frac{Eh}{R} Y_1(z)\right]W_0 + \left[\frac{Eh}{\alpha R} Y_2(z)\right]\theta_0 - \left[4\alpha^2 R Y_3(z)\right]M_0$$

$$-\left[4\alpha R Y_4(z)\right]Q_0 + \frac{R\gamma}{\alpha}\left[z - Y_2(z)\right]$$

............(3.115)

We shall make use of the boundary conditions in equations (3.111) through (3.115).

At z = 0, M(z) = W(z) = 0 $\Rightarrow W_0 = M_0 = 0$ ....(3.116)

At z = αL, W(z) = θ(z) = 0 ....(3.117)

Substitution of the above two equations (3.116) and (3.117) into equations (3.111) and (3.112) provides two (2) equations with two (2) unknowns (ie. θ_0 and Q_0).

$$\frac{Y_2(\alpha L)}{\alpha}\theta_0 - \left[\frac{4\alpha R^2 Y_4(\alpha L)}{Eh}\right]Q_0 + \frac{R^2\gamma}{\alpha Eh}\left[\alpha L - Y_2(\alpha L)\right] = 0 \quad \ldots\ldots\ldots\ldots\ldots(3.118)$$

$$Y_1(\alpha L)\theta_0 - \left[\frac{4\alpha^2 R^2}{Eh}Y_3(\alpha L)\right]Q_0 + \frac{R^2\gamma}{Eh}\left[1 - Y_1(\alpha L)\right] = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.119)$$

Solving equations (3.118) and (3.119) to obtain the initial conditions as:

$$\theta_0 = \frac{\gamma R^2\left[Y_4(\alpha L) + Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L) - \alpha L Y_3(\alpha L)\right]}{Eh\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]} \quad \ldots\ldots(3.120)$$

$$Q_0 = \frac{\gamma\left[Y_2(\alpha L) - \alpha L Y_1(\alpha L)\right]}{4\alpha^2\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.121)$$

$W_0 = 0$ and M_0

Substitute for W_0, θ_0, Q_0 and M_0 into equations (3.111) through (3.115) to get:

$$W_Z = \frac{\gamma R^2\left[Y_4(\alpha L) + Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L) - \alpha L Y_3(\alpha L)\right]Y_2(z)}{\alpha Eh\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]}$$

$$- \frac{R^2\gamma\left[Y_2(\alpha L) - (\alpha L)Y_1(\alpha L)\right]}{\alpha Eh\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]}Y_4(z)$$

$$+ \frac{R^2\gamma}{\alpha Eh}\left[z - Y_2(z)\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.122)$$

$$\theta(z) = \frac{\gamma R^2\left[Y_4(\alpha L) + Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L) - \alpha L Y_3(\alpha L)\right]Y_1(z)}{Eh\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]}$$

$$- \frac{R^2\gamma\left[Y_2(\alpha L) - (\alpha L)Y_1(\alpha L)\right]}{Eh\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]}Y_3(z)$$

$$+\frac{\gamma R^2}{Eh}\left[1-Y_1(z)\right]\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.123)$$

$$M(z)=\frac{\gamma\left[Y_4(\alpha L)+Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)-\alpha LY_3(\alpha L)\right]}{\alpha^3\left[Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)\right]}Y_4(z)$$

$$+\frac{\gamma\left[Y_2(\alpha L)-(\alpha L)Y_1(\alpha L)\right]}{4\alpha^3\left[Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)\right]}Y_2(z)$$

$$-\frac{\gamma}{\alpha^3}Y_4(z)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.124)$$

$$Q(z)=\frac{\gamma\left[Y_4(\alpha L)+Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)-\alpha LY_3(\alpha L)\right]}{\alpha^2\left[Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)\right]}Y_3(z)$$

$$+\frac{\gamma\left[Y_2(\alpha L)-\alpha LY_1(\alpha L)\right]}{4\alpha^2\left[Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)\right]}Y_1(z)$$

$$-\frac{\gamma}{\alpha^2}Y_3(z)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.125)$$

$$N(z)=\frac{\gamma R\left[Y_4(\alpha L)+Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)-\alpha LY_3(\alpha L)\right]}{\alpha\left[Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)\right]}Y_2(z)$$

$$-\frac{\gamma R\left[Y_2(\alpha L)-\alpha LY_1(\alpha L)\right]}{\alpha\left[Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)\right]}Y_4(z)$$

$$+\frac{\gamma R}{\alpha}\left[z-Y_2(z)\right]\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.126)$$

The Initial value general solutions obtained in equation (3.122) through (3.126) are used to evaluate the performance of the cylindrical shell structure under hydrostatic pressure through the determination of deflection and stresses in the subsequent chapter.

3.2.2 Case of Circular Cylindrical Shell subject to Food Grain Pressure

In case of grain pressure, the distribution of forces (see Fig. 3.6) is also triangular as hydrostatic pressure.

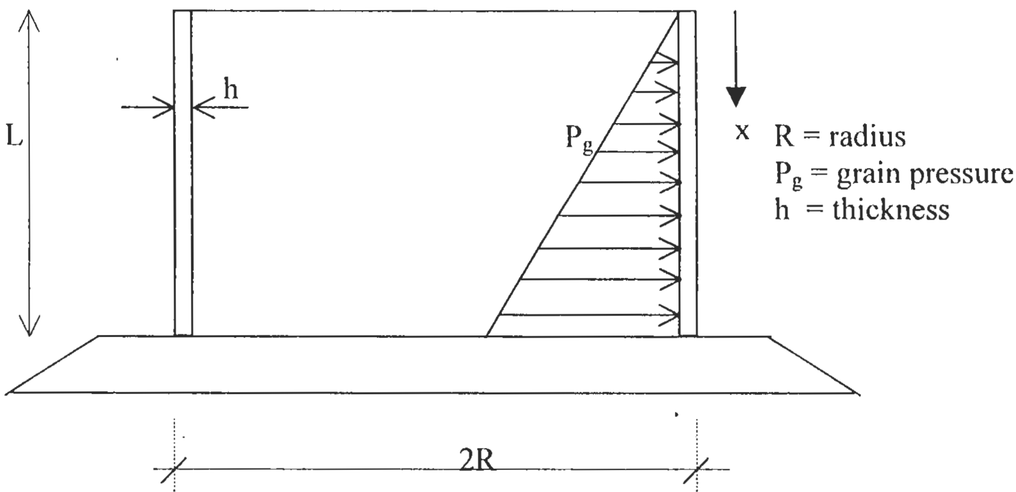


Fig. 3.6: Grain Pressure Distribution

The static differential equation is given by:

$$W^{iv} + 4\alpha^4 W = \frac{P_g}{D} \qquad (3.127)$$

where W, α, and D have the same meaning as in the previous sections and

$P_g = k\gamma x$,

where; k is the lateral-to-vertical pressure ratio given by Rankine as

$k = \tan^2\left[45^\circ - \frac{\phi}{2}\right]$, ϕ = angle of internal friction of grain

γ = unit weight of grain (density)

The differential equation can be rewritten as

$$W^{iv} + 4\alpha^4 W = \frac{k\gamma x}{D} \qquad (3.128)$$

3.2.2.1 Classical Solution

The homogeneous solution is given as: $W^{iv} + 4\alpha^4 W = 0$, its solution has already been found in section (3.2.1.1.1)

3.2.2.1.1 The Particular Solution

Assuming the particular solution to be in the form:

$$W_p(x) = ax + b \qquad (3.129)$$

And substituting into equation (3.128) to get:

$$4\alpha^4 ax + 4\alpha^4 b = \frac{k\gamma x}{D}$$

$$\Rightarrow \begin{cases} a = 0 \\ b = \dfrac{k\gamma x}{4D\alpha^4} \end{cases}$$

Therefore the particular solution is:

$$W_p(x) = \frac{k\gamma x}{4\alpha^4 D} \qquad (3.130)$$

3.2.2.1.2 The General Solution

The general solution is obtained by summing homogeneous and particular solutions:

$$W(x) = A_1 \cosh\alpha x \cos\alpha x + A_2 \cosh\alpha x \sin\alpha x + A_3 \sinh\alpha x \cos\alpha x + A_4 \sinh\alpha x \sin\alpha x + \frac{k\gamma x}{4\alpha^4 D} \quad \text{.................(3.131)}$$

where A_1, A_2, A_3, and A_4 are arbitrary constants to be determined using the boundary conditions.

Expressions for rotation θ(x), moment M(x), shear force Q(x) and normal force N(x) are as follows:

$$\theta(x) = \frac{dw}{dx} = \alpha(A_1 + A_4)\sinh\alpha x \cos\alpha x + \alpha(A_4 - A_1)\cosh\alpha x \sin\alpha x + \alpha(A_2 - A_3)\sinh\alpha x \sin\alpha x + \alpha(A_2 + A_3)\cosh\alpha x \cos\alpha x + \frac{k\gamma}{4D\alpha^4}$$

$$\text{..(3.132)}$$

$$M(x) = -\frac{Dd^2w}{dx^2} = \frac{Eh^3\alpha^2}{6(1-\mu^2)} A_1 \sinh\alpha x \sin\alpha x - \frac{Eh^3\alpha^2}{6(1-\mu^2)} A_2 \sinh\alpha x \cos - \frac{Eh^3\alpha^2}{6(1-\mu^2)} A_3 \cosh\alpha x \sin\alpha x - \frac{Eh^3\alpha^2}{6(1-\mu^2)} A_4 \cosh\alpha x \cos\alpha x$$

$$\text{..(3.133)}$$

$$Q(x) = \frac{dm}{dx} = \frac{Eh^3\alpha^3}{6(1-\mu^2)}(A_1 + A_4)\cosh\alpha x \sin\alpha x + \frac{Eh^3\alpha^3}{6(1-\mu^2)}(A_1 - A_4)\sinh\alpha x \cos\alpha x + \frac{Eh^3\alpha^3}{6(1-\mu^2)}(A_3 - A_2)\cosh\alpha x \cos\alpha x + \frac{Eh^3\alpha^3}{6(1-\mu^2)}(A_2 + A_3)\sinh\alpha x \sin\alpha x$$

$$\text{..(3.134)}$$

$$N(x) = \frac{Eh}{R}w(x) = \frac{Eh}{R}A_1 \cosh\alpha x \cos\alpha x + \frac{Eh}{R}A_2 \cosh\alpha x \sin\alpha x$$
$$+ \frac{Eh}{R}A_3 \sinh\alpha x \cos\alpha x + \frac{Eh}{R}A_4 \sinh\alpha x \sin\alpha x + k\gamma Rx$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.135)$$

3.2.2.2 Application of Initial Value Method in the defining constant of Integration

Let $z = \alpha x$

In a similar manner as section (3.2.1.2), equation (3.128) can be put as:

$$\frac{d^4w}{dz^4} + 4W(z) = \frac{4k\gamma R^2}{Eh\alpha}z \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.136)$$

Therefore the homogeneous equation is:

$\frac{d^4w}{dz^4} + 4W(z) = 0$ whose solution has been obtained in section (3.2.1.2.1)

3.2.2.2.1 The Initial value Particular Solution

Consider the following figure:

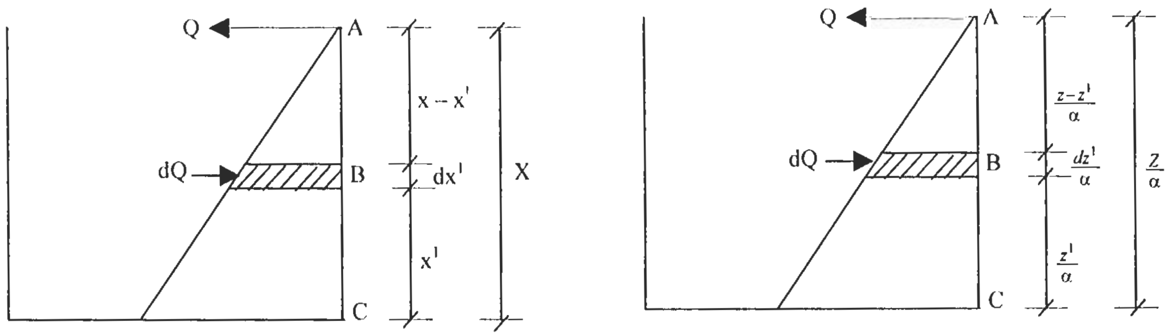


Fig. 3.7: Elemental Force Acting on Side of Grain Silo

To find the particular solution, an origin transformation from A to B is assumed, introducing at the same time a new variable x'.

Let $z' = \alpha x' \Rightarrow dx' = \dfrac{dz'}{\alpha}$

The distributed load at new origin B is given by

$$q(z') = \frac{z - z'}{\alpha} k\gamma$$

Therefore the elemental force can be expressed as

$$dQ = q(z').\frac{dz'}{\alpha} = \frac{k\gamma(z - z')}{\alpha^2} dz' \qquad (3.137)$$

To get the particular solution, we make use of the homogeneous solution in which only parameters similar to dQ are considered. Therefore the coefficients of Q_0 in the homogeneous solution are used with the appropriate sign to formulate the particular solutions:

$$dW_p = \frac{4\alpha R^2}{Eh} Y_4(z').\frac{k\gamma(z-z')}{\alpha^2} dz'$$

$$\Rightarrow dW_p = \frac{4k\gamma R^2}{Eh\alpha}(z-z')Y_4(z')dz' \ldots\ldots\ldots\ldots\ldots(3.138)$$

$$d\theta_p = \frac{4\alpha^2 R^2}{Eh} Y_3(z').\frac{k\gamma(z-z')}{\alpha^2} dz'$$

$$\Rightarrow d\theta_p = \frac{4k\gamma R^2}{Eh}(z-z')Y_3(z')dz' \ldots\ldots\ldots\ldots\ldots(3.139)$$

$$dM_p = -\frac{Y_2(z')}{\alpha}.\frac{k\gamma(z-z')}{\alpha^2} dz'$$

$$\Rightarrow dM_p = -\frac{k\gamma}{\alpha^3}(z-z')Y_2(z')dz' \ldots\ldots\ldots\ldots\ldots(3.140)$$

$$dQ_p = -Y_1(z').\frac{k\gamma(z-z')}{\alpha^2} dz'$$

$$\Rightarrow dQ_p = -\frac{k\gamma}{\alpha^2}(z-z')Y_1(z')dz' \ldots\ldots\ldots\ldots\ldots(3.141)$$

$$dN_p = 4\alpha R Y_4(z').\frac{k\gamma(z-z')}{\alpha^2} dz'$$

$$\Rightarrow dN_p = \frac{4Rk\gamma}{\alpha}(z-z')Y_4(z')dz' \ldots\ldots\ldots\ldots\ldots(3.142)$$

Integrating the above expressions gives the actual particular solutions:

$$W_p = \frac{4k\gamma R^2}{Eh\alpha}\left[Z\int_0^z Y_4(z')dz' - \int_0^z Z' Y_4(z')dz'\right]$$

Consider the term $\int_0^z Z'Y_4(z')dz'$, which will be solved using integration by parts.

Given that $\int vdu = uv - \int udv$,

Let $v = z'; dv = dz'$

$$du = Y_4(z'); u = -\tfrac{1}{4}Y_1(z')$$

$$\int_0^z z'Y_4(z')dz' = \left[-\tfrac{1}{4}z'Y_1(z') + \tfrac{1}{4}\int Y_1(z')dz'\right]_0^z$$

$$= \left[-\tfrac{1}{4}z'Y_1(z') + \tfrac{1}{4}Y_2(z')\right]_0^z$$

$$W_p = \frac{4k\gamma R^2}{Eh\alpha}\left[-\tfrac{1}{4}zY_1(z') + \tfrac{1}{4}z'Y_1(z') - \tfrac{1}{4}Y_2(z')\right]_0^z$$

$$\Rightarrow W_p = \frac{k\gamma R^2}{Eh\alpha}\left[z - Y_2(z)\right] \quad \text{.........................(3.143)}$$

$$\theta_p = \frac{4k\gamma R^2}{Eh}\left[Z\int_0^z Y_3(z')dz' - \int_0^z Z'Y_3(z')dz'\right]$$

Integration by parts to get:

$$\int_0^z z'Y_3(z')dz' = \left[z'Y_4(z') - \int Y_4(z')dz'\right]_0^z$$

$$= \left[z'Y_4(z') + \tfrac{1}{4}Y_1(z')\right]_0^z$$

$$\theta_p = \frac{4k\gamma R^2}{Eh}\left[zY_4(z') - z'Y_4(z') - \tfrac{1}{4}Y_1(z')\right]_0^z$$

$$\Rightarrow \theta_p = \frac{k\gamma R^2}{Eh}\left[1 - Y_1(z)\right] \quad \text{.......................(3.144)}$$

$$M_p = -\frac{k\gamma}{\alpha^3}\left[zY_2(z')dz' - \int_0^z z'Y_2(z')dz'\right]$$

Integration by parts to get:

$$\int_0^z z'Y_2(z')dz' = \left[z'Y_3(z') - \int Y_3(z')dz'\right]_0^z$$

$$= \left[z'Y_3(z') - Y_4(z')\right]_0^z$$

$$M_p = -\frac{k\gamma}{\alpha^3}\left[zY_3(z') - z'Y_3(z') + Y_4(z')\right]_0^z$$

$$\Rightarrow M_p = -\frac{k\gamma}{\alpha^3}Y_4(z) \quad \text{........................(3.145)}$$

$$Q_p = -\frac{k\gamma}{\alpha^2}\left[z\int_0^z Y_1(z')dz' - \int_0^z z'Y_1(z')dz'\right]$$

Integration by parts:

$$\int_0^z z'Y_1(z')dz' = \left[z'Y_2(z') - \int Y_2(z')dz'\right]_0^z$$

$$= \left[z'Y_2(z') - Y_3(z')\right]_0^z$$

$$Q_p = -\frac{k\gamma}{\alpha^2}\left[zY_2(z') - z'Y_2(z') + Y_3(z')\right]_0^z$$

$$\Rightarrow Q_p = -\frac{k\gamma}{\alpha^2}Y_3(z) \quad \text{........................(3.146)}$$

$$N_p = \frac{Eh}{R} W_p = \frac{Eh}{R} \cdot \frac{k\gamma R^2}{Eh\alpha} [z - Y_2(z)]$$

$$\Rightarrow N_p = \frac{k\gamma R}{\alpha} [z - Y_2(z)] \qquad (3.147)$$

3.2.2.2.2 The Initial Value General Solution

It is obtained by adding homogeneous and particular solutions:

$$W(z) = Y_1(z) W_0 + \left[\frac{Y_2(z)}{\alpha} \right] \theta_0 - \left[\frac{4\alpha^2 R^2}{Eh} Y_3(z) \right] M_0 - \left[\frac{4\alpha R^2}{Eh} Y_4(z) \right] Q_0 + \frac{k\gamma R^2}{Eh\alpha} [z - Y_2(z)] \qquad (3.148)$$

$$\theta(z) = -4\alpha Y_4(z) W_0 + Y_1(z) \theta_0 - \left[\frac{4\alpha^3 R^2}{Eh} Y_2(z) \right] M_0 - \left[\frac{4\alpha^2 R^2}{Eh} Y_3(z) \right] Q_0 + \frac{k\gamma R^2}{Eh} [1 - Y_1(z)] \qquad (3.149)$$

$$M(z) = \left[\frac{Eh}{\alpha^2 R^2} Y_3(z) \right] W_0 + \left[\frac{Eh}{\alpha^3 R^2} Y_4(z) \right] \theta_0 + Y_1(z) M_0 + \frac{Y_2(z)}{\alpha} Q_0 - \frac{k\gamma}{\alpha^3} Y_4(z) \qquad (3.150)$$

$$Q(z) = \left[\frac{Eh}{\alpha R^2} Y_2(z) \right] W_0 + \left[\frac{Eh}{\alpha^2 R^2} Y_3(z) \right] \theta_0 - 4\alpha Y_4(z) M_0 + Y_1(z) Q_0 - \frac{k\gamma}{\alpha^2} Y_4(z) \qquad (3.151)$$

$$N(z)=\left[\frac{Eh}{R}Y_1(z)\right]W_0+\left[\frac{Eh}{\alpha R}Y_2(z)\right]\theta_0-\left[4\alpha^2 RY_3(z)\right]M_0- \qquad \ldots\ldots\ldots\ldots(3.152)$$
$$\left[4\alpha RY_4(z)\right]Q_0+\frac{k\gamma R}{\alpha}\left[z-Y_2(z)\right]$$

Boundary conditions are applied in equations (3.148) through (3.152).

At Z = 0, {M (z) = W (z)} = 0 ⇒ $W_0 = M_0 = 0$

At Z = L, W(z) = θ(z) = 0 .... (3.153)

Substitution of z = αL and equation (3.153) into equations (3.148) and (3.149) which gives:

$$\frac{Y(\alpha L)}{\alpha}\theta_0-\frac{4\alpha R^2}{Eh}Y_4(\alpha L)Q_0+\frac{4\alpha R^2}{Eh\alpha}\left[\alpha L-Y_2(\alpha L)\right]=0\ldots\ldots(3.154)$$

$$Y_1(\alpha L)\theta_0-\frac{4\alpha^2 R^2}{Eh}Y_3(\alpha L)Q_0+\frac{k\gamma R^2}{Eh}\left[1-Y_1(\alpha L)\right]=0\ldots\ldots\ldots(3.155)$$

Solving equations (3.154) and (3.155), to obtain:

$$\theta_0=\frac{k\gamma R^2\left[Y_4(\alpha L)+Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)-\alpha L Y_3(\alpha L)\right]}{Eh\left[Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)\right]}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.156)$$

$$Q_0=\frac{k\gamma\left[Y_2(\alpha L)-\alpha L Y_1(\alpha L)\right]}{4\alpha^2\left[Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)\right]}\ldots\ldots\ldots(3.157)$$

Substituting for W_0, M_0, θ_0, and Q_0 in equations (3.148) through (3.152) to get:

$$W(z) = \frac{k\gamma R^2\left[Y_4(\alpha L) + Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L) - \alpha L Y_3(\alpha L)\right]}{\alpha Eh\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]} Y_2(z)$$

$$- \frac{k\gamma R^2\left[Y_2(\alpha L) - \alpha L Y_1(\alpha L)\right]}{\alpha Eh\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]} Y_4(z)$$

$$+ \frac{k\gamma R^2}{\alpha Eh}\left[z - Y_2(z)\right] \qquad (3.158)$$

$$\theta(z) = \frac{k\gamma R^2\left[Y_4(\alpha L) + Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L) - \alpha L Y_3(\alpha L)\right]}{Eh\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]} Y_1(z)$$

$$- \frac{k\gamma R^2\left[Y_2(\alpha L) - \alpha L Y_1(\alpha L)\right]}{Eh\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]} Y_3(z)$$

$$+ \frac{k\gamma R^2}{Eh}\left[1 - Y_1(z)\right] \qquad (3.159)$$

$$M(z) = \frac{k\gamma\left[Y_4(\alpha L) + Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L) - \alpha L Y_3(\alpha L)\right]}{\alpha^3\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]} Y_4(z)$$

$$+ \frac{k\gamma\left[Y_2(\alpha L) - \alpha L Y_1(\alpha L)\right]}{4\alpha^3\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]} Y_2(z)$$

$$- \frac{k\gamma}{\alpha^3} Y_4(z) \qquad (3.160)$$

$$Q(z) = \frac{k\gamma\left[Y_4(\alpha L) + Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L) - \alpha L Y_3(\alpha L)\right]}{\alpha^2\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]} Y_3(z)$$

$$+ \frac{k\gamma\left[Y_2(\alpha L) - \alpha L Y_1(\alpha L)\right]}{4\alpha^2\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]} Y_1(z)$$

$$- \frac{k\gamma}{\alpha^2} Y_3(z) \qquad (3.161)$$

$$N(z) = \frac{k\gamma R\left[Y_4(\alpha L) + Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L) - \alpha L Y_3(\alpha L)\right]}{\alpha\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]} Y_2(z)$$

$$- \frac{k\gamma R\left[Y_2(\alpha L) - \alpha L Y_1(\alpha L)\right]}{\alpha\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]} Y_4(z)$$

$$+\frac{k\gamma R}{\alpha}\left[z-Y_2(z)\right]\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.162)$$

The Initial value general solutions given in equations (3.158) through (3.162) are used to evaluate the performance of the cylindrical shell structure under foodgrain pressures.

CHAPTER FOUR

MATERIALS AND METHODOLOGY

The materials of construction for the intended shell of laterized concrete are obtained from the rational combination of cement, laterite, gravel and water. Due to the fact that variation in proportion of any of the component materials of laterized concrete results in variation in its strength characteristics, there is need to optimize the mix ratio Thus, a mathematical model, which relates the strength of the laterized concrete and its component ratio, need to be developed. To achieve this, Scheffe (1958) simplex lattice design employing a second order polynomial for quaternary mixtures, as adopted by Nwakonobi et. al (1998), was used to formulate the optimal model.

The outlines of the model and the method of determination of various structural characteristics of laterized concrete are presented in this chapter.

4.1 The Outline of the Laterized Concrete Mix Ratio Optimization Model

4.1.1 Simple Lattice Design

In mixture experiments involving the study of properties of a q-component mixture, which are dependent on the component ratio only, the factor space is a regular, (q – 1)–simplex (Scheffe, 1958). The relationship that holds for the component of the mixture is as shown in equation (4.1) below:

$$\sum_{i=1}^{q} X_i = 1 \qquad (4.1)$$

where:

$X_i \geq 0$ = the component concentration

q = the number of components

For binary systems q = 2, the simplex is a straight-line segment. Each end of the straight line represents straight component. When q = 3, the simplex is an equilateral triangle with its interior. Vertices of the triangle represent straight substances, and sides represent binary systems. Each point in the triangle corresponds to a certain composition of the ternary system. For quaternary system, q = 4, the regular simplex is a tetrahedron where each vertex represents a straight component, an edge represents a binary system, and a face a ternary system. Points inside the tetrahedron correspond to quaternary systems (see Fig. 4.1).

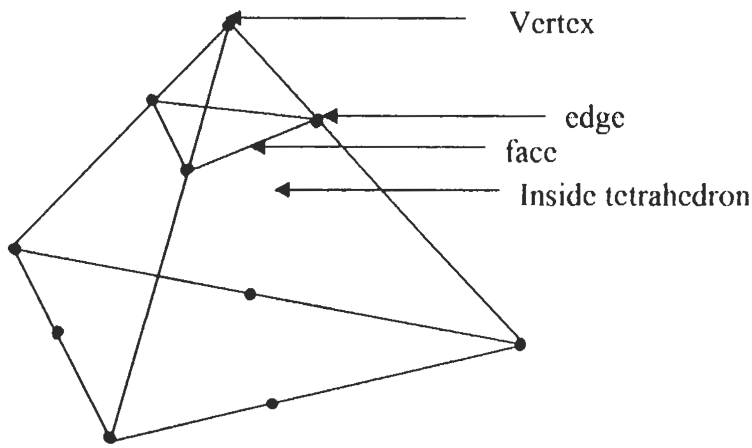


Fig. 4.1: Tetrahedron and representative Points

The componeint X_1 is therefore, absent in the face X_2, X_3, X_4, but as tetrahedron sectons parallel to the face approach vertex X_1, component X_1 in them grows in concentration.

The response function (property) in a multi-component system can be approximated by a polynomial. To describe such a function adequately, high degree polynomials are required and hence a great many experimental trials. A polynomial of degree n in q variables has $C_q^n + n$ coefficients

$$\hat{y} = b_0 + \sum_{1\le i\le q} b_i X_i + \sum_{1\le i\le j\le q} b_{ij} X_i X_j + \sum_{1\le i\le j\le k\le q} b_{ijk} X_i X_j X_k + \ldots + \sum b_{i,i2 \ldots i_n} x_{i_1} x_{i_2}\, x_{i_n} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.2)$$

The relationship given in equation (4.1) enables the equation components to be eliminated and the number of coefficients reduced to $C_q^n + n - 1$. But it is required that all the q components be introduced into the model.

Scheffe (1958) suggested to describe mixture properties by reduced polynomials from Equation (4.2) subject to the normalization condition of Equation (4.1) for a sum of independent variables. The reduced second-degree polynomial for a quaternary system is derived as follows:

$$\hat{Y} = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_{12}X_1X_2 + b_{13}X_1X_3$$

$$+ b_{14}X_1X_4 + b_{23}X_2X_3 + b_{24}X_2X_4 + b_{34}X_3X_4 + b_{11}X_1^2$$

$$+ b_{22}X_2^2 + b_{33}X_3^2 + b_{44}X_4^2 \quad \ldots\ldots (4.3)$$

$$\text{since } X_1 + X_2 + X_3 + X_4 = 1 \quad \ldots\ldots (4.4)$$

Then $b_0X_1 + b_0X_2 + b_0X_3 + b_0X_4 = b_0$ ...... (4.5)

Multiplying Equation (4.4) by X_1, X_2, X_3, and X_4 in succession gives

$$\left.\begin{aligned} X_1^2 &= X_1 - X_1X_2 - X_1X_3 - X_1X_4 \\ X_2^2 &= X_2 - X_1X_2 - X_2X_3 - X_2X_4 \\ X_3^2 &= X_3 - X_1X_3 - X_2X_3 - X_3X_4 \\ X_4^2 &= X_4 - X_1X_4 - X_2X_4 - X_3X_4 \end{aligned}\right\} \quad \ldots\ldots (4.6)$$

Substituting Equation (4.5) and Equation (4.6) in Equation (4.3) and transforming, it is obtained as:

$$\hat{Y} = (b_0 + b_1 + b_{11})X_1 + (b_0 + b_2 + b_{22})X_2 + (b_0 + b_3 + b_{33})X_3 + (b_0 + b_4 + b_{44})X_4$$

$$+ (b_{12} - b_{11} - b_{22})X_1X_2 + (b_{13} - b_{11} - b_{33})X_1X_3 + (b_{14} - b_{11} - b_{44})X_1X_4$$

$$+ (b_{23} - b_{22} - b_{33})X_2X_3 + (b_{24} - b_{22} - b_{44})X_2X_4 + (b_{34} - b_{33} - b_{44})X_3X_4 \qquad \text{.......................(4.7)}$$

Designating:

$$\beta_i = b_0 + b_i + b_{ii}\,;\ \beta_{bj} = b_{ii} - b_{jj} \qquad \text{..........................(4.8)}$$

The reduced second-degree polynomial in four variables is thus arrived at as follows:

$$\hat{Y} = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{23}X_2X_3 + \beta_{24}X_2X_4 + \beta_{34}X_3X_4 \qquad \text{.......................... (4.9)}$$

Thus the number of coefficients is reduced from fifteen to ten. The reduced second-degree polynomial in q variables is

$$\hat{Y} = \sum_{1\le i\le q} \beta_iX_i + \sum_{1\le i\le j\le q} \beta_{ij}X_iX_j \qquad \text{...................................(4.10)}$$

Thus, the minimum number of points required to determine the coefficients of a polynomial of degree 'n' in 'q' variables is equal to $C_q^n + n - 1$ (Table 4.1).

Table 4.1: Number of Experiments Required to obtain the Coefficients of various degrees

No. of Components	Degree		
	2	3	4
3	6	10	15
4	10	20	35
5	15	35	70
6	21	41	126
8	36	92	330
10	55	175	715

Source: Akhnazarova and Kafarou (1982)

4.1.2 Scheffe's Simplex-Lattice Experimental Designs

The most common simplex lattice experimental designs were proposed by Scheffe (1958) and Zedginidze (1971). These designs provide a uniform scatter of points over the (q – 1) – simplex. The points form a (q, n) – lattice on the simplex where 'q' is the number of mixture components, 'n' is the degree of polynomial. Simplex-lattice designs are saturated. For each component there exist (n + 1) similar levels, $x_i = 0, {}^1\!/_n, {}^2\!/_n, \ldots, 1$, and all possible combinations are derived with such values of component concentration. Employing a quadratic lattice (q, n), where 'n' the number of degree of polynomial (n = 2), the levels of every factor that must be used are 0, ½ and 1. The (4, 2) – lattice is shown in Fig. 4.2

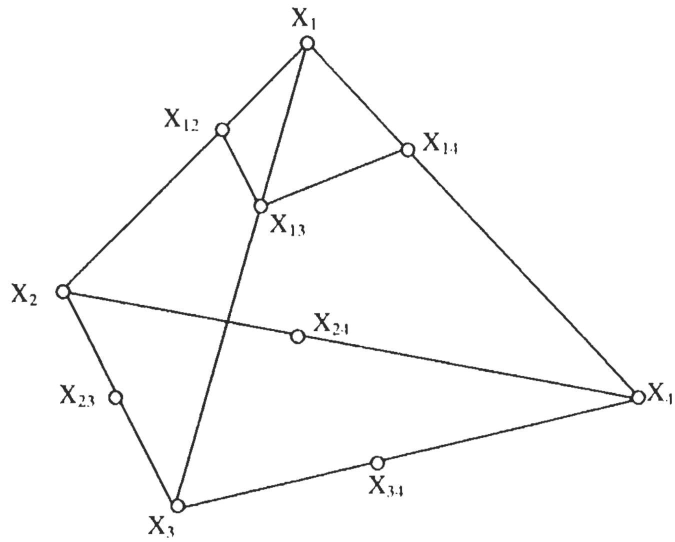


Fig. 4.2: The (4, 2) – Lattice
Source: Akhnazarova and Kafarou (1982)

Having written the coordinates of points of the simplex lattice, the design matrix is obtained as shown in Table 4.2 for the (4, 2) – lattice.

From this Table, the subscripts of the mixture property symbols represented by Y_{exp} indicate the relative content of each component in the mixture. For example, the mixture 1 contains the component X_1 alone, the property of the mixture is denoted by Y_1, mixture 5 contain (½) X_1 and (½) X_2, the property is designated as Y_{12}.,etc.

4.1.3 Derivation of the Coefficients of the Polynomial

Coefficients of this polynomial are derived using the design saturation property. To obtain the coefficients of the polynomial of Equation (4.9),

Table 4.2: Experimental Design Matrix for a (4, 2) –Lattice

N	X_1	X_2	X_3	X_4	Y_{exp}
1	1	0	0	0	Y_1
2	0	1	0	0	Y_2
3	0	0	1	0	Y_3
4	0	0	0	1	Y_4
5	0.5	0.5	0	0	Y_{12}
6	0.5	0	0.5	0	Y_{13}
7	0.5	0	0	0.5	Y_{14}
8	0	0.5	0.5	0	Y_{23}
9	0	0.5	0	0.5	Y_{24}
10	0	0	0.5	0.5	Y_{34}

source : Akhnazarova and Kafarou (1982)

coordinates of all the ten points of the design matrix shown in Table 4.2 above are substituted in succession into the equation.

Substituting the coordinates of the first point ($X_1 = 1, X_2 = 0, X_3 = 0, X_4 = 0$) gives

$$Y_1 = \beta_1 \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.11)$$

The coordinates of the second point ($X_1 = 0, X_2 = 1, X_3 = 0, X_4 = 0$) when substituted gives

$$Y_2 = \beta_2 \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.12)$$

Similarly, $Y_3 = \beta_3$ and $Y_4 = \beta_4$.................. (4.13)

This implies that $Y_i = \beta_i$..................................(4.14)

The substitution of the coordinates of the fifth point yields:

$$Y_{12} = \tfrac{1}{2}\beta_1 + \tfrac{1}{2}\beta_2 + \tfrac{1}{4}\beta_{12} \quad \text{.................................. (4.15)}$$

But $\beta_1 = Y_i$, therefore

$$Y_{12} = \tfrac{1}{2}Y_1 + \tfrac{1}{2}Y_2 + \tfrac{1}{4}\beta_{12} \quad \text{.................................. (4.16)}$$

$$\text{Thus, } \beta_{12} = 4Y_{12} - 2Y_1 - 2Y_2 \quad \text{..(4.17)}$$

$$\text{Similarly, } \beta_{13} = 4Y_{13} - 2Y_1 - 2Y_3 \quad \text{..(4.18)}$$

$$\beta_{14} = 4Y_{14} - 2Y_1 - 2Y_4 \quad \text{..(4.19)}$$

$$\beta_{23} = 4Y_{23} - 2Y_2 - 2Y_3 \quad \text{..(4.20)}$$

$$\beta_{24} = 4Y_{24} - 2Y_2 - 2Y_4 \quad \text{..(4.21)}$$

$$\text{and , } \beta_{34} = 4Y_{34} - 2Y_3 - 2Y_4 \quad \text{..(4.22)}$$

The coefficients of the reduced second-degree polynomial for the q-component mixture

$$Y = \sum_{1 \le i \le q} \beta_i X i + \sum_{1 \le i \le j \le q} \beta_{ij} X i X_j$$

are determined in a similar manner, as

$$\beta_i = Y_i,\ \beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j \quad \text{..................................(4.23)}$$

4.1.4 Adequacy of the Regression Model

Having derived the coefficients of the regression equation, the statistical analysis becomes necessary. This equation and the response values predicted should be tested for adequacy of fit. In experimentation employing simplex-lattice design there are no degrees of freedom to test the equation for adequacy because the designs are saturated. Thus, to test the adequacy the experiments are run at additional so-called test points. The number of the control points and their coordinates are conditioned by the problem formulation and experiment nature.

Besides, the control points are sought so as to improve the model in case of inadequacy. The accuracy of response prediction is dissimilar at different points of the simplex. The variance of response $S^2_{\bar{y}}$ is obtainable from the accumulation law. For a second degree polynomial

$$S^2_{\bar{y}} = S^2_y \left(\sum_{1 \le i \le q} \frac{a_i^2}{n_i} \qquad \sum_{1 \le i \le j \le q} \frac{a_{ij}^2}{n_{ij}} \right) \ldots\ldots\ldots\ldots (4.24)$$

$$\text{where } a_i = X_i\,(2X_i - 1), \quad a_{ij} = 4X_i\,X_j \ldots\ldots\ldots\ldots\ldots\ldots (4.25)$$

If the number of replicate observation at all the points of the design is equal, ie. $n_i = n_{ij} = n$, then all the relations for $S^2_{\bar{y}}$ will take the form

$$S^2_{\bar{y}} = S^2_y \frac{\xi}{n} \ldots\ldots \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.26)$$

where for the second degree polynomial

$$\xi = \sum_{1 \le i \le q} a_i^2 + \sum_{1 \le i \le j \le q} a_{ij}^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.27)$$

Adequacy is tested at each control point using the following t-statistic

$$t = \frac{\Delta Y\sqrt{n}}{S_y^2\sqrt{1+\xi}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.28a)$$

$$\text{where } \Delta Y = \left|\widehat{Y}_{\exp} - Y_{theory}\right| \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.28b)$$

n = number of parallel observations at every point.

The t-statistic has the student distribution and it is compared with the tabulated value of $t_\alpha/L_{(v)}$ at a level of significance α, where L is the number of control points and V is the number of degrees of freedom for the replication variance.

The null hypothesis that the equation is adequate is accepted if $t_{expt} < t_{table}$ for all the control points. Adequacy can also be tested at several points using the χ^2 test (Chemleva and Mikeshina, 1969).

More formally the Hypotheses are:

(1) The null hypothesis, H_0; there is no significant difference between the theoretical and experimental results.

(2) The alternative hypothesis, or H_A: there is a significant difference between the theoretical and experimental results.

The confidence interval for response values denoted by

CONF $(Y_1 \leq Y \leq Y_2)$ is

$$\hat{Y} - \Delta \leq Y \leq \hat{Y} + \Delta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.29)$$

Akhnazarova and Kafarou (1982) give 'Δ' as;

$$\Delta = t_{\alpha/k}, V_e \frac{S_y}{\sqrt{n}} \xi^{½} \text{..............................} \textbf{(4.30)}$$

where,

k is the number of polynomial coefficient determined.

n is the number of parallel observations.

4.1.5 Component Transformation

It can easily be understood that the real component concentrations of laterized concrete mixtures cannot be simply transformed to determine some X_1, X_2, X_3, X_4 that is subject to

$$\sum_{i=1}^{4} X_i = 1$$

This will be accomplished by a transformation from actual components (0.650, 1, 1, 2) to pseudo-components (X_1, X_2, X_3, X_4) for the N_{th} experimental point.

The arbitrarily chosen component concentration of the tetrahedron vertices are A_1 (0.650, 1, 1, 2), A_2 (0.75, 1, 2, 4), A_3 (1.10, 1, 3, 6), A_4 (1.14, 1, 3, 8). See Fig. 4.3

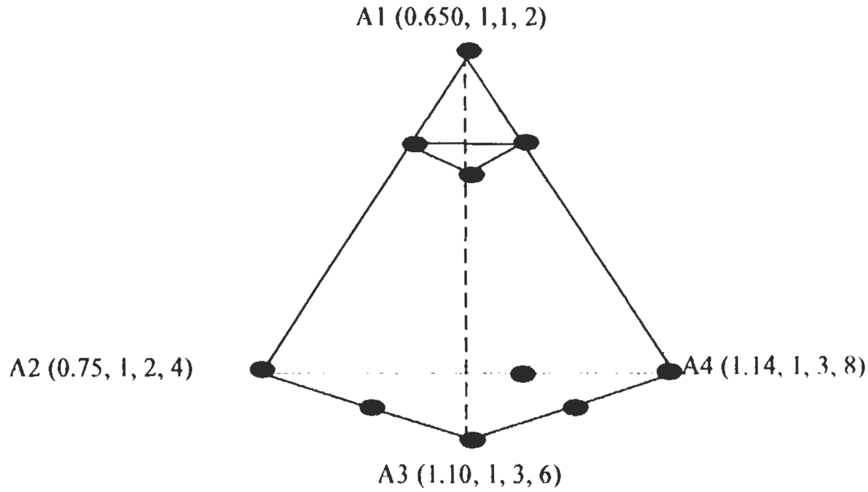


Fig. 4.3: Tetrahedron Vertices for (4, 2) Lattice

From Table 4.2, it can be seen that the required transformation can be depicted as follows:

$$Z\begin{bmatrix} 0.65 & 0.75 & 1.10 & 1.14 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 3 \\ 2 & 4 & 6 & 8 \end{bmatrix} \xrightarrow{\textit{transforming}} X\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus, if D denotes the transformation matrix, then the relation can be better expressed as:

$$D.Z = X = I \quad \text{..............................(4.31)}$$

$$D = I.Z^{-1} \quad \text{..............................(4.32)}$$

$$D = Z^{-1} \quad \text{..............................(4.33)}$$

The pseudo-component can be depicted giving the transformation matrix B and any actual component (Z_1, Z_2, Z_3, Z_4). This transformation will yield the

X_i values for the first four experimental points in Table 4.2. Since the X_i values for the remaining six points are known it would be necessary to determine their Z_i values (ie. actual components). The inverse transformation from pseudo components to actual components is expressed as

$$A.X = Z \qquad (4.34)$$

where A is the inverse transformation matrix

$$A = Z.X^{-1} \qquad (4.35)$$

But X = DZ from Equation (4.31)

$$A = Z.(Z.D)^{-1}$$

$$A = Z.Z^{-1}.D^{-1}$$

$$A = I.D^{-1}$$

$$A = D^{-1} \qquad (4.36)$$

Thus for any pseudo-component

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

The actual component is given by

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{bmatrix} = \begin{bmatrix} 0.650 & 0.750 & 1.10 & 1.14 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 3 \\ 2 & 4 & 6 & 8 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} \qquad \ldots\ldots\ldots\ldots 4.37$$

This is employed to determine the components for points 5 to 10 (see Table 4.3). It is limited in that the first four points are chosen arbitrarily. When X_is and Z_is are known, the inverse matrix is used to determine the actual components, Z_is, known X_is (the pseudo components).

This work is limited to ten control points. These were chosen so that they could be incorporated in the new design eg. the (4, 3) lattice should the (4, 2) lattice not fit adequately, thus the model can be refined.

The pseudo components chosen and their actual components (determined from the inverse transformation) are given in Tables 4.3 and 4.4).

Table 4.3: Actual Components for the Experimental Points 1 to 10

N	X_1	X_2	X_3	X_4	Y_{exp}	Z_1	Z_2	Z_3	Z_4
1	1	0	0	0	Y_1	0.65	1	1	2
2	0	1	0	0	Y_2	0.75	1	2	4
3	0	0	1	0	Y_3	1.10	1	3	6
4	0	0	0	1	Y_4	1.14	1	3	8
5	0.5	0.5	0	0	Y_{12}	0.70	1	1.5	3
6	0.5	0	0.5	0	Y_{13}	0.875	1	2	4
7	0.5	0	0	0.5	Y_{14}	0.895	1	2	5
8	0	0.5	0.5	0	Y_{23}	0.925	1	2.5	5
9	0	0.5	0	0.5	Y_{24}	0.945	1	2.5	6
10	0	0	0.5	0.5	Y_{34}	1.12	1	3	7

Table 4.4: Actual Components for the Test Points

N	X_1	X_2	X_3	X_4	C_{exp}	Z_1	Z_2	Z_3	Z_4
1	0.75	0.25	0	0	C_1	0.675	1	1.25	2.5
2	0	0.75	0	0.25	C_2	0.848	1	2.25	5
3	0.25	0	0.75	0	C_3	0.988	1	2.5	5
4	0	0.25	0.75	0	C_4	1.013	1	2.75	5.5
5	0.5	0.25	0.25	0	C_5	0.788	1	1.75	3.5
6	0.5	0	0.25	0.25	C_6	0.885	1	2	4.5
7	0.25	0	0.5	0.25	C_7	0.998	1	2.5	5.5
8	0	0.25	0.5	0.25	C_8	1.023	1	2.75	6
9	0.25	0	0.25	0.5	C_9	0.998	1	2.5	6
10	0.25	0.25	0.25	0.25	C_{10}	0.910	1	2.25	5

4.2 Optimization and Structural Characteristics Determination Tests

4.2.1 Sources of Material

The lateritic soil for this investigation was collected from Nsukka, the project area. The coarse aggregate used was crushed granite of igneous origin with a size range of 9 – 14mm. Ordinary Portland Cement the properties which conform to the British Standard (BS 110) part 2 of 1970 was used in this study while water was drawn from the nearest clean water source. The determination tests were carried out in the Laboratory of Civil Engineering Department, University of Nigeria, Nsukka.

4.2.2 Sample Preparation

The lateritic soil was sieved so as to exclude the clay content of lateritic soil as well as the coarse aggregate. The sample was passed through two sieves with different size opening of 4.75mm and 0.3m respectively. The two sets of sieves used were separated and rigidly fitted on a wooden frame as shown in Fig. 4.4. The size range of the sample needed was passed through the upper sieve and retained above the lower sieve. The crushed granite with almost uniform size of 12 mm was prepared to BS 1017: part 1 and 2.

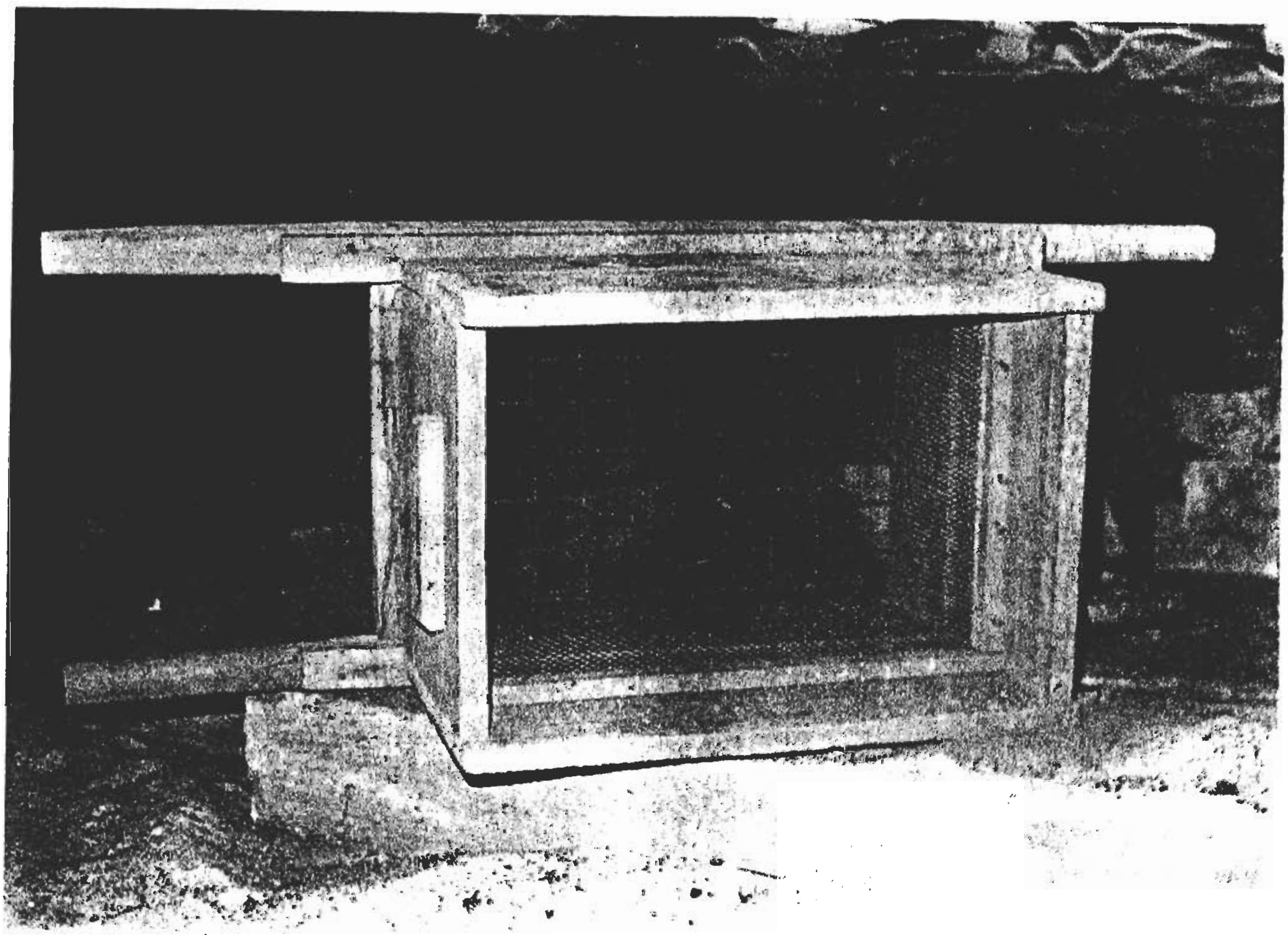


Plate 4.1: Wire Mesh on a Wooden Frame

4.2.3 Mixture Design

Scheffe's simplex lattice experimental design was used to obtain data for fitting a regression equation which relates the compressive strength of the laterized concrete, Y and its component ratios of cement (X_1), laterite (X_2), gravel (X_3) and water cement ratio (X_4). In this design the following relationship exists:-

$$\sum_{i=1}^{4} X_i = 1 \;;\; X_i \geq 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.38)$$

This relationship was transformed to establish the actual component concentration as earlier explained in the preceding section. The number of test points in the reduced second-degree polynomial for a quaternary mixture is ten (10). Thus, ten mixtures of different compositions were established. Ten additional points were selected inside the tetrahedron for test of adequacy of the fitted regression model that was developed.

4.2.4 Batching and Mixing of Specimens

The laterized concrete was obtained from the predetermined mix proportions of water – cement ratio, cement, fine aggregate and coarse aggregate. Batching was by weight using Avery weighing scale. Different mixtures of cement, lateritic soil and gravel, were prepared and 'worked' manually using shovel to stir. The working process involved the gradual addition of predetermined quantities of water to the mixtures already made and the continuous stirring with a shovel until a workable mix was obtained.

Tests for Determination of Structural Characteristics

The test for strength in compression was chosen for obtaining the optimal model. This is because it is relatively simple to perform and compressive strength is commonly used in the construction industry for the purpose of specification and quality control (Jackson, 1983; Neville, 1996).

The mix ratio for the optimum strength was therefore used to prepare the laboratory specimens which enabled the determination of some other structural characteristics of laterized concrete which include: modulus of elasticity, shear modulus, flexural strength, Poisson's ratio and density. Another mix ratio of lesser strength was also used for the determination of the above structural characteristics of laterized concrete for relative comparative reasons.

4.2.5.1 Compressive Strength Determination

The compressive strength of laterized concrete is taken as the maximum compressive load it can carry per unit area. According to Neville

(1996), the most common of all tests on hardened concrete is the compressive strength test, partly because it is an easy test to perform, and partly because many, of the desirable characteristics of concrete are qualitatively related to its strength, but mainly because of the intrinsic importance of the compressive strength of concrete in structural design. In this test, cube specimens of size 150 mm x 150 mm were made and tested for compressive strength. The preparation of the test cubes was done in accordance with BS 1881: part 108:1983. Each specimen was made by filling each mould in three layers and compacting manually with 25 mm diameter rod. On each layer, 35 strokes were delivered. Demoulding was performed in accordance with BS 1881: part 111: 1983. The specimens after being demoulded were submerged in a water tank. The curing was in accordance with BS 1881: part 111:1983. The cubes specimens were cured for 28 days.

The cubes while still wet after removal from curing tank were placed with the cast face in contact with the platens of the testing machine as shown in Fig. 4.4. That is, the position of the cubes when tested was at right angle to that as-cast. This was in accordance with the specification of BS 1881: part 116:1983. The load was applied without shock at a loading rate of 15 N/mm^2.min until no greater load could be sustained.

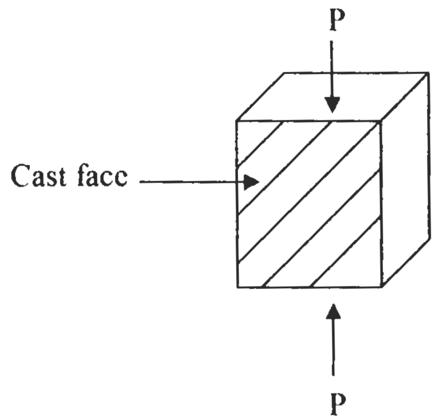


Fig. 4.4: Position of Cube for compressive strength test.

Two replicate samples were prepared for each batch. Therefore for the ten experimental points and ten control points, a total of 40 cubes were cast and tested. The compressive strength (Y) of each cube was obtained from the ratio:

$$Y = \frac{Maximum\ Load}{Cross\ Sectional\ Area}\ {}^{N}\!/\!_{mm^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.39)$$

4.2.5.2 Flexural Strength Test

Flexural Strength (tensile strength in bending) of laterized concrete is of considerable importance in resisting cracking due to drying or temperature variation. Values of the flexural strength are needed in some method of design of unreinforced laterized concrete, as reliance will only be placed on its flexural strength to distribute concentrated loads over a wide area.

The measurement of the strength of concrete material in direct tension is difficult and indirect methods have been developed for assessing this property. The flexure test is most commonly used and is employed in this study. In this test, beam specimens of size 100 x 100 x 500 mm were moulded and tested for tensile strength. The specimens have a slenderness ratio of 5. The making, curing and the method of test were done in accordance with BS 1881: part 118:1983. Each specimen was made by filling each mould in three layers. Each layer was compacted manually by using 25mm diameter rod to deliver 150 strokes on the layer. The specimens were water-cured to standard curing age of 28 days.

For the testing of the specimens, the third-point loading method was used (Fig. 4.5). The load was applied through two rollers at the third points of the span until the specimen broke. Three replications were used for each sample of mix ratio. Six beam specimens were therefore tested for the two mix ratios. The flexural strength of laterized concrete was calculated from the relation:

$$F_{tb} = PL/bd^2 \qquad (4.40)$$

where: F_{tb} = flexural strength, N/mm^2

P = maximum total load on beam, N

L = length of beam specimen, mm

b = width of the beam, mm

d = depth of the beam, mm

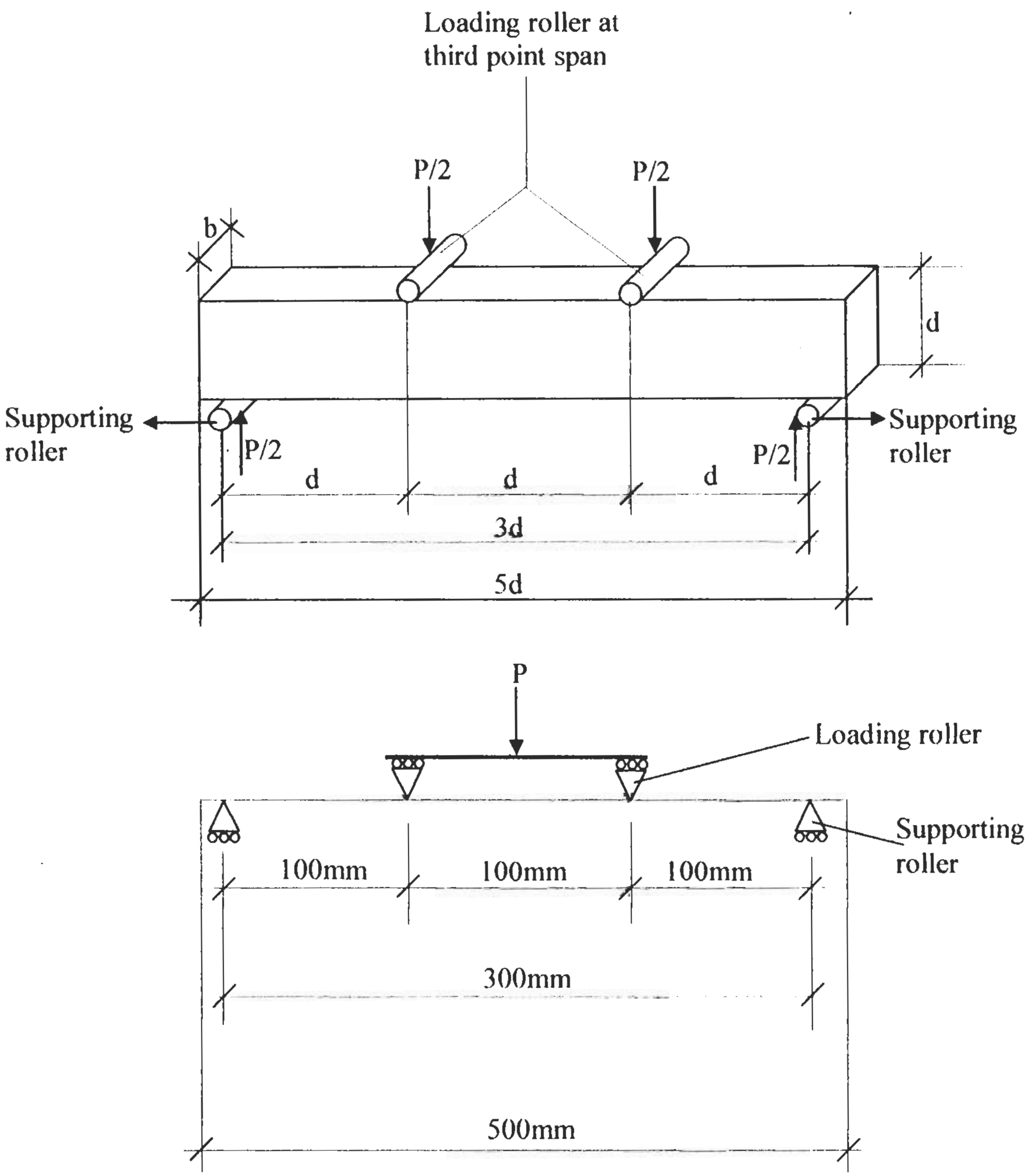


Fig. 4.5: Arrangement for Flexure Test

4.2.5.3 Static Modulus of Elasticity Test

Static modulus of elasticity gives the strain response to an applied stress of known intensity (Neville, 1996). It is a measure of the elastic property of a material. For practical purposes, it is usual to determine the modulus at an arbitrarily selected stress, applied at a defined rate, which takes into account movement due to creep. BS 1881:part 121:1983 prescribes that the basic loading level should be 0.5 N/mm^2 and the upper loading level, to be one-third of the compressive strength for determination of the modulus, which is known as the secant modulus of elasticity.

In this test, cylinder specimens were made, cured and stored in accordance with BS 1881:part 121:1983. For the three replications used for each mix ratio, a total of six specimens were cast and tested. All specimens were tested in moist condition, i.e. immediately after removal from the water tank. The test specimens were placed centrally in the compression machine and the strain gauge fixed accordingly (see Fig. 4.6). The basic load of 0.5 N/mm^2 (σ_b), was applied and the dial/strain gauge readings were taken both in lateral and in axial or longitudinal direction. The load was increased at a constant rate of 1 $N/mm^2.s$ until the load equal to one-third of the predetermined compressive strength of laterized concrete was reached (upper loading level). The load was maintained for 60 seconds and strain readings were recorded within 30 seconds. One additional preloading cycle

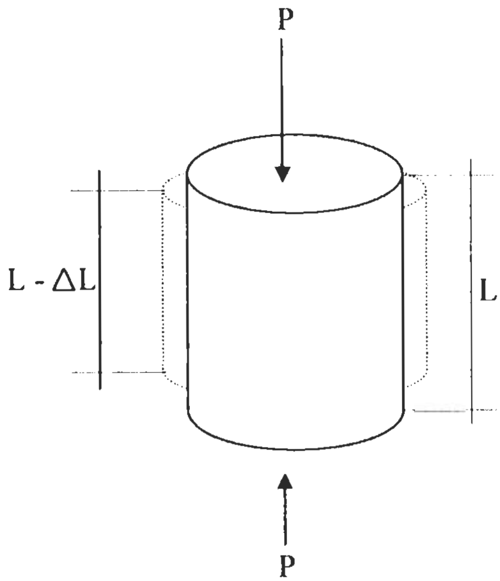


Fig. 4.6: Deformation of Test Cylinder

was carried out by using the same loading and unloading rate. The specimen was reloaded to upper loading level (σ_a) at the specified rate and the strain readings were recorded within 30 seconds. The static modulus of elasticity in compression was estimated using below formula:

$$E_{LC} = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{\sigma_a - \sigma_b}{\epsilon_a - \epsilon_b} \qquad \text{(4.41)}$$

where, E_c = modulus of elasticity, N/mm^2

σ_a = the upper loading stress in N/mm^2 ($\sigma_a = \frac{F_c}{3}$)

σ_b = the basic or initial loading level (ie. 0.5 N/mm^2)

ϵ_a = mean strain under the upper loading stress

ϵ_b = mean strain under the basic stress

F_c = compressive strength of laterized concrete.

4.2.5.4 Poisson's Ratio

When a uniaxial load is applied to a concrete specimen it produces a longitudinal strain in the direction of the applied load and, at the same time, a lateral strain of opposite sign. The ratio of the lateral strain to the longitudinal strain is called poisson's ratio; the sign of the ratio is ignored. Normally, we are usually interested in the consequences of an applied compression so that we have axial contraction and lateral extension. The value of poisson's ratio under tensile load is reported to be the same in compression (Neville, 1996).

The design and analysis of some structures require the knowledge of poisson's ratio (Neville and Brooks, 1987). In this case the lateral strain accompanying the axial strain to the applied axial compressive stress were recorded in the static modulus of elasticity tests (BS 1881:part 121:1983). The poisson's ratio was calculated as the ratio of lateral strain to longitudinal strain (see Fig. 4.6).

Thus,

$$Poisson's\ ratio(\mu) = \frac{lateral\ strain}{longitudinal\ strain} \qquad (4.42)$$

4.2.5.5 Modulus of Rigidity Test

Shear stress is related to shear strain by the modulus of rigidity. It is the ratio of shear stress to the shear strain and is usually obtained by plotting torque on a cylinder versus the resulting angle of twist (MWPS, 1983). In practice, shearing of concrete is always accompanied by compression and tension caused by bending, and even in testing, it is impossible to eliminate an element of bending. Thus, it is not common for shear strength to be tested. However, a relationship between elasticity modulus and modulus of rigidity was adopted in order to estimate shear modulus of rigidity. It is given as:

$$G = \frac{E}{2(\mu + 1)} \qquad (4.43)$$

where: G = modulus of rigidity, N/mm^2

E = modulus of elasticity, N/mm^2

μ = poisson's ratio

4.2.5.6 Density Determination

The weights of tested specimens were obtained using the weighing scale. Three replicate specimen samples were made. The value of density was estimated from the relation:

$$Density(\gamma) = \frac{Mass(kg)}{Volume(m^3)} \quad \text{..} (4.44)$$

CHAPTER FIVE

RESULTS OF OPTIMIZATION AND STRUCTURAL CHARACTERISTICS DETERMINATIONS

The experimental data obtained was used to fit the regression equation and the steps for developing the mathematical model and optimization method are presented in this chapter. The results of determination of the various structural characteristics of laterized concrete at optimum mix proportion are also presented in this chapter.

5.1 Regression Equation for the compressive strength of laterized concrete

Table 5.1 shows the results of two parallel observations each, of the 10 design points and the 10 tests or control points of the (4, 2) lattice and replication variance of response.

The compressive strength (response, Y) of each cube was obtained from equation (4.39)

$$Y = \frac{Maximum\ Load}{Cross\ Sectional\ Area}\ {}^{N}\!/_{mm^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(5.1)$$

Number of degrees of freedom for replication variance,

$$V_e = \sum_{i=1}^{20} V_i = \sum_{i=1}^{20}(n_i - 1) = 20 \ldots\ldots\ldots\ldots\ldots(5.2)$$

where: n = 2 (number of parallel observation)

Replication Variance, $$S_y^2 = \sum_{i=1}^{20} \frac{S_i^2 V_i}{V_e} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.3)$$

Table 5.1:Compressive Strength (response)and Replication variance of Response

Exp. No (N)	Repli-Cation	Response (Y_1) N/mm^2	Response Designa-tion	$\sum_{i=1}^{n} Y_i$	$\bar{Y} = \frac{(\sum_{i=1}^{n} Y_i)}{n}$	$\sum_{i=1}^{n} Y_i^2$	$s_i^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n} Y_i^2 - \frac{1}{n}(\sum_{i=1}^{n} Y_i)^2\right]$
1	1 2	26.1 28.0	Y_1	54.1	27	1465.2	1.8
2	1 2	18.6 18.2	Y_2	36.8	18.4	677.2	0.1
3	1 2	16.4 16.8	Y_3	33.2	16.6	551.2	0.1
4	1 2	6.6 8.0	Y_4	14.6	7.3	107.56	1.02
5	1 2	25.7 25.2	Y_{12}	50.9	25.5	1295.5	0.1
6	1 2	18.0 17.6	Y_{13}	35.6	17.8	633.8	0.12
7	1 2	17.5 17.5	Y_{14}	35	17.5	612.5	0.0
8	1 2	16.9 17.2	Y_{23}	34.1	17	581.5	0.1
9	1 2	16.8 17.0	Y_{24}	33.8	16.9	571.2	0.02
10	1 2	8.8 9.7	Y_{34}	18.5	9.3	171.5	0.4
11	1 2	25.7 27.4	Yc_1	53.1	26.6	1411.3	1.5
12	1 2	17.4 17.2	Yc_2	34.6	17.3	598.6	0.02
13	1 2	16.8 17.2	Yc_3	34	17	578.1	0.1
14	1 2	16.5 16.8	Yc_4	33.3	16.7	554.5	0.1
15	1 2	23.0 23.5	Yc_5	46.5	23.3	1081.3	0.2
16	1 2	18.1 17.1	Yc_6	35.2	17.6	620.0	0.5
17	1 2	15.0 15.7	Yc_7	30.7	15.4	471.5	0.3
18	1 2	17.0 15.0	Yc_8	32.0	16.0	514.0	2.0
19	1 2	14.6 12.8	Yc_9	27.4	13.7	377.0	1.6
20	1 2	15.7 18.4	Yc_{10}	34.1	17.1	585.1	3.7
							$\sum_{i=1}^{20} S_i^2 = 13.78$

$$= \sum_{i=1}^{20} \frac{S_i^2}{V_e} = \frac{13.78}{20} = 0.689$$

Replication error, $S_y = \sqrt{0.689}$; $S_y = 0.830$

5.1.1 Determination of the Regression Equation

From Equation (4.23) and Table 5.1 the coefficients of the second degree equation are:

$\beta_1 = 27$, $\beta_2 = 18.4$, $\beta_3 = 16.6$, $\beta_4 = 7.3$

$\beta_{12} = 4Y_{12} - 2Y_1 - 2Y_2$

$= 4(25.5) - 2(27) - 2(18.4) = 11.2$

$\beta_{13} = 4(17.8) - 2(27) - 2(16.6) = -16$

$\beta_{14} = 4(17.5) - 2(27) - 2(7.3) = 1.4$

$\beta_{23} = 4(17) - 2(18.4) - 2(16.6) = -2$

$\beta_{24} = 4(16.9) - 2(18.4) - 2(7.3) = 16.2$

$\beta_{34} = 4(9.3) - 2(16.6) - 2(7.3) = -10.6$

Thus from Equation (4.9)

$$\hat{Y} = 27X_1 + 18.4X_2 + 16.6X_3 + 7.3X_4 + 11.2X_1X_2 - 16X_1X_3 + 1.4X_1X_4 - 2X_2X_3 + 16.2X_2X_4 - 10.6X_3X_4 \quad \text{...........(5.4)}$$

Equation (5.4) is the regression equation for the compressive strength of laterized concrete, as obtained in this study.

5.1.2 Tests of Adequacy of the Regression Model

The model is statistically analysed using t-statistic and χ^2-test. The adequacy of the model is tested against the experimental results of the control points. The hypothesis made include:

1. Null Hypothesis (H_0): there is no significant difference between the experimental and the theoretically expected results.
2. Alternative Hypothesis (H_A): there is a significant difference between the experimental and the theoretically expected results.

5.1.2.1 The t-Statistic for the Test Points

The theoretical prediction of the response (Y) is obtained by substituting the values of X_i for the test results (from Table 4.3) into equation (5.4). This is compared with the experimental results (Table 5.1) for adequacy of fit. For each test point the t-statistic is constructed from Equation (4.28a)

$$t = \frac{\Delta Y \sqrt{n}}{S_y^2 \sqrt{1+\xi}} \qquad (5.5)$$

S_y^2, ξ and ΔY are evaluated using equations (4.26), (4.27) and (4.28b) respectively. The number of replicate observation at each point is n = 2. The replicate error, $S_y = 0.830$. Table 5.2 shows the t-statistic of the test points. For number of degrees of freedom (d.f)

$V_e = 20$, number of control points, L = 10, significance level, $\alpha = 0.05$.

$$t_{\alpha/L}\,(V_e) = t_{0.05/10}(20)$$

$t_{0.005}(20) = 3.15$

Since t-table is » t-calculated in all the ten test points, the model formulated is very adequate. The null hypothesis that there is significant difference between the theoretical response (Y_{theory}) and the experimental results (Y_{exp}) should therefore be rejected.

5.1.2.2 χ^2-Test

χ^2-test was also used to test the adequacy of the model at the ten test points.

$$\chi^2 = \sum \frac{(O-E)^2}{E} \qquad (5.6)$$

where, O = observed result (Y_0)

E = expected result (Y_E)

$$= \frac{(26.6-26.95)^2}{26.95} + \frac{(17.3-18.7)^2}{18.7} + \frac{(17-16.2)^2}{16.2} + \frac{(16.7-16.6)^2}{16.6}$$

$$+ \frac{(23.3-21.5)^2}{21.5} + \frac{(17.6-17)^2}{17} + \frac{(15.4-13.6)^2}{13.6} + \frac{(16-14.2)^2}{14.2}$$

$$+ \frac{(13.7-12.4)^2}{12.4} + \frac{(17.1-17.3)^2}{17.3}$$

$= 0.005 + 0.105 + + 0.04 + + 0.001 + 0.151 + 0.021 + 0.238 + 0.228 + 0.105 + 0.002 = 0.896$

Table 5.2: t-Statistic for the Test Points

N	CN	i	j	a_i	a_{ij}	$\sum a_i^2$	$\sum a_{ij}^2$	ξ	Y_0	Y_E	ΔY	t
		1	2	0.375	0.75							
		1	3	0.375	0							
		1	4	0.375	0							
1	C_1	2	3	-0.125	0							
		2	4	-0.125	0							
		3	4	0	0							
		4	-	0	-	0.4531	0.5625	1.0156	26.6	26.95	0.35	0.41
		1	2	0	0							
		1	3	0	0							
		1	4	0	0							
2	C_2	2	3	0.375	0							
		2	4	0.375	0.75							
		3	4	0	0							
		4	-	-0.125	-	0.2969	0.5625	0.8594	17.3	18.7	1.4	1.7
		1	2	-0.125	0							
		1	3	-0.125	0.75							
		1	4	-0.125	0							
3	C_3	2	3	0	0							
		2	4	0	0							
		3	4	0.375	0							
		4	-	0	-	0.1875	0.5625	0.75	17	16.2	0.8	1.03
		1	2	0	0							
		1	3	0	0							
		1	4	0	0							
4	C_4	2	3	-0.125	0.75							
		2	4	-0.125	0							
		3	4	0.375	0							
		4	-	0	-	0.1719	0.5625	0.7344	16.7	16.6	0.1	0.13
		1	2	0	0.5							
		1	3	0	0.5							
		1	4	0	0							
5	C_5	2	3	-0.125	0.25							
		2	4	-0.125	0							
		3	4	-0.125	0							
		4	-	0	-	0.0469	0.5625	0.6094	23.3	21.5	1.8	2.4
		1	2	0	0							
		1	3	0	0.5							
		1	4	0	0.5							
6	C_6	2	3	0	0							
		2	4	0	0							
		3	4	-0.125	0.25							
		4	-	-0.125	-	0.0313	0.5625	0.5938	17.6	17.0	0.6	0.81
		1	2	-0.125	0							
		1	3	-0.125	0.5							
		1	4	-0.125	0.25							
7	C_7	2	3	0	0							
		2	4	0	0							
		3	4	0	0.5							
		4	-	-0.125	-	0.0625	0.5625	0.625	15.4	13.6	1.8	2.4
		1	2	0	0							
		1	3	0	0							
		1	4	0	0							
8	C_8	2	3	-0.125	0.5							
		2	4	-0.125	0.25							
		3	4	0	0.5							
		4	-	-0.125	-	0.0469	0.5625	0.6094	16.0	14.2	1.8	2.4
		1	2	-0.125	0							
		1	3	-0.125	0.25							
		1	4	-0.125	0.5							
9	C_9	2	3	0	0							
		2	4	0	0							
		3	4	-0.125	0.5							
		4	-	0	-	0.0625	0.5625	0.625	13.7	12.4	1.3	1.7
		1	2	-0.125	0.25							
		1	3	-0.125	0.25							
		1	4	-0.125	0.25							
10	C_{10}	2	3	-0.125	0.25							
		2	4	-0.125	0.25							
		3	4	-0.125	0.25							
		4	-	-0.125	-	0.1094	0.375	0.4844	17.1	17.3	0.2	0.28

number of degrees of freedom = 2

significance level, $\alpha = 0.05$

From χ^2-table = 6.0

χ^2-table > χ^2-calculated.

The model is found adequate in both student's t-test and χ^2-tests.

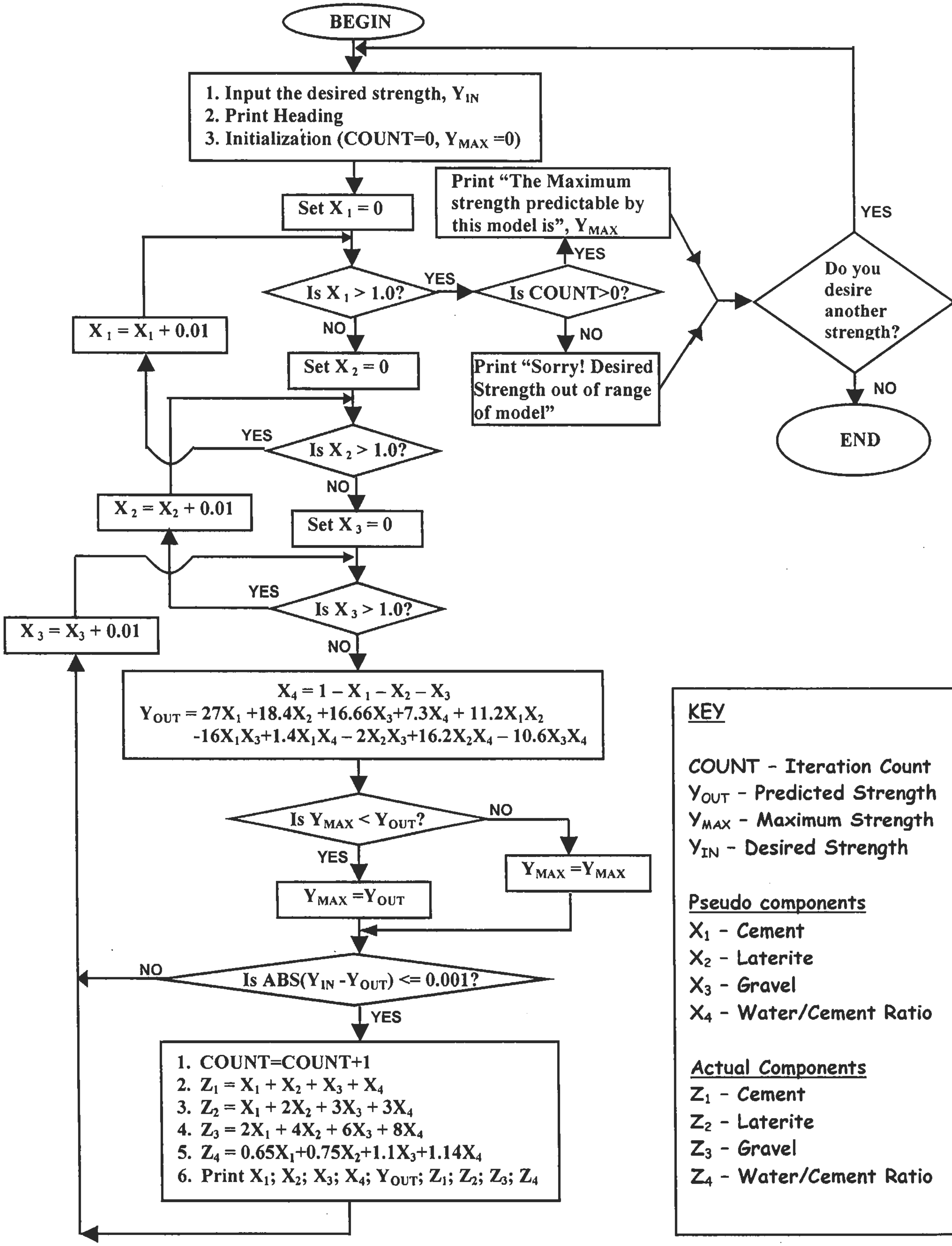
5.2 Development of a Flowchart

A flowchart for computation of laterized concrete mix proportions corresponding to a desired strength is shown in Fig. 5.1. The computation commenced by inputting a desired strength, Y_{IN}, which was pprinted as a heading. At the beginning, iteration count and maximum strength were set to zero. The loop for the first variable X_1 was entered and then that for the other variables X_2, X_3 and X_4 (setting them to zero). During the process of running the loops and any of the variables (X_1, X_2, X_3) was found to be greater than one, the iteration count was checked to know whether it was zero or not in case of X_1. But in the case of X_2 and X_3 being greater than one, the preceding variable was increased by 0.01.

On the other hand, when none of the variables (X_1, X_2, X_3) was greater than one,

Then; $X_4 = 1-X_1-X_2-X_3$ and,

$$Y_{out} = 27X_1 + 18.4X_2 + 16.6X_3 + 7.3X_4 + 11.2X_1X_2 - 16X_1X_3 + 1.4X_1X_4 - 2X_2X_3 + 16.2X_2X_4 - 10.6X_3X_4.$$



KEY

COUNT - Iteration Count
Y_{OUT} - Predicted Strength
Y_{MAX} - Maximum Strength
Y_{IN} - Desired Strength

Pseudo components
X_1 - Cement
X_2 - Laterite
X_3 - Gravel
X_4 - Water/Cement Ratio

Actual Components
Z_1 - Cement
Z_2 - Laterite
Z_3 - Gravel
Z_4 - Water/Cement Ratio

Fig. 5.1: Flow Chart for Computation of Laterized Concrete Mix Proportions Corresponding to a desire Strength

When the maximum strength, Y_{max}, was not less than output strength, Y_{out}, this implies that

$Y_{max} = Y_{max}$

But when it was; implies that $Y_{max} = Y_{out}$.

The difference between input and output strength was then computed and when this was less or equal to 0.001, the output results become acceptable and therefore;

Count = count +1 (more iteration count)

$Z_1 = X_1+X_2+X_3+X_4$ (cement ratios)

$Z_2 = X_1+2X_2+3X_3+3X_4$ (laterite ratios)

$Z_3 = 2X_1+4X_2+6X_3+8X_4$ (gravel ratios)

$Z_4 = 0.65X_1+0.75X_2+1.1X_3+1.14X_4$ (water/cement ratios))

The values of X_1,X_2,X_3,X_4, Y_{out}, Z_1, Z_2, Z_3 and Z_4 where printed out. Then, the next loop was entered by increasing the variable, X_3 by 0.01 and the loops of the other variables X_2,X_1,or X_4 were also entered depending on whether the variable X_3 is greater or less than one respectively.

However, for iteration count not being greater than zero implies that the desired strength inputted was out of range of model (ie outside the range of data used for its estimation). The computation was ended or another desired strength inputted as required. On the other hand, for the iteration count being greater than zero, the maximum strength, Y_{max}, predictable by the model was printed out. Another desired strength was inputted and the computation process started again till the end.

5.2.1 Program Testing and Test Results

The optimization was achieved by a computer code written in Q-Basic.(see Appendix A). The input data of strength tested including that of optimum strength are 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28N/mm^2 (see Appendix B). The desired strength of 17N/mm^2 has the highest number of matching combinations (49) and 27N/mm^2 has only one matching combination. Thus, from the optimization carried out, the optimum strength of laterized concrete is 27N/mm^2 corresponding to an optimum mix proportion of 1:1:2 of cement, laterite, and gravel respectively at a water-cement ratio of 0.650. For a desired strength that has more than one matching combinations, the choice of selecting any one of the combination may be based on such criteria as cost, workability and/ or light weight.

On the basis of workability, relevant data are synthesized from the raw printed matching combinations for each desired strength and are shown in Table 5.3

Table 5.3: Program Test Results

Desired Strength N/mm^2	Classification	Mix Proportions(kg)			
		Cement	laterite	gravel	water-cement ratio
13	Low strength	1	2.860	6.280	1.062
14	,,	1	2.820	5.920	1.065
15	,,	1	2.910	5.980	1.082
16	,,	1	2.710	5.540	1.008
17	Moderate strength	1	2.440	4.880	0.956
18	,,	1	2.320	5.600	0.921
19	,,	1	2.180	5.200	0.892
20	Very moderate strength	1	2.030	4.640	0.855
21	,,	1	1.940	4.480	0.836
22	,,	1	1.850	4.140	0.806
23	,,	1	1.720	3.780	0.779
24	,,	1	1.640	3.500	0.751
25	High strength concrete	1	1.400	2.980	0.721
26	,,	1	1.360	2.760	0.697
27	,,	1	1.000	2.000	0.650

It can be clearly observed from Table 5.3 that the low strength laterized concrete (13 to 16 N/mm^2) requires an approximate mix proportion of 1:3:6 (cement, laterite and gravel) at a water-cement ratio of 1.10. Laterized concrete having moderate strength (17 to 19 N/mm^2) is obtained with an approximate mix proportion of 1:2:5 (cement, laterite and gravel) at a water-cement ratio of 0.9. But for a very moderate strength (20 to 24 N/mm^2) an approximate mix proportion of 1:2:4 (cement, laterite and gravel) at water-cement ratio of 0.8 is needed. But, high strength laterized concrete (25 to 27 N/mm^2) will require an approximate mix proportion of 1:1:2 (cement, laterite and gravel) at a water-cement ratio of 0.650.

5.3 Results of the Structural Characteristics Determinations

The determined structural characteristics of laterized concrete are shown in table 5.4.

Table 5.4: Structural Characteristics of Laterized Concrete

Material Property	Mix Ratio	
	1:1:2: 0.650	1:2:4: 0.791
Density (kg/m^3)	2,400	2,474
Flexural strength (N/mm^2)	4.12	3.17
Compressive strength (N/mm^2)	27	20
Modulus of Elasticity (N/mm^2)	18,888.9	10,163.9
Modulus of Rigidity (N/mm^2)	7,495.6	4,199.9
Poisson's ratio	0.26	0.21

5.3.1 Flexural Strength

Recall equation (4.40) for calculation of flexural strength of laterized concrete given in the previous chapter

$$F_{tb} = PL/bd^2 \quad \text{..........} (5.7)$$

Where, F_{tb} = flexural strength, N/mm^2

P= maximum total load on beam, N

L = length of the beam specimen, mm

b = width of the beam, mm

d = depth of the beam, mm

For the mix ratio of 1:1:2 with water cement ratio of 0.650

maximum total load, ρ = 8240.8 N

length of beam specimen, L = 500 mm

width of the beam, b = 100 mm

depth of the beam, d = 100 mm

Flexural strength (F_{tb}) is obtained using the above relationship

$$\text{Therefore, } F_{tb} = \frac{8240.8x500}{100x(100)^2}$$

$$= 4.12 \; N/mm^2$$

5.3.2 Calculation of Static Modulus of Elasticity

The static modulus of elasticity in compression was estimated using equation (4.41) given as:

$$E_{LC} = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{\sigma_a - \sigma_b}{\epsilon_a - \epsilon_b}$$

where, E_c = modulus of elasticity, $N/_{mm^2}$

σ_a = the upper loading stress in $N/_{mm^2}$ ($\sigma_a = \frac{F_c}{3}$)

σ_b = the basic or initial loading level (ie. 0.5 $N/_{mm^2}$)

$\in_a$ = mean strain under the upper loading stress

$\in_b$ = mean strain under the basic stress

F_c = compressive strength of laterized concrete.

E_c for the mix ratio of 1:1:2 with water cement ratio of 0.650. The dial gauge readings for the three replicate samples are 0.27, 0.21, and 0.25 mm respectively at the upper loading stress. The strain value was obtained using the formula:

$$\in = \frac{\Delta L}{L}$$

where, $\in$ = strain under the loading stress

ΔL = change in length

L = original length of the specimen

The length of the specimen was 200 mm. Therefore, the strain values for each of the three tested specimen are 0.00135, 0.00105, and 0.00125 respectively. The mean strain value under the upper loading stress $\in_a$ is therefore obtained as:

$$\in_a = \frac{0.00135 + 0.00105 + 0.00125}{3}$$

$$= 0.00122$$

Similarly, the mean strain under the basic stress was calculated as:

$$\epsilon_b = \frac{0.00065 + 0.00085 + 0.0008}{3}$$

$$= 0.00077$$

The upper loading stress, $\sigma_a = 9 \ N/_{mm^2}$

The basic loading stress, $\sigma_b = 0.5 \ N/_{mm^2}$

Therefore, $E_c = \dfrac{9 - 0.5}{0.00122 - 0.00077}$

$$= 18{,}888.9 \ N/_{mm^2}$$

5.3.3 Estimation of Poisson's Ratio

The Poisson's ratio was calculated from the following relationship as given in equation (4.42).

$$\textit{Poisson's ratio}(\mu) = \frac{\textit{lateral strain}}{\textit{longitudinal strain}} \quad \text{.......................................(5.9)}$$

For the 1:1:2 mix ratio and 0.650 water cement ratio:

The corresponding mean lateral strain measured in the static modulus of elasticity test was 0.0002.

Therefore, $\mu = \dfrac{0.0002}{0.00077} = 0.26$

Both the lateral and the longitudinal strain were measured at the basic loading stress.

5.3.4 Modulus of Rigidity

A relationship between elasticity modulus and modulus of rigidity was adopted in order to estimate shear modulus. Recall equation (4.43) in the preceding chapter

$$G = \frac{E}{2(\mu + 1)} \quad \text{.................................(5.10)}$$

where, G = shear modulus, N/mm^2

E = modulus of elasticity, N/mm^2

μ = Poisson's ratio

For 1:1:2 mix ratio with water cement ratio of 0.650

$$G = \frac{18{,}888.9}{2(0.26+1)} = 7{,}495.6\ N/mm^2$$

5.3.5 Estimation of Density

The value of density of laterized concrete was estimated from equation (4.44) of the previous chapter

$$\text{Density } (\gamma) = \frac{Mass(kg)}{Volume(m^3)} \quad \text{.......................................(5.11)}$$

For 1:1:2 mix ratio and 0.650 water cement ratio

Average weight of specimen = 8.1 kg

Volume of specimen = 0.003375 m^3

$$\gamma = \frac{8.1}{0.003375} = 2400 \, {}^{kg}\!/_{m^3}$$

The values obtained for the material properties (for optimum ratio) are adopted in performing the static analysis of the circular cylindrical shell structure under the action of hydrostatic and foodgrain pressures using Pasternack's equations formulated on the basis of cylindrical theory of shell of revolution.

CHAPTER SIX

ANALYSIS AND DESIGN OF A CYLINDRICAL SHELL BUILT OF LATERIZED CONCRETE

The general solution of the initial value method given in equations (3.122) Through (3.126) and equations (3.158) through (3.162) are used to evaluate the performance of a cylindrical shell structure under hydrostatic and foodgrain pressures respectively.

6.1 Cylindrical Reservoir under Hydrostatic Pressure

Recall the Initial value solutions obtained in section 3.2.1.2.3

$$W_Z = \frac{\gamma R^2[Y_4(\alpha L)+Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)-\alpha L Y_3(\alpha L)]Y_2(z)}{\alpha Eh[Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)]}$$

$$-\frac{R^2\gamma[Y_2(\alpha L)-(\alpha L)Y_1(\alpha L)]}{\alpha Eh[Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)]}Y_4(z)$$

$$+\frac{R^2\gamma}{\alpha Eh}[z-Y_2(z)]\text{...(6.1)}$$

$$\theta(z) = \frac{\gamma R^2[Y_4(\alpha L)+Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)-\alpha L Y_3(\alpha L)]Y_1(z)}{Eh[Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)]}$$

$$-\frac{R^2\gamma[Y_2(\alpha L)-(\alpha L)Y_1(\alpha L)]}{Eh[Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)]}Y_3(z)$$

$$+\frac{\gamma R^2}{Eh}[1-Y_1(z)]\text{...(6.2)}$$

$$M(z) = \frac{\gamma[Y_4(\alpha L)+Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)-\alpha L Y_3(\alpha L)]}{\alpha^3[Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)]}Y_4(z)$$

$$+\frac{\gamma[Y_2(\alpha L)-(\alpha L)Y_1(\alpha L)]}{4\alpha^3[Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)]}Y_2(z)$$

$$-\frac{\gamma}{\alpha^3}Y_4(z)\ldots\ldots(6.3)$$

$$Q(z)=\frac{\gamma[Y_4(\alpha L)+Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)-\alpha LY_3(\alpha L)]}{\alpha^2[Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)]}Y_3(z)$$

$$+\frac{\gamma[Y_2(\alpha L)-\alpha LY_1(\alpha L)]}{4\alpha^2[Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)]}Y_1(z)$$

$$-\frac{\gamma}{\alpha^2}Y_3(z)\ldots\ldots(6.4)$$

$$N(z)=\frac{\gamma R[Y_4(\alpha L)+Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)-\alpha LY_3(\alpha L)]}{\alpha[Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)]}Y_2(z)$$

$$-\frac{\gamma R[Y_2(\alpha L)-\alpha LY_1(\alpha L)]}{\alpha[Y_2(\alpha L)Y_3(\alpha L)-Y_1(\alpha L)Y_4(\alpha L)]}Y_4(z)$$

$$+\frac{\gamma R}{\alpha}[z-Y_2(z)]\ldots\ldots(6.5)$$

The sizes of structural members are selected, and the values of the properties of the laterized concrete as determined in chapter five are adopted, and inputted into the above equations (6.1) through (6.5) to evaluate the deflection and stresses generated due to hydrostatic pressure.

The following values were therefore adopted in this case for the different parameters:

Diameter, D = 10 m

Radius of reservoir, R = 5 m

Height of reservoir, L = 5 m

Thickness of reservoir, h = 0.05 m

Unit weight of water, $\gamma_w = 9.81$ KN/m^3

Yong Modulus of laterized concrete, E = 18,888.9 KN/m^2

Poisson's ratio, $\mu = 0.26$

Therefore:

$$\alpha^4 = \frac{3(1-\mu^2)}{R^2h^2} = \frac{3(1-0.26^2)}{5^2(0.05)^2} = 44.7552 \approx 44.76$$

$\Rightarrow \alpha = 2.586490428$

$\alpha L = 12.93245214$

Substitution of the values for α, L, γ, R, E and h as defined previously, equations (6.1) through (6.5) are put as:

$W(z) = -\ 4.7753 \times 10\text{-}9Y_2(z) + 2.219996445 \times 10\text{-}8Y_4(z)$

$+\ 1.003971695 \times 10\text{-}4z$..................................(6.6)

$\theta(z) = -\ 1.23513 \times 10\text{-}8Y_1(z) + 5.74199955 \times 10\text{-}8Y_3(z)$

$+\ 2.596763178 \times 10\text{-}4$......................................(6.7)

$M(z) = -\ 3.134059229 \times 10\text{-}5Y_2(z) - 2.6967 \times 10\text{-}5Y_4(z)$........(6.8)

$Q(z) = -\ 8.106214195 \times 10\text{-}5Y_1(z) - 6.9747 \times 10\text{-}5Y_3(z)$........

.(6.9)

$N(z) = -\ 9.02 \times 10\text{-}4Y_2(z) + 4.193329085 \times 10\text{-}3Y_4(z)$

$+\ 18.96392094z$...(6.10)

The values of W(z), M(z), θ(z), Q(z) and N(z) are computed for each of the ten (10) segments of the height of reservoir and are tabulated as shown in Table 6.1

Table 6.1: Initial-Value Solution (Reservoir thickness = 0.05 m)

X m	Z	$Y_1(z)$	$Y_2(z)$	$Y_3(z)$	$Y_4(z)$	W(z) m	θ(z) Radian	M(z) KN-m	Q(z) KN	N(z) KN
0	0	1	0	0	0	0	0.0002597	0	-0.000081	0
0.5	1.29325	0.536895	1.173111	0.810314	0.35569	0.000129	0.0002597	-0.0000464	-0.00010	24.5255
1.0	2.58649	-5.67627	-1.046082	1.740226	2.28297	0.000259	0.0002598	-0.0000288	0.000338	49.0605
1.5	3.87974	-17.91308	-17.09665	-8.140796	0.40058	0.000389	0.0002594	0.000525	0.00202	73.59218
2.0	5.17298	39.21304	-38.36489	-39.511991	-29.5599	0.000519	0.0002569	0.001999	-0.000423	98.01063
2.5	6.46623	316.15675	187.34057	29.262821	-64.4073	0.000647	0.0002575	-0.00413	-0.0276	122.18601
3.0	7.75947	110.58688	638.594	583.3005	264.0037	0.000782	0.0002918	-0.0271	-0.0496	147.68102
3.5	9.05272	-3978.662	-1213.029	776.30184	1382.8164	0.000945	0.0003534	0.000722	0.26837	178.5678
4.0	10.34596	-9415.255	-10905.168	-6197.541	-744.9569	0.00107	0.0000201	0.36186	1.19548	202.9125
4.5	11.63921	34044.257	-5667.679	-22689.81	-11346.650	0.000944	-0.00146	0.4836	-1.17715	178.2571
5.0	12.93245	193057.50	133533.96	37005.219	-29761.765	6.53E-09	4.72E-09	-3.3824	-18.2306	1.4526E-03

6.1.1 Reservoir thickness of 100 mm

Since the selected wall thickness of 0.05 m was found inadequate; a higher value of reservoir thickness was adopted (ie. 100 mm). Other parameters are as previously defined:

Therefore:

$$\alpha^4 = \frac{3(1-\mu^2)}{R^2h^2} = \frac{3(1-0.26^2)}{5^2 x\ (0.1)^2} = 11.1888 \approx 11.19$$

$$\Rightarrow \alpha = 1.828924921$$

$$\alpha L = 9.144624606$$

Substituting these values into the equations of initial value general solutions, Equations (6.1) through (6.5) are now put as follows:

$$W(z) = -\ 5.791409 \text{ x } 10^{-8}Y_2(z) - 5.122472595 \text{ x}10^{-7}Y_4(z)$$
$$+\ 7.099151935 \text{ x } 10^{-5}z\ldots\ldots\ldots\ldots(6.11)$$

$$\theta(z) = -\ 1.059205 \text{ x } 10^{-7}Y_1(z) - 9.368617782 \text{ x}10^{-7}Y_3(z)$$
$$+\ 1.298381589 \text{ x } 10^{-4}\ldots\ldots\ldots\ldots(6.12)$$

$$M(z) = 2.8926411 \text{ x } 10^{-3}Y_2(z) - 1.308155 \text{ x}10^{-3}Y_4(z)\ldots\ldots\ldots(6.13)$$

$$Q(z) = 5.290423433 \text{ x } 10^{-3}Y_1(z) - 2.392517 \text{ x}10^{-3}Y_3(z)\ldots\ldots\ldots(6.14)$$

$$N(z) = -\ 0.02187869Y_2(z) - 0.193515746Y_4(z)$$
$$+\ 26.8190342z\ldots\ldots\ldots\ldots(6.15)$$

The values of W(z), M(z), θ(z), Q(z) and N(z) are computed for each of the ten (10) segments of the height of reservoir and are put in tabular form (see Table 6.2).

$I = 10{,}416{,}666.67\ mm^4$

$y = 25\ mm$

$M_{(max)} = -3.3824\ KN\text{-}m$

$$\therefore \delta_{max} = \frac{3.3824x10^6 x25}{10{,}416{,}666.67} = 8.118\ {}^{N}\!/_{mm^2}$$

The maximum stress developed is greater than the allowable stress of the laterized concrete material, which is 4.12 N/mm^2. The thickness of 50 mm is therefore not adequate.

Hoop Tension:

$$N_{max}(z) = 202.9125\ {}^{KN}\!/_{m}$$

To get the stress that will be developed per unit area, force per unit length is divided by the total thickness.

Thus:

$$\frac{202.9125x10^3}{10^3 x50} = 4.06\ {}^{N}\!/_{mm^2} \approx 4.1\ {}^{N}\!/_{mm^2}$$

the stress developed due to hoop tension is almost equal to the allowable stress of the laterized concrete material. The thickness of the reservoir should therefore be increased for safety.

Table 6.2: Initial-Value method (Thickness of Reservoir = 100 mm)

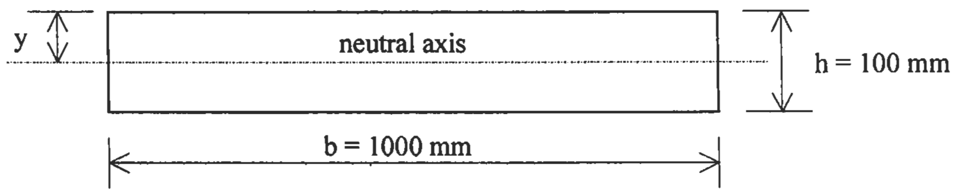
X m	Z	$Y_1(z)$	$Y_2(z)$	$Y_3(z)$	$Y_4(z)$	W(z) m	θ(z) Radian	M(z) KN-m	Q(z) KN	N(z) KN
0	0	1	0	0	0	0	0.000129	0	0.005290	0
0.5	0.914462	0.88364	0.893166	0.414873	0.12703	0.0000648	0.000129	0.002417	0.00368	24.51995
1.0	1.8289249	-0.81531	1.156878	1.466403	0.96559	0.000129	0.000128	0.00208	-0.00782	48.83783
1.5	2.743387	-7.191503	-2.05343	1.50018	2.53938	0.000194	0.000129	-0.00926	-0.04164	73.12851
2.0	3.657849	-16.87324	-13.21404	-4.78227	1.81838	0.000259	0.000136	-0.0406	-0.07782	97.45899
2.5	4.572312	-6.756034	-27.33479	-23.9524	-10.29010	0.000331	0.000153	-0.0656	0.02156	125.21434
3.0	5.486775	84.4312	-0.941608	-43.1543	-42.68496	0.000411	0.000161	0.0531	0.54992	155.4308
3.5	6.401237	299.1988	167.34154	17.7429	-65.92781	0.000479	0.0000815	0.5703	1.54044	180.77184
4.0	7.315699	385.45104	515.49592	322.7703	65.0226	0.000456	-0.000213	1.4061	1.26697	172.3387
4.5	8.230162	-689.2684	527.8786	451.8765	608.5735	0.000242	-0.000221	0.73085	-4.72764	91.40715
5.0	9.1446246	-4499.449	-1602.433	647.291	1448.508	-1.01E-11	-1.51E-10	-6.5301	-25.3526	2.854E-05

Checks for Adequate Cross-Section

Checks for the adequacy of the cross-section were performed by adopting the maximum values of bending moment and Hoop tension (see Table 6.2) which occur at a segment along the height of the reservoir:

Bending Moment:

$M_{max}(z) = -6.5301KN\text{-}m$



$$\delta_{max} = \frac{My}{I}$$

$$I = \frac{bh^3}{12} = \frac{1000x100^3}{12} = 83{,}333{,}333.33mm^4$$

$$\therefore \delta_{max} = \frac{6.5301x10^6x50}{83{,}333{,}333.33} = 3.92 \, {N}/{mm^2}$$

The maximum stress developed by the section due to bending moment is less than the allowable stress of laterized concrete material (ie. 4.12 N/mm^2). The reservoir thickness of 100 mm is therefore adequate.

Hoop Tension:

$$N_{max}(z) = 180.77184 \, {KN}/{m}$$

Dividing by reservoir thickness

$$= \frac{180.77184x10^3}{10^3 x100} = 1.81 \frac{N}{mm^2}$$

The stress developed due to Hoop tension is also far less than the allowable stress of the laterized concrete material. Thickness of 100 mm is safe and adequate.

From Table 6.2, deflection, rotation, bending moment, shear force,, and hoop tension diagrams are plotted (see Figs. 6.1a, b, c, d, and e).

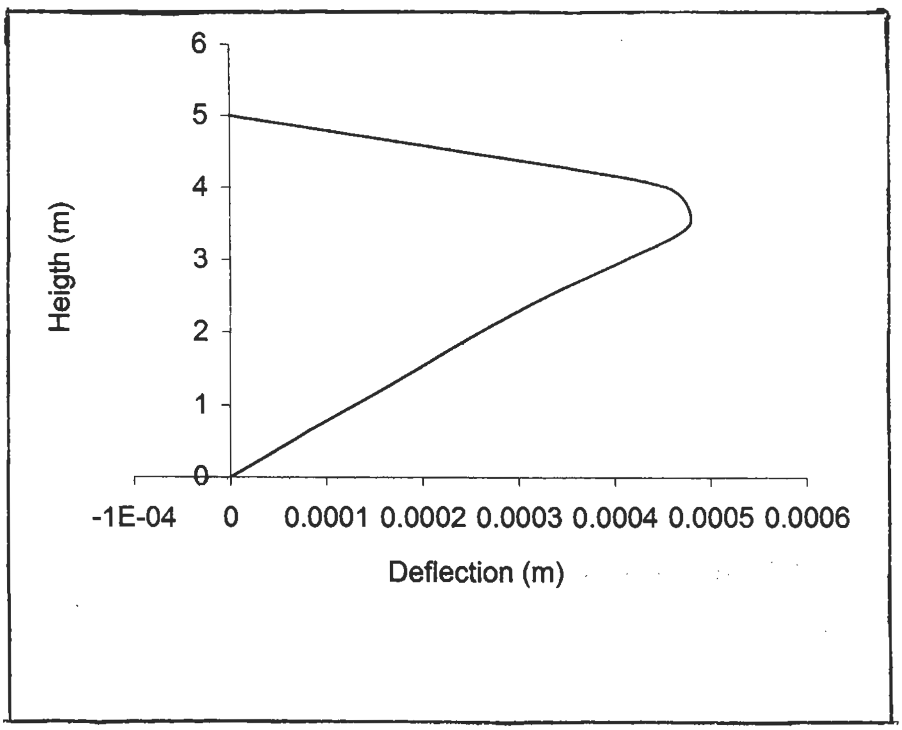


Fig. 6.1a: Deflection diagram

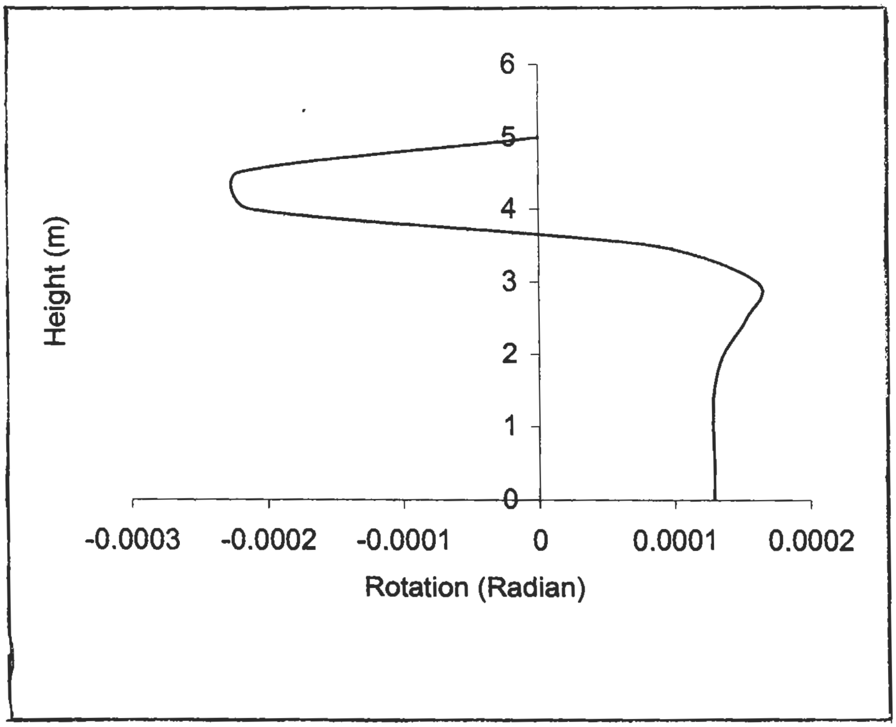


Fig.6.1b: Rotation diagram

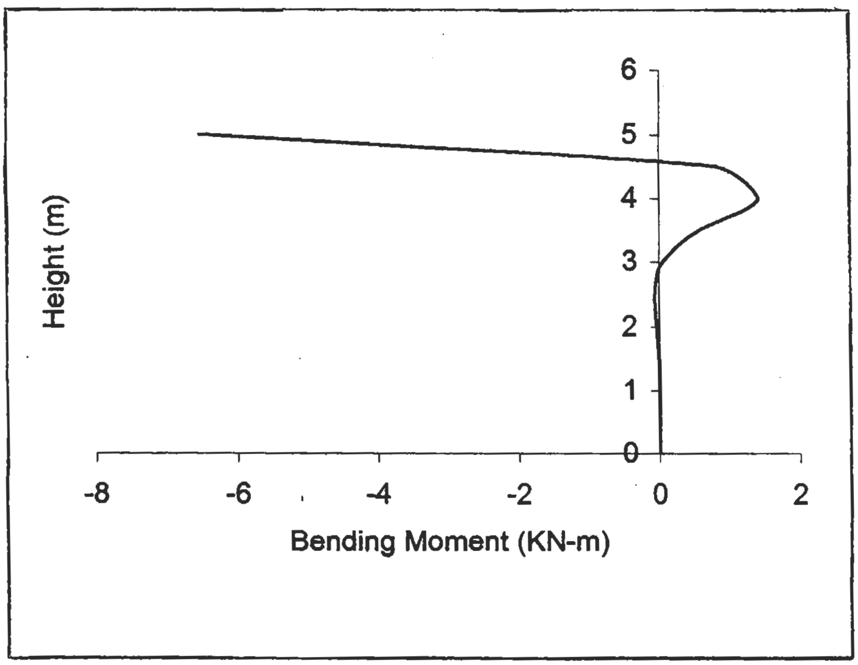


Fig.6.1c: Bending Moment diagram

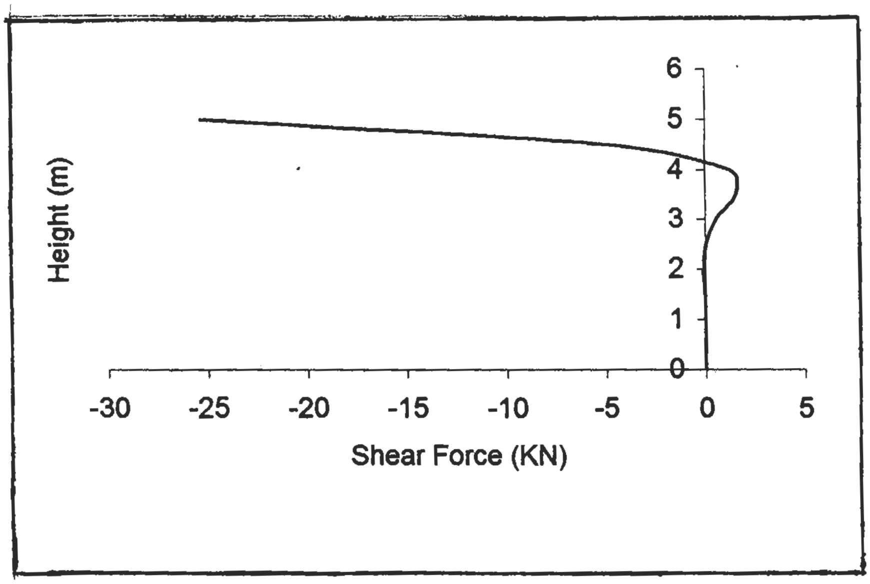


Fig. 6.1d: Shear force diagram

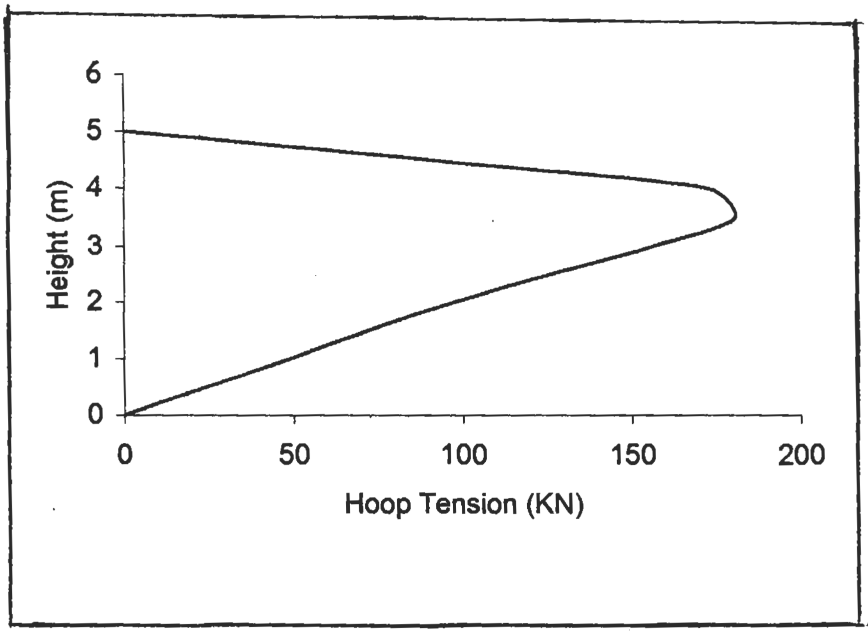


Fig. 6.1e: HoopTension diagram

6.2 Cylindrical Silo under Foodgrain Pressure

Recalling the Initial value solutions provided in section 3.2.2.2.2;

$$W(z) = \frac{k\gamma R^2[Y_4(\alpha L) + Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L) - \alpha L Y_3(\alpha L)]}{\alpha Eh[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)]}Y_2(z)$$

$$-\frac{k\gamma R^2[Y_2(\alpha L) - \alpha L Y_1(\alpha L)]}{\alpha Eh[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)]}Y_4(z)$$

$$+\frac{k\gamma R^2}{\alpha Eh}[z - Y_2(z)] \qquad (6.16)$$

$$\theta(z) = \frac{k\gamma R^2[Y_4(\alpha L) + Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L) - \alpha L Y_3(\alpha L)]}{Eh[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)]}Y_1(z)$$

$$-\frac{k\gamma R^2[Y_2(\alpha L) - \alpha L Y_1(\alpha L)]}{Eh[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)]}Y_3(z)$$

$$+\frac{k\gamma R^2}{Eh}[1 - Y_1(z)] \qquad (6.17)$$

$$M(z) = \frac{k\gamma[Y_4(\alpha L) + Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L) - \alpha L Y_3(\alpha L)]}{\alpha^3[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)]}Y_4(z)$$

$$+\frac{k\gamma[Y_2(\alpha L) - \alpha L Y_1(\alpha L)]}{4\alpha^3[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)]}Y_2(z)$$

$$-\frac{k\gamma}{\alpha^3}Y_4(z) \qquad (6.18)$$

$$Q(z) = \frac{k\gamma[Y_4(\alpha L) + Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L) - \alpha L Y_3(\alpha L)]}{\alpha^2[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)]}Y_3(z)$$

$$+\frac{k\gamma[Y_2(\alpha L) - \alpha L Y_1(\alpha L)]}{4\alpha^2[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)]}Y_1(z)$$

$$-\frac{k\gamma}{\alpha^2}Y_3(z) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6.19)$$

$$N(z) = \frac{k\gamma R[Y_4(\alpha L) + Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L) - \alpha L Y_3(\alpha L)]}{\alpha[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)]}Y_2(z)$$

$$-\frac{k\gamma R[Y_2(\alpha L) - \alpha L Y_1(\alpha L)]}{\alpha[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)]}Y_4(z)$$

$$+\frac{k\gamma R}{\alpha}[z - Y_2(z)] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6.20)$$

To determine the deflection and stresses developed due to foodgrain pressure within the silo shell, the following values are adopted for various parameters:

Diameter of silo, D = 20 m

Radius of silo, R = 10 m

Height of silo, L = 8 m

Thickness of silo wall, h = 0.1 m

Unit weight of soybean grain, γ_g = 11.7 KN/m^3

Young Modulus of laterized concrete, E and Poisson's ratio, μ are as previously defined.

Therefore:

$$\alpha^4 = \frac{3(1-\mu^2)}{R^2h^2} = \frac{3(1-0.26^2)}{10^2 x\, 0.1^2} = 2.7972 \approx 2.80$$

$\Rightarrow \alpha = 1.293245214$

αL = 10.34596171

$K = \tan^2(45^\circ - \frac{\phi}{2}); \phi = 29^\circ$ for soybean grain

$\therefore K = 0.347$

Substituting for the above different parameters into equations (6.16) through (6.20) to obtain the equations of initial value general solutions as:

$$W(z) = 1.738911 \times 10^{-7} Y_2(z) - 2.37355334 \times 10^{-7} Y_4(z) + 1.661987639 \times 10^{-4} z \quad (6.21)$$

$$\theta(z) = 2.248832 \times 10^{-7} Y_1(z) - 3.069586486 \times 10^{-7} Y_3(z) + 2.149357559 \times 10^{-4} \quad (6.22)$$

$$M(z) = 6.701683451 \times 10^{-4} Y_2(z) + 1.963906 \times 10^{-3} Y_4(z) \quad (6.23)$$

$$Q(z) = 8.666920054 \times 10^{-4} Y_1(z) + 2.539814 \times 10^{-3} Y_3(z) \quad (6.24)$$

$$N(z) = 0.03284601 Y_2(z) - 0.044833811 Y_4(z) + 31.3931183 z \quad (6.25)$$

W(z), M(z), θ(z), Q(z) and N(z) are computed for each of the ten (10) segments of silo height and put in tabular form (see Table 6.3).

Table 6.3: Initial Value Solutions (Grain Silo thickness = 100 mm)

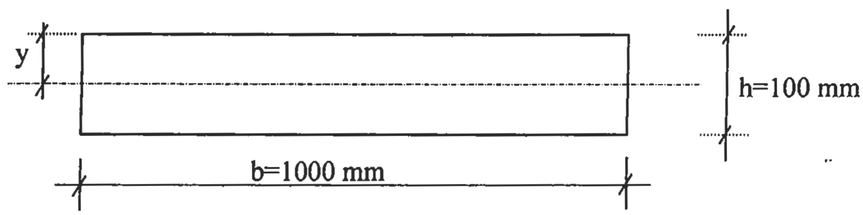
X m	Z	$Y_1(z)$	$Y_2(z)$	$Y_3(z)$	$Y_4(z)$	W(z) m	θ(z) Radian	M(z) KN-m	Q(z) KN	N(z) KN
0	**0**	**1**	**0**	**0**	**0**	**0**	**0.000215**	**0**	**0.000866**	**0**
0.8	**1.034596**	**0.809565**	**0.995144**	**0.528387**	**0.183564**	**0.000172**	**0.000215**	**0.001027**	**0.002044**	**32.5036**
1.6	**2.069192**	**-1.922754**	**0.835326**	**1.7110565**	**0.4149202**	**0.000344**	**0.000214**	**0.001375**	**0.002679**	**64.96722**
2.4	**3.103789**	**-11.15557**	**-5.344402**	**0.210113**	**2.883162**	**0.000514**	**0.000212**	**0.002081**	**-0.009135**	**97.13281**
3.2	**4.138385**	**-17.02774**	**-21.67602**	**-13.15978**	**-2.328471**	**0.000685**	**0.000215**	**-0.019099**	**-0.048181**	**129.30923**
4.0	**5.172981**	**39.213159**	**-19.909229**	**-39.51201**	**-29.55993**	**0.000863**	**0.000236**	**-0.071395**	**-0.066367**	**163.06735**
4.8	**6.207577**	**247.540**	**114.393158**	**-9.37584**	**-66.57249**	**0.00107**	**0.000273**	**-0.054079**	**0.190728**	**201.61726**
5.6	**7.242173**	**401.2193**	**486.5353**	**285.9255**	**42.658174**	**0.00128**	**0.000217**	**0.409837**	**1.073931**	**241.42261**
6.4	**8.276769**	**-806.549**	**493.0506**	**896.3251**	**649.79987**	**0.00131**	**-0.000242**	**1.606573**	**1.577469**	**246.89532**
7.2	**9.3113655**	**-5495.986**	**-2434.993**	**312.9998**	**1530.4964**	**0.000761**	**-0.00112**	**1.373896**	**-3.968366**	**143.71501**
8.0	**10.34596**	**-9415.255**	**-10905.169**	**-6197.541**	**-744.9569**	**-6.58E-09**	**-8.107E-09**	**-8.77132**	**-23.90073**	**-8.69E-05**

Check For Adequate Cross-Section

Check for the adequacy of the cross-section using the same method as in the previous section:

Bending Moment: From Table 6.3,

$M_{max}(z) = -8.77132$ KN-m



$$\delta_{max} = \frac{My}{I}$$

$$I = \frac{bh^3}{12} = \frac{1000x100^3}{12} = 83{,}333{,}333.33\,mm^4$$

$$\therefore \delta_{max} = \frac{8.77132x10^6 x50}{83{,}333{,}333.33} = 5.26\,{}^{N}\!/_{mm^2}$$

The maximum stress developed by the section due to the bending moment is greater than the allowable stress of the laterized concrete material (allowable = 4.12 N/mm^2). The silo thickness is therefore not adequate. Greater thickness is required.

6.2.1 Silo Thickness of 150 mm

The thickness of the silo wall was increased to 150 mm (0.15 m) for adequate strength. This redefines the parameter;

$$\alpha^4 = \frac{3(1-\mu^2)}{R^2h^2} = \frac{3(1-0.26^2)}{10^2 x\, 0.15^2} = 1.2432 \approx 1.24$$

$$\Rightarrow \alpha = 1.055930296$$

$$\alpha L = 8.447442365$$

Other parameters remain as defined in section (6.2).

Substituting for the values of the different parameters into equations (6.16) through (6.20) to obtain equation of initial-value general solutions as:

$$W(z) = -7.343747 \text{ x } 10^{-7}Y_2(z) - 1.131201312 \text{ x}10^{-6}Y_4(z)$$
$$+ 1.357007224 \text{ x } 10^{-4}z \qquad (6.26)$$

$$\theta(z) = -7.754484 \text{ x } 10^{-7}Y_1(z) - 1.194469736 \text{ x}10^{-6}Y_3(z)$$
$$+ 1.432905039 \text{ x } 10^{-4} \qquad (6.27)$$

$$M(z) = 7.18633294 \text{ x } 10^{-3}Y_2(z) - 0.018661439Y_4(z) \qquad (6.28)$$

$$Q(z) = 7.588266668 \text{ x } 10^{-3}Y_1(z) - 0.019705179Y_3(z) \qquad (6.29)$$

$$N(z) = -0.20807294Y_2(z) - 0.320507226Y_4(z)$$
$$+ 38.44856062z \qquad (6.30)$$

The values of W(z), θ(z), M(z), Q(z) and N(z) are computed for each of the ten (10) segments of the height of silo and are tabulated as shown on Table 6.4.

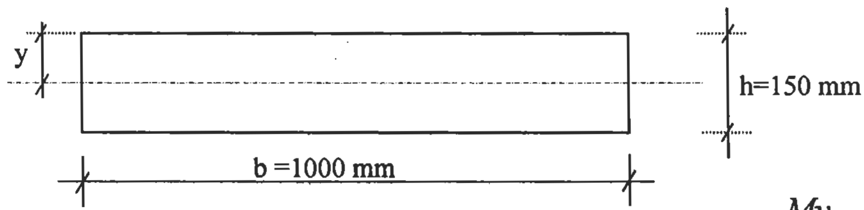
Table 6.4: Initial Value Solutions (Thickness of Grain Silo = 150 mm)

X m	Z	$Y_1(z)$	$Y_2(z)$	$Y_3(z)$	$Y_4(z)$	W(z) m	θ(z) Radian	M(z) KN-m	Q(z) KN	N(z) KN
0	0	1	0	0	0	0	0.0001425	0	0.00758826	0
0.8	0.844744	0.915234	0.830415	0.35478	0.100223	0.000114	0.0001422	0.0040973	-0.0000459	32.27428
1.6	1.689488	-0.331635	1.235591	1.29882	0.77268	0.000227	0.00014199	-0.0055399	-0.028110	64.45364
2.4	2.53423	-5.208610	-0.76174	1.78736	2.190865	0.000342	0.00014519	-0.046359	-0.074745	96.893805
3.2	3.37897	-14.27545	-8.84805	-1.72288	2.69713	0.000462	0.0001564	-0.113917	-0.074376	130.893123
4.0	4.22372	-16.032149	-23.08953	-15.07042	-3.532125	0.000594	0.0001737	-0.100014	0.175309	168.33233
4.8	5.068465	27.70255	-23.39149	-37.23872	-25.54593	0.000734	0.0001663	0.308625	0.9444009	207.92997
5.6	5.913209	172.4327	52.77729	-33.43731	-59.82644	0.000831	0.0000495	1.495723	1.9673535	235.5476
6.4	6.75795	382.8320	289.7985	98.38270	-46.51626	0.000757	-0.0002711	2.950649	0.966383	214.4430
7.2	7.602698	249.0944	609.7146	485.1673	180.310180	0.000379	-0.0006294	1.0167647	-7.663666	107.65697
8.0	8.44744	-1303.882	314.504	966.445	809.19294	-1.856E-10	-5.59E-09	-12.84057	-28.93818	-4.749E-05

Check for the Adequacy of the Cross-Section

Bending Moment: The maximum bending moment from Table 6.4 =

-12.84057 KN-m



$$\delta_{max} = \frac{My}{I}$$

$$I = \frac{bh^3}{12} = \frac{1000x150^3}{12} = 281250000\,mm^4$$

$$\therefore \delta_{max} = \frac{12.84057x10^6 x75}{281250000} = 3.42\ N/_{mm^2}$$

$$\delta_{max} = 3.42\ N/_{mm^2}$$

The maximum stress developed by the section due to the bending moment is less than the allowable stress of the laterized concrete material, which is 4.12 N/mm^2. The thickness of 150 mm of silo is adequate and safe.

Hoop Tension: From Table 6.4

$$N_{max}(z) = 235.5476\ KN/_m$$

Dividing by the silo thickness

$$= \frac{235.5476x10^3}{10^3 x150} = 1.57\ N/_{mm^2}$$

The stress developed due to Hoop tension is also far less than the allowable stress of the laterized concrete material. The thickness of the silo is therefore adequate.

From Table 6.4, deflection, rotation, bending moment, shear force, and hoop tension diagrams are plotted (see Figs. 6.2a, b, c, d and e).

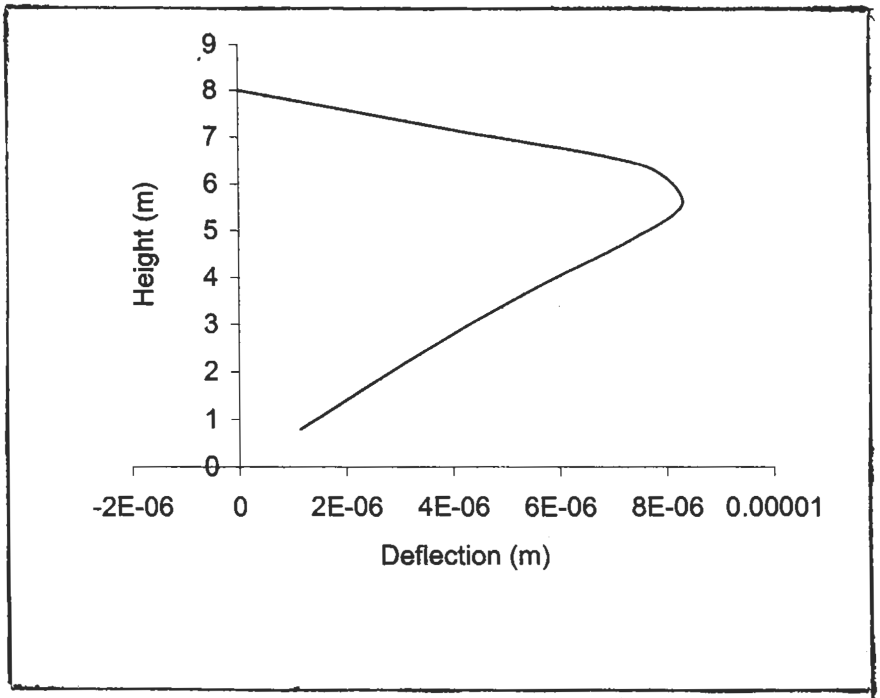


Fig. 6.2a: Deflection diagram

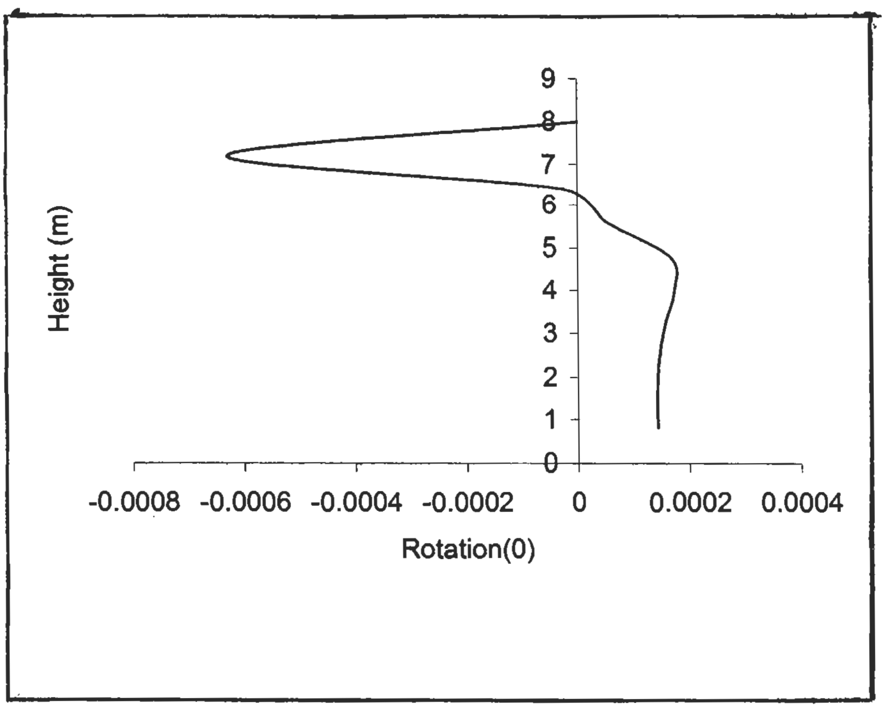


Fig. 6.2b: Rotation diagram

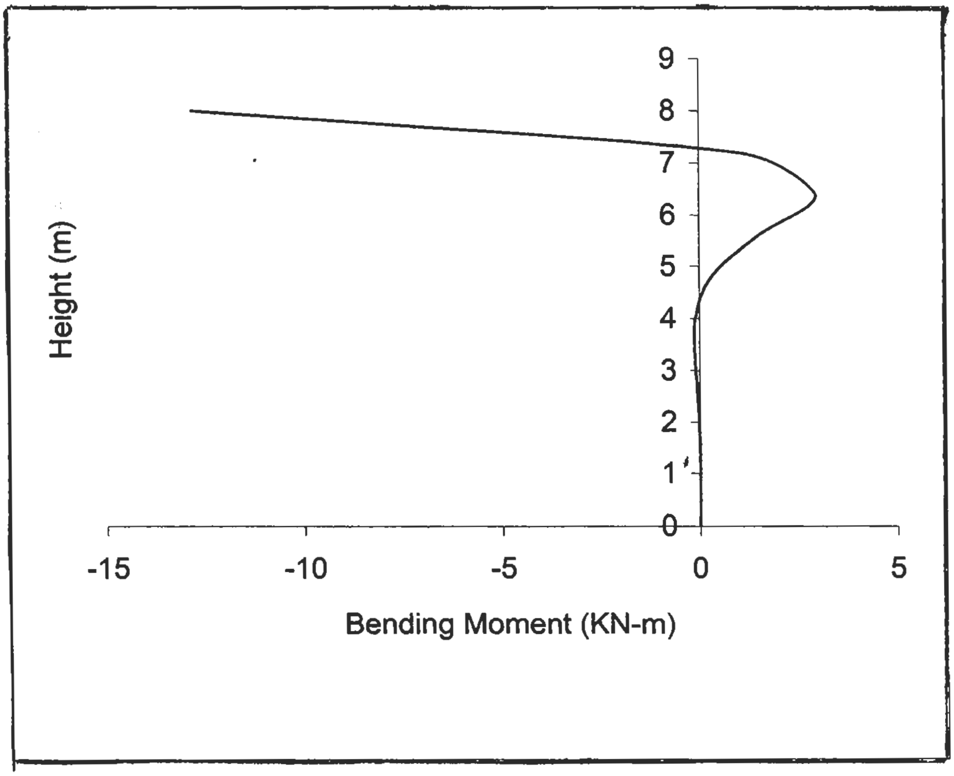


Fig. 6.2c: Bending Moment diagram

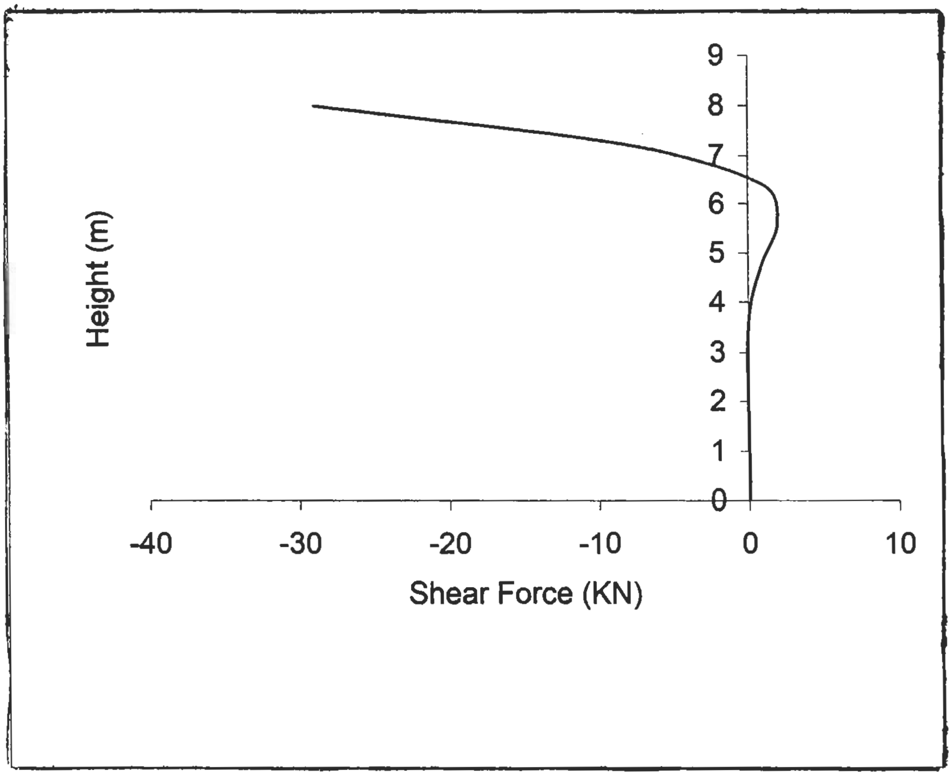


Fig. 6.2d: Shear force diagram

CHAPTER SEVEN

DISCUSSION

7.1 The Regression Equation

The experimental data are very well fitted to the regression model, which satisfies the student-t and chi-square tests. The model parameters estimated are therefore acceptable. For optimization of strength using this equation, the optimum value of compressive strength estimated is 27N/mm^2. This corresponds to an optimum mix proportion of 1:1:2:0.650 (cement, laterite, gravel and water-cement ratio). Thus, for optimum strength, the mixture must contain 1 part by weight of cement, 1 part of laterite and 2 parts of gravel (of the total weight) at a water-cement ratio of 0.650. This optimum mix ratio is mainly recommended when laterite component of laterized concrete is obtained from a borrow pit situated at Ugwu-oye, Nsukka of Enugu State (source of laterite sample).

7.2 Structural Characteristics of Laterized Concrete

Table 5.4 shows the results obtained from the determination of the material properties using two different mix ratios of laterized concrete. The values obtained are viz: flexural strength, 4.12 N/mm^2; compressive strength, 27 N/mm^2; young modulus, 18,888.9 N/mm^2; shear modulus, 7495.6 N/mm^2 and poisson's ratio, 0.26 are higher for the optimum mix ratio of 1:1:2 at water cement ratio of 0.650 when compared with those of the mix ratio 1:2:4 at water cement ratio of 0.791 as shown on the table. Only density is lesser for the former mix ratio than the latter. The results in Table 5.4 indicate that the optimum mix ratio of 1:1:2:0.650, which has the highest strength, reflects on the quality of other properties of the laterized concrete material. Thus, results in Table 5.4 provide data needed for the accurate analysis and design of structures such as silos, water reservoirs, etc built of laterized concrete.

7.3 Analytical Technique

7.3.1 Static Equilibrium

There are many theories that can be used for the analysis of shell structures. In this study, the Half-moment theory was used in the derivation of the static equilibrium for a circular cylindrical shell. The static equilibrium is a differential equation to the fourth derivative.

7.3.2 Solution to Static Equilibrium

Initial value method was used in the analysis of the static equilibrium using the relation $Z = \alpha x$

General solution was developed for direct deflection W(Z), rotation Θ (Z), bending moment M(Z), shear force Q (Z) and hoop tension N (Z) which were made use of in the determination of deflection and stresses and then the adequate reservoir/silo thickness was determined.

7.4 Performance Evaluation of a Cylindrical Shell

Tables 6.1 and 6.2 show the results of the evaluation of stresses and deflection within a reservoir of thickness 50 mm and 100 mm respectively, while Tables 6.3 and 6.4 show that for grain silo shells of thickness 100 mm and 150 mm respectively. The values obtained are used to plot diagrams of the various components or parameters.

7.4.1 Direct deflection

The deflection curve indicates a parabolic-like shape. The deflection is zero at both ends of reservoir as well as silo. The maximum value of deflection is 0.479mm and occurs at a height of 3.5 m from the top of reservoir, which is 5 m high. But for food grain silo which is 8m high, the maximum value of deflection is 0.831mm and occurs at a height of 5.6 m from the top.

7.4.2 Rotation

For the reservoir, the rotation is 0.000129 radians at the top end and zero at the bottom end. The maximum rotation is negative of 0.000221 radians and occurs at height 4.5 m from the top. In the case of grain silo, the rotation is 0.0001425 radians at the top end and is zero at the bottom end. The maximum rotation is negative of 0.0006294 radians and occurs at a height 7.2 m from the top.

7.4.3 Bending Moment

For both reservoir and grain silo, the bending moment is zero at the top. In case of reservoir, the maximum sagging moment is 1.4061 KN-M and occurs at height 4 m while the maximum hogging moment is 6.5301 KNM and occurs at the bottom due to cantilever action. In case of food grain silo, the maximum sagging moment is 2.9506 KNM and occurs at height 6.4 m from the top while the maximum hogging moment is 12.8405 KNM which occurs also at the bottom.

7.4.4 Shear Force

For the reservoir, the shear force is 0.005290 KN at the top and is –25.35262 KN at the bottom end of reservoir. The maximum positive shear is 1.54044 KN and occurs at height 3.5 m from top of reservoir. For food grain silo, the shear force is 0.007588 KN at the top end and is –28.93818 KN at the bottom end of the silo. The maximum positive shear is 1.96735 KN and occurs at height 5.6 m from the top of the silo.

7.4.5 Hoop Tension

The hoop tension curve is similar to the deflection curve in both reservoir and silo cases. For reservoir, the maximum hoop tension is 180.77184 KN and occurs at 3.5 m height from the top. For grain silo, the maximum hoop tension is 235.5476 KN and occurs at height 5.6m from the top of the silo.

7.5 Determination of Cross-Section

7.5.1 Reservoir Thickness

The results of the analysis indicate that the maximum stresses developed due to hydrostatic loading is greater than the strength of the laterized concrete in reservoir thickness of 50 mm, 10 m in diameter and 5 m in height. The wall thickness of 100 mm was found adequate for reservoir of similar dimensions. This reservoir thickness can therefore retain water safely without reinforcement. Minimum reinforcement may only be provided to take care of cracks due to thermal effects.

7.5.2 Silo Thickness

The maximum stress developed due to soybean grain loading is greater than the material strength in silo thickness of 100 mm, 20 m in diameter and 8 m in height. The wall thickness of 150 mm was found adequate. This silo thickness can therefore retain soybean grain safely without reinforcement.

7.6 Characteristic Strengths

It is found that the results of concrete cube tests provided that the concrete has been mixed and the cubes have been made under conditions of strict constraint control follow closely the Normal distribution curve. This curve represents the distribution, which is obtained if the variation in the results is due only to chance or random events.

If the extent of variability of the laterized concrete strength as measured by the standard deviation is known, it is possible to calculate the relationship of the mean strength to the minimum for any given percentage of results falling below the minimum. This is given by

$$f_{cu} = T - KS_y$$

where ;

f_{cu} = minimum strength of laterized concete

T = mean strength of laterized concrete

K = factor depending on the percentage of results which are allowed to fall below the minimum

S_y = standard deviation of the laterized concrete samples.

When K = 1.64, the percentage of results falling below the minimum strength is 5 percent (see Table 2.5). The standard deviation of the model results is 0.830. Therefore the optimum charateristic strength from the model is:

$$f_{cr} = 27 - 1.64 \times 0.830 = 25.6 \text{ N/mm}^2$$

This minimum strength of laterized concrete is referred to as the characteristic strength.

CHAPTER EIGHT

CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions

The implication of this study is that there is distinction in the properties of laterized concrete resulting from the variations in the mix proportions of the constituent materials. Thus, for a job where a high strength concrete is specified, particularly in construction of heavy loaded storage structures such as water reservoirs, silos etc, the data provided for the optimum mix proportion in this study can therefore be adopted in the analysis and design of such structures.

From the cylindrical shell analysis using initial-value method, a wall thickness of 50mm was found inadequate to retain water to a reservoir height of 5m and 10m in diameter without reinforcement. A wall thickness of 100mm was found adequate. For a foodgrain silo, a wall thickness of 100mm was found inadequate to retain soybean grains to a silo height of 8m and 20m in diameter without reinforcement. A wall thickness of 150mm was found adequate. Thus, the possibilities of using laterized concrete in constructing cylindrical storage structures were successfully demonstrated. Theory of shell structures can therefore be applied in the analyses and design of structures built of laterized concrete.

8.2 Recommendations

In a reservoir built of laterized concrete, minimum reinforcement may be provided to prevent cracks due to thermal effect or temperature variations. A special type of cement called water-proofing cement should be used in plastering the inner surface of the reservoir to ensure water tight structure. Cement plaster may also be applied on the outer surface of the reservoir for more durability and whether protection especially if built outside or exposed to harsh environment.

For a silo, cement plaster should be applied on both inner and outer surface for more environmental protection and longevity.

It is therefore recommended that for any future work or investigation in this area of study, a prototype of the cylindrical shell structure built of laterized concrete should be constructed and tested in order to validate the above analytical results. Attention should also be focused on the analyses and design of dome or cover as well as floor of cylindrical shell built of laterized concrete using the theory of shell structures. Thermal properties of laterized concrete materials also need to be determined in order to ensure adequate environmental protection.

REFERENCES

Aboaba, F.O. (1976) Storage and processing of Major Crops in Nigeria AFP symposium proceedings, 1976. Pp 131-135.

Ahire, O. (2001). Effect of Stabilized Laterite as lining Material in Irrigation Channels. B. Eng. Project report submitted to the Department of Agric. Engineerig, University of Agriculture, Makurdi, March, 2001.

Akhnazarova, S. and Kafarou, V. (1982). Experiment Optimization in Chemistry and Chemical Engineering. Translated from Russian by Vladimir M. Matskousley and Repyev, Alexander P., Moscow. MIR Publishers pp. 241-280..

Amadou, A. (2002). Application of the Initial value method in the Analysis of Circular Cylindrical shell under Hydrostatic Pressure and Ring Forces. M.Eng. Thesis submitted to the Department of Civil Engineering, University of Nigeria, Nsukka. December, 2002.

Appert, J. (1987). The Storage of food grains and seeds. CTA Macmillan Press Ltd London and Basingstoke.

Bagchi, A. and Paramasivam, V. (1996). Stability Analysis of Axisymmetric Thin shells. J. Engrg. Mech., Volume 122, Issue 3, pp. 278 – 281.

Bakker-Arkema, F.W. (1999): Grain and Grain Quality. In CIGR Handbook of Agric Engr. Vol. IV, edited by CIGR – The International Commission of Agric. Engr. Publ. ASAE. St. Joseph MI.

Bartali, E.H, (1999a). Masonry and Blocks. In chapter I of CIGR Handbook of Agric Engr. Vol. II, edited by CIGR – The International Commission of Agric. Engr. Publ. ASAE. St. Joseph MI.

Bartali, E.H. (1999b). Concrete and Steel. In CIGR Handbook of Agric Engr. Vol. II, edited by CIGR – The International Commission of Agric. Engr. Publ. ASAE. St. Joseph MI.

Beard, R. (1982). Grain Silos in Reinforced Brickwork. Proceedings of ICE Conference on Reinforced and Prestressed Masonry, May, 1982, London, Pub. Thomas telford Ltd, London.

Boumans, G. (1985). Grain Handling and Storage. Elsevier Science Pub. Amsterdan.

Brand, E.W. and Hangenoi, M. (1969). Effect of Method of Preparation of compaction Strength characteristics of Lateritic Soils. Proceeding of specialty Conference of Engineering Properties of Lateritic Soils. VIII CSMFE Mexico City, Mexico, vol. 1 pp. 107 – 116.

Britannica (2004). Shell Structure. Britannica Concise Encylopedia Article.

Brooker, D.B., Bakker-Arkema, F.W., Hall, C.W.(1992). Drying and Storage of Grains and Oilseeds. New York: Van Nostrand Reinhold

BS 110: Part 2(1970). Method of Testing Concrete. British Standards Institution.

BS 1881: Parts 108 –121 (1983). Testing Concrete. British Standards Institution.

Calladine, C.R. (1983). Theory of shell structures. Cambridge University Press. New York . 763 Pp.

Carles, A. B. (1983). Structures and Equipment. Oxford Tropical Handbooks; Oxford University Press, New York.

Chandrashekhera, K. and Rao, K.S.N. (1998). Static Analysis of Infinite Laminated Cylindrica Shell Panel. J. Aerosp. Engrg. Volume II, Issue 1, pp. 18.

Chandrashekhera, K. and Rao, K.S.N. (1996). Analysis of a Long thick Orthotropic Circular Cylindrical shell Panel. J. Engrg. Mech. Volume 122, Issue 6, pp. 575 – 579.

Chemleva T. and Mikeshina, N. (1969). In collected papers: New Ideas in Experimental Design, Edited by V. Nalimov, Nauka, Moscow.

Christensen, C. M. and Kaufman, H. H. (1969). Grain Storage: The Role of Fungi in Quality Loss. Minneapolis, MN: Univeristy of Minnesota Press.

Christensen, C.M. and Meronuck, R. A. (1986). Quality Maintenance in Stored Grains and Seeds. Minneapolis, MN: University of Mennesota Press.

Crop Storage Unit (1989). Improved Indoor Metal bins for On Farm Grain Storage, Federal Department of Agriculture and Rural Development. Nigeria.

Crop Storage Unit (1992). Improved Brick Masonry and Concrete Structures for Grain Storage. Federal Department of Agriculture and Rural Development. Nigeria.

Denttartog, J.P. (1952). Advanced Strength of Materials. McGraw Hill Book Co. Inc. New York. 379 Pps.

De Sousa e Silva, J. and Berbert, P.A. (1999) Grain Drying and Storage in the Tropics. CIGR Handbook of Agric Engr. Vol. IV. Edited by CIRG – The International Commission of Agric Engr. Publ. ASAE.St. Joseph MI.

Dewar, J.D. (1986). Ready-mixed Concrete Mix Design. Mun. Engr., 3, pp. 35 – 43.

Ding, X; Coleman, R., Rotter, J.M. (1996). Technique for Precise Measurement of Large-scale Silos and Tanks. J. Surv. Engrg., Vol. 122, Issue 1, pp. 14 – 25.

Dinnie, A. and Beard, R. (1970). Reinforced Brickwork Silos for Grain Storage. Proceedings of the British Ceramic Society. No. 17.

Department of the Environment (1988). Design of Normal Concrete Mixes, HMSO.

Dolby, C. (1999) Wood as a Construction Material for Farm Buildings. In Chapter one of CIGR Handbook of Agric Engr. Vol. II, edited by CIGR - the international commission of Agric Engr. Published ASAE.St.Joseph MI.

Gasga (1996). Risks and Consequences of the Misuse of Pesticides in the treatment of stored products. Technical Bulletin No.2, Wageningen, the Netherlands: CTA.

Gidigasu, M. D. (1976). Laterite Soil Engineering. American Elsevier Publishing Co. Inc, New York.

Haeger, J.M. and Sarafian, S.S. (1967). A New Concept of Storage Bin Construction. A.C.I. Journal.

Hall, C. W. (1957). Drying Farm Crops. Ann Arbor, MIL: Edward Brothers.

Harte, R., Kratzig, W.B., Montag, U. (2001). Shape Optimization, Design and Construction of the 200 m Niederaussem Cooling Tower Shell. A Structural Engineering Odyssey, Structures Congress, Edited by Peter C. Chang, May 21 – 23, 2001, Washington, D.C., USA.

Igbeka, J.C. and Olumeko, D. O. (1991). An appraisal of village – level Grain Practices in Nigeria. Agricultural Mechanization in Asia, Africa and Latin America. Vol. 27, No. 2 Pp 29-33.

Jackson, N. (1983). Civil Engineering Materials Macmillan Pub.

Johnson, G.D. (1982). Design and Construction of a Reinforced Brickwork Tank. Proceedinngs of ICE Conference on Reinforced and Prestressed Masonry, Pub. Thomas Tilford Ltd. London.

Krylov, A.N. (1931). On the Analysis of Beams on Elastic foundations, publication of the Acad. Sci. USSR.

Lasisi , F. and Osunade, J.A. (1984). Comparative Study of cement stabilized and unstablized laterite soils with grain size and optimum moisture content as factors. West Indian Journal of Engineering, West Indies.

Lasisi, F. and Osunade, J. A. (1990). Laterized Concrete Masonry as an Alternative in Building and Rural Infrastructures. Proceeding of International Conference on Structural Engineering, Seam 2, Kumasi, Ghana.

Mallagh, T.J.S. (1982). Prestressed Blockwork Silos. Proceeding of Institution of Civil Engineers, Conference on Reinforced and Prestressed Masonry, Pub. Thomas Telford Ltd, London.

Masida, E.A. (1978). On the utilization of lateritic clays for rural bricks. M.Sc. Thesis, Department of Geology, University of Ife, Nigeria.

Merritt, F. S. (1976). Standard Handbook for Civil Engineers, New York: McGraw-Hill.

Mijinyawa, Y and Dahunsi, D. (1997). The Use of Local Building Materials for the Construction of Farm Structures in Western Nigeria. NSE Technical Transactions, 32(3), pp 11-16.

Mohammed, B. (2003). Major Problems of Construction Materials for Livestock Structures in Borno State, Nigeria. Proceeding of the 4th Internation Conference of NIAE, Vol. 25, pp 294-300.

Montes, P., Bremner, T.W., Mrawira, D. (2005). Effects of Calcium Nitrile-Based Corrosion Inhibitor and Fly-Ash on Compressive Strength of High-performance Concrete, Materials Journals, ACI International, Vol. 102, pp. 3 – 8.

Montross, J.E; Montross, M.D. and Bakker-Arkema, F.W. (1999). Grain Storage. In chapter one of CIGR Handbook of Agric Engr. Vol IV, edited by CIGR – the International Commission of Agric Engr, published ASAE.St.Joseph MI.

Muhammad, A.K. (2000). Analysis of Thin Cylindrical shell subjected to Local and continuous Axi-symmetric loads. Unpublished Project Report submitted to the Department of Civil Engineering, KNUST Kumasi-Ghana, August, 2000.

Murdock, L.J., Brook, K.M., Dewar, J.D. (1991). Concrete Materials and Practice.. Pub. Edward Arnold, AuckLand, 470 pp.

Midwest Plan Service (1983). Farm Structures and Environment handbook. 11th Ed. Amsterdam.

Naaman, A.E., Wongtanakitcharoen, T. and Hanser, G (2005). Influence of Different Fibres on Plastic Shrinkage Cracking of Concrete. Materials Journal, ACI international, Vol. 102, pp. 49 – 58.

Nehdi, M. and Ladanchuk, J.D. (2004). Fibre Synergy in Fiber-reinforced Self-Consolidating Concrete. Materials Journal. ACI International, Vol. 101, pp. 508 –517.

Neville, A.M. (1996). Properties of Concrete, Addison Wesley, Longman Ltd. England.

Neville, A.M. and Brooks, J.J. (1987). Concrete Technology. Longman Group Ltd, Singapore.

Newill, D. and Dowling, J.W.F. (1964). Laterites in Western Malaysia and Northern Nigeria. Specialty Session and Engineering Properties of Lateritic Soils Proceedings. pp. 133 - 150. Bangkok, Thailand.

Nwakonobi, T.U.; Idike, F.I. Osadebe, N.N; Echiegu, E.A. (1998). Computer Optimization Models for Mix Proportioning of Clay-Rice Husk Cement Mixture for Partition Walls of Animal Buildings. NSAE Conference Proceedings of 1998, Lagos, Vol. 20, pp. 315 – 319.

Ola, S.A. (1985). Strength and Structural Properties of some Stabilized/unstabilized Compressed Kwara Soils. Seminar on the use of Clay Bricks and blocks for the provision of Cheaper and Durable Housing. Proceedings, pp. 51 – 78. Nigeria.

Olumeko, D.O. (1991). Technical Survey, Evaluation, and Economic Review of storage structures in Nigeria. Unpublished M. Phil. Thesis. University of Ibadan, Nigeria Pp. 110.

Olumeko, D.O. (1996): Assessment of On-farm Storage Structures in Nigeria. Paper Presented at NCAM National Training Workshops, NIR/AI, Ilorin, Kwara State.

Osunade, J.A.; Adeyefa, P.O.; and Lasisi, F. (1990): Shear and Tensile Properties of Unreinforced Laterized Concrete. Ife, Journal of Technology, Vol. 2 No. 1, Pp 8-17.

Osunade, J.A. (1993). The Compressive Strength of Laterized concrete. The Effect of Types and sizes of coarse Aggregates. Masonry International Journal Bulletin, Vol. 7, No. 1, Pp 104.

Osunade, J.A. (1994). Effect of grain size ranges of laterite fine aggregate on the shear and tensile strengths of laterized concrete. International journal for Housing Science and its Application, USA, Vol. 18 No. 2 Pp 91 – 104.

Osunade, J.A. (1997). Laterized Concrete Masonry as an alternative in Building And Rural Infrastructures. International Conference Proceedings On Structural Engineering, Vol. 1, Pp 151 – 161.

Pedersen, J. R. (1992). Insects. Identification, Damage and Detection. In Storage of Cereal Grains and their Products. Ed. D. B. Sauer. St Paul, MN: AACC.

Pircher, M. and Bridge, R.Q. (2001). Buckling of Thin-Walled Silos and Tanks under Axial Load – Some new Aspects. J. Struct. Engrg., Vol. 127, Issue 10, pp. 1129 – 1136.

Pitt, J. I. (1995). Under what conditions are Mycotoxins produced? Austrialia Mycotoxin Newsletter 6(2) : 1-2.

Quality Scheme for Ready Mixed Concrete (1989). Technical Regulations, Quality Scheme for Ready Mixed Concrete, Hampton.

Reimbert, M.L. (1976). Silos Theory and Practice. Trans. Tech. Pub. 250 pp.

Ren, D. and Kuan-Chen, F. (2001). Solution of Complete Circular Cylindrical Shell under Concentrated Loads. J. Engrg. Mech., Volume 127, Issue 3, pp. 248 – 253.

Salau, M. A. (1998). Structural Engineering Properties of some Plantation Timbers. NSE Technical Transactions, 33(3), pp 75-92.

Shelton, D.P. and Harper, J.M. (2003). An Overview of Concrete as a Building Material. Publ. Cooperative Extension, Institute of Agriculture & Natural Resources. University of Nebraska, Lincoln.

Scheffe, H. (1958) . Experiments with mixtures. Journal of the Royal Statistical Society, Ser. B. (20) Pp. 344 – 360.

Sechler, E.E. (1974). The historical development of Shell Research and Design. In thin shell structures, ed. Y.C. Fung and E. E. Sechler, pp. 3-25. Englewood Cliffs, N.J. Prentice – Hall, Inc.

Shi, Z. and Ohtsu, M. (2001). Application of Linear Programming to the Limit Analysis of Conical-shaped Steel Water Tanks. J. Struct. Engrg., Vol. 127,issue 11, pp. 1316 –1323.

Song, H., Shim, S., Byun, K. (2002). Failure Analysis of Reinforced Concrete Shell Structures using Layered Shell Element with Pressure Mode. J. Struct. Engrg., Vol. 128,issue 5, pp. 655 – 664.

Stroeven, P. and Bui, D.D. (1997). Effect of Rice-husk Ash on Compressive Strength of Gap-graded Concrete containing very Fine Sand. Proceeding of International congress on Structural Engineering, Seam 4.

Timoshenko, S. and Woinosky-krieyer S. (1959). Theory of Plate and Shell . McGraw Hill Book. Inc. New York.

Whitaker, J.H. (1979). Agricultural Buildings and Structures. Reston Publishing Company, Reston, Virginia, 530pp.

Zedginidze, I. (1971). Mathematical Design of Experiment to Study and optimize mixture properties, Metsniereba Publishers, Ibilisi.

Zingoni, A.J.; Nwakali A. and Salahuddin, A. (2000). Theory and Analysis of Structures: Trusses, Beams, Frames, Plates, and Shells. UNESCO Publ. Chap 5, pp. 165 –214

APPENDIX A

Q-BASIC PROGRAM

```
10 REM A QBasic program that optimizes the proportion of laterised concrete mixes
20 REM Variables Used
30 REM z1, z2, z3, z4, x1, x2, x3, x4, ymax, yout, yin

31 REM PersonalData
32 Name: NWAKONOBI, T. U. (MRS)
33 REG. NO: PG/Ph.D/98/
34 DEPT: AGRIC ENGINEERING
35 REM

40 REM Begin Main Program
50 LET Count = 0
60 CLS
70 GOSUB 100
80 END
90 REM End of Main Program

100 REM Procedure Begin
110 LET ymax = 0
120 PRINT
130 PRINT
140 PRINT "A Computer Model for the Computation of Laterized Concrete Mix
Proportions"
150 PRINT
160 PRINT "Corresponding to a Desired Strength"
170 PRINT
180 INPUT "Enter Desired Strength"; yin

190 GOSUB 400
200 FOR x1 = 0 TO 1 STEP .01
210 FOR x2 = 0 TO 1 - x1 STEP .01
220 FOR x3 = 0 TO 1 - x1 - x2 STEP .01
230 LET x4 = 1 - x1 - x2 - x3
240 LET yout = 27 * x1 + 18.4 * x2 + 16.6 * x3 + 7.3 * x4 + 11.2 * x1 * x2 - 16 * x1
* x3 + 1.4 * x1 * x4 - 2 * x2 * x3 + 16.2 * x2 * x4 - 10.6 * x3 * x4
250 GOSUB 500

260 IF (ABS(yin - yout) <= .001) THEN 270 ELSE 290
270 LET Count = Count + 1
280 GOSUB 600
290 NEXT x3
291 NEXT x2
292 NEXT x1
```

```
295 PRINT
300 IF (Count > 0) THEN GOTO 310 ELSE GOTO 340
310 PRINT "The Maximum value of strength predictable by this model is "; ymax;
"N/sq.mm"
320 SLEEP (2)
330 GOTO 360
340 PRINT "Sorry! Desired Strength out of range of model"
350 SLEEP (2)
360 RETURN

400 REM Procedure PrintHeading
410 PRINT
420 PRINT "Count  x1    x2    x3     x4     y    z1    z2    z3    z4"
430 PRINT

440 RETURN

500 REM Procedure CheckMax
510 IF ymax < yout THEN ymax = yout ELSE ymax = ymax
520 RETURN

600 REM Procedure OutResults
610 LET z1 = x1 + x2 + x3 + x4
620 LET z2 = x1 + 2 * x2 + 3 * x3 + 3 * x4
630 LET z3 = 2 * x1 + 4 * x2 + 6 * x3 + 8 * x4
640 LET z4 = .65 * x1 + .75 * x2 + 1.1 * x3 + 1.14 * x4
650 PRINT TAB(1); Count; USING "####.###"; x1; x2; x3; x4; yout; z1; z2; z3; z4
660 RETURN
```

APPENDIX B

PROGRAM TESTING AND TEST RESULTS

The Desired strength is 13

Count	x1	x2	x3	x4	y	z1	z2	z3	z4
1	0.000	0.140	0.580	0.280	12.999	1.000	2.860	6.280	1.062
2	0.040	0.170	0.430	0.360	12.999	1.000	2.750	6.220	1.037
3	0.040	0.210	0.260	0.490	12.999	1.000	2.710	6.400	1.028
4	0.050	0.070	0.630	0.250	12.999	1.000	2.830	6.160	1.063
5	0.100	0.150	0.010	0.740	13.000	1.000	2.650	6.780	1.032
6	0.110	0.150	0.120	0.620	12.999	1.000	2.630	6.500	1.023
7	0.130	0.130	0.080	0.660	13.000	1.000	2.610	6.540	1.022
8	0.170	0.090	0.270	0.470	12.999	1.000	2.570	6.080	1.011
9	0.180	0.010	0.540	0.270	12.999	1.000	2.630	5.800	1.026
10	0.190	0.080	0.150	0.580	13.000	1.000	2.540	6.240	1.010
11	0.190	0.080	0.160	0.570	13.001	1.000	2.540	6.220	1.009
12	0.230	0.040	0.030	0.700	13.001	1.000	2.500	6.400	1.010
13	0.250	0.030	0.140	0.580	13.000	1.000	2.470	6.100	1.000

The Maximum value of strength predictable by this model is 27.15071 N/sq.mm

The Desired strength is 14

Count	x1	x2	x3	x4	y	z1	z2	z3	z4
1	0.010	0.300	0.080	0.610	14.000	1.000	2.680	6.580	1.015
2	0.030	0.200	0.520	0.250	13.999	1.000	2.740	5.980	1.026
3	0.040	0.100	0.680	0.180	14.000	1.000	2.820	6.000	1.054
4	0.050	0.190	0.500	0.260	14.001	1.000	2.710	5.940	1.021
5	0.050	0.260	0.230	0.460	14.000	1.000	2.640	6.200	1.005
6	0.090	0.000	0.770	0.140	14.000	1.000	2.820	5.920	1.065
7	0.090	0.040	0.710	0.160	14.001	1.000	2.780	5.880	1.052
8	0.090	0.110	0.590	0.210	14.000	1.000	2.710	5.840	1.029
9	0.090	0.130	0.550	0.230	13.999	1.000	2.690	5.840	1.023
10	0.110	0.200	0.120	0.570	14.000	1.000	2.580	6.300	1.003
11	0.120	0.170	0.360	0.350	14.001	1.000	2.590	5.880	1.000
12	0.150	0.160	0.090	0.600	14.001	1.000	2.540	6.280	1.000
13	0.160	0.070	0.550	0.220	14.001	1.000	2.610	5.660	1.012
14	0.170	0.140	0.260	0.430	14.000	1.000	2.520	5.900	0.992
15	0.230	0.090	0.120	0.560	13.999	1.000	2.450	6.020	0.987
16	0.290	0.040	0.140	0.530	14.000	1.000	2.380	5.820	0.977
17	0.300	0.030	0.100	0.570	14.000	1.000	2.370	5.880	0.977

The Maximum value of strength predictable by this model is 27.15071 N/sq.mm

The Desired strength is 15

Count	x1	x2	x3	x4	y	z1	z2	z3	z4
1	0.000	0.340	0.430	0.230	14.999	1.000	2.660	5.780	0.990
2	0.010	0.370	0.260	0.360	15.000	1.000	2.610	5.940	0.980
3	0.020	0.290	0.490	0.200	15.000	1.000	2.670	5.740	0.998
4	0.020	0.360	0.170	0.450	15.001	1.000	2.600	6.100	0.983
5	0.020	0.360	0.220	0.400	14.999	1.000	2.600	6.000	0.981
6	0.040	0.010	0.870	0.080	15.000	1.000	2.910	5.980	1.082
7	0.040	0.300	0.410	0.250	14.999	1.000	2.620	5.740	0.987
8	0.040	0.320	0.340	0.300	15.001	1.000	2.600	5.800	0.982
9	0.040	0.330	0.100	0.530	15.000	1.000	2.590	6.240	0.988
10	0.050	0.270	0.460	0.220	15.000	1.000	2.630	5.700	0.992
11	0.050	0.320	0.270	0.360	15.000	1.000	2.580	5.880	0.980
12	0.080	0.020	0.820	0.080	14.999	1.000	2.820	5.800	1.060
13	0.080	0.140	0.650	0.130	15.000	1.000	2.700	5.660	1.020
14	0.080	0.170	0.600	0.150	14.999	1.000	2.670	5.640	1.011
15	0.080	0.290	0.180	0.450	15.000	1.000	2.550	6.000	0.980
16	0.080	0.290	0.200	0.430	15.000	1.000	2.550	5.960	0.980
17	0.100	0.260	0.300	0.340	15.000	1.000	2.540	5.760	0.978
18	0.120	0.220	0.390	0.270	15.000	1.000	2.540	5.620	0.980
19	0.120	0.240	0.080	0.560	14.999	1.000	2.520	6.160	0.984
20	0.130	0.140	0.570	0.160	15.000	1.000	2.600	5.520	0.999
21	0.140	0.200	0.390	0.270	15.001	1.000	2.520	5.580	0.978
22	0.170	0.180	0.350	0.300	15.000	1.000	2.480	5.560	0.973
23	0.230	0.020	0.640	0.110	15.001	1.000	2.520	5.260	0.994
24	0.230	0.120	0.370	0.280	14.999	1.000	2.420	5.400	0.966
25	0.240	0.050	0.560	0.150	15.000	1.000	2.470	5.240	0.980
26	0.250	0.120	0.230	0.400	14.999	1.000	2.380	5.560	0.961
27	0.260	0.110	0.120	0.510	14.999	1.000	2.370	5.760	0.965
28	0.280	0.080	0.350	0.290	14.999	1.000	2.360	5.300	0.958
29	0.290	0.080	0.290	0.340	14.999	1.000	2.340	5.360	0.955
30	0.300	0.040	0.450	0.210	15.000	1.000	2.360	5.140	0.959
31	0.310	0.060	0.050	0.580	15.000	1.000	2.320	5.800	0.963
32	0.330	0.010	0.470	0.190	15.001	1.000	2.330	5.040	0.956
33	0.360	0.010	0.370	0.260	14.999	1.000	2.270	5.060	0.945
34	0.380	0.010	0.160	0.450	15.001	1.000	2.230	5.360	0.944

The Maximum value of strength predictable by this model is 27.15071 N/sq.mm

The Desired strength is 16

Count	x1	x2	x3	x4	y	z1	z2	z3	z4
1	0.000	0.390	0.500	0.110	16.001	1.000	2.610	5.440	0.968
2	0.030	0.230	0.680	0.060	16.000	1.000	2.710	5.540	1.008
3	0.030	0.260	0.640	0.070	15.999	1.000	2.680	5.500	0.998
4	0.040	0.400	0.330	0.230	16.000	1.000	2.520	5.500	0.951
5	0.050	0.380	0.070	0.500	16.000	1.000	2.520	6.040	0.965
6	0.070	0.190	0.680	0.060	15.999	1.000	2.670	5.460	1.004
7	0.080	0.240	0.590	0.090	16.001	1.000	2.600	5.380	0.984
8	0.110	0.320	0.170	0.400	16.001	1.000	2.460	5.720	0.955
9	0.130	0.290	0.100	0.480	16.000	1.000	2.450	5.860	0.959
10	0.150	0.240	0.430	0.180	16.000	1.000	2.460	5.280	0.956
11	0.160	0.120	0.660	0.060	15.999	1.000	2.560	5.240	0.988
12	0.170	0.250	0.130	0.450	16.000	1.000	2.410	5.720	0.954
13	0.180	0.120	0.630	0.070	16.000	1.000	2.520	5.180	0.980
14	0.190	0.170	0.510	0.130	16.001	1.000	2.450	5.160	0.960
15	0.190	0.210	0.390	0.210	16.001	1.000	2.410	5.240	0.949
16	0.190	0.230	0.140	0.440	16.001	1.000	2.390	5.660	0.952
17	0.220	0.160	0.460	0.160	16.000	1.000	2.400	5.120	0.951
18	0.220	0.200	0.260	0.320	15.999	1.000	2.360	5.360	0.944
19	0.250	0.080	0.590	0.080	16.001	1.000	2.420	5.000	0.963
20	0.250	0.160	0.050	0.540	16.000	1.000	2.340	5.760	0.953
21	0.260	0.160	0.130	0.450	16.001	1.000	2.320	5.540	0.945
22	0.270	0.150	0.280	0.300	16.000	1.000	2.310	5.220	0.938
23	0.310	0.110	0.090	0.490	15.999	1.000	2.270	5.520	0.942
24	0.320	0.090	0.380	0.210	16.000	1.000	2.270	4.960	0.933
25	0.320	0.100	0.080	0.500	16.001	1.000	2.260	5.520	0.941
26	0.350	0.060	0.400	0.190	16.000	1.000	2.240	4.860	0.929
27	0.350	0.080	0.160	0.410	16.000	1.000	2.220	5.260	0.931
28	0.390	0.020	0.430	0.160	16.001	1.000	2.200	4.720	0.924

The Maximum value of strength predictable by this model is 27.15071 N/sq.mm

The Desired strength is 17

Count	x1	x2	x3	x4	y	z1	z2	z3	z4
1	0.000	0.500	0.500	0.000	17.000	1.000	2.500	5.000	0.925
2	0.000	0.580	0.180	0.240	17.000	1.000	2.420	5.320	0.907
3	0.020	0.490	0.030	0.460	17.001	1.000	2.470	5.860	0.938
4	0.020	0.540	0.190	0.250	17.000	1.000	2.420	5.340	0.912
5	0.030	0.490	0.420	0.060	16.999	1.000	2.450	5.020	0.917
6	0.040	0.410	0.530	0.020	16.999	1.000	2.510	5.060	0.939

7	0.060	0.410	0.480	0.050	17.000	1.000	2.470	5.040	0.932
8	0.060	0.430	0.030	0.480	17.000	1.000	2.450	5.860	0.942
9	0.060	0.470	0.200	0.270	17.001	1.000	2.410	5.360	0.919
10	0.070	0.390	0.490	0.050	17.001	1.000	2.470	5.040	0.934
11	0.070	0.430	0.400	0.100	17.000	1.000	2.430	5.060	0.922
12	0.090	0.400	0.400	0.110	17.000	1.000	2.420	5.060	0.924
13	0.100	0.310	0.560	0.030	17.000	1.000	2.490	5.040	0.948
14	0.100	0.410	0.210	0.280	17.001	1.000	2.390	5.340	0.923
15	0.100	0.410	0.270	0.220	17.000	1.000	2.390	5.220	0.920
16	0.110	0.390	0.150	0.350	16.999	1.000	2.390	5.480	0.928
17	0.130	0.360	0.340	0.170	17.001	1.000	2.380	5.100	0.922
18	0.140	0.240	0.600	0.020	17.001	1.000	2.480	5.000	0.954
19	0.150	0.340	0.300	0.210	16.999	1.000	2.360	5.140	0.922
20	0.160	0.330	0.190	0.320	16.999	1.000	2.350	5.340	0.925
21	0.170	0.220	0.580	0.030	17.001	1.000	2.440	4.940	0.948
22	0.170	0.260	0.500	0.070	17.001	1.000	2.400	4.940	0.935
23	0.170	0.280	0.450	0.100	17.000	1.000	2.380	4.960	0.929
24	0.180	0.240	0.520	0.060	17.000	1.000	2.400	4.920	0.937
25	0.190	0.210	0.560	0.040	17.001	1.000	2.410	4.900	0.943
26	0.200	0.270	0.080	0.450	16.999	1.000	2.330	5.560	0.933
27	0.200	0.280	0.150	0.370	17.000	1.000	2.320	5.380	0.927
28	0.210	0.140	0.650	0.000	16.999	1.000	2.440	4.880	0.956
29	0.230	0.250	0.190	0.330	17.001	1.000	2.290	5.240	0.922
30	0.240	0.220	0.040	0.500	17.000	1.000	2.300	5.600	0.935
31	0.240	0.240	0.220	0.300	17.000	1.000	2.280	5.160	0.920
32	0.240	0.240	0.230	0.290	17.000	1.000	2.280	5.140	0.920
33	0.250	0.160	0.540	0.050	16.999	1.000	2.340	4.780	0.933
34	0.260	0.210	0.100	0.430	17.000	1.000	2.270	5.400	0.927
35	0.260	0.210	0.340	0.190	17.000	1.000	2.270	4.920	0.917
36	0.300	0.170	0.340	0.190	17.001	1.000	2.230	4.840	0.913
37	0.310	0.130	0.470	0.090	16.999	1.000	2.250	4.680	0.919
38	0.320	0.140	0.030	0.510	17.000	1.000	2.220	5.460	0.927
39	0.340	0.140	0.200	0.320	17.001	1.000	2.180	5.000	0.911
40	0.340	0.140	0.240	0.280	17.000	1.000	2.180	4.920	0.909
41	0.350	0.130	0.180	0.340	17.001	1.000	2.170	5.020	0.911
42	0.350	0.130	0.260	0.260	17.000	1.000	2.170	4.860	0.907
43	0.360	0.110	0.070	0.460	16.999	1.000	2.170	5.260	0.918
44	0.390	0.050	0.500	0.060	17.000	1.000	2.170	4.460	0.909
45	0.430	0.020	0.490	0.060	16.999	1.000	2.120	4.360	0.902
46	0.430	0.050	0.080	0.440	17.001	1.000	2.090	5.060	0.907

47	0.440	0.050	0.180	0.330	16.999	1.000	2.070	4.800	0.898
48	0.460	0.030	0.130	0.380	17.000	1.000	2.050	4.860	0.898
49	0.490	0.000	0.080	0.430	17.000	1.000	2.020	4.900	0.897

The Maximum value of strength predictable by this model is 27.15071 N/sq.mn

The Desired strength is 18

Count	**x1**	**x2**	**x3**	**x4**	**y**	**z1**	**z2**	**z3**	**z4**
1	0.000	0.710	0.090	0.200	18.000	1.000	2.290	4.980	0.860
2	0.010	0.730	0.140	0.120	18.000	1.000	2.250	4.740	0.845
3	0.010	0.830	0.160	0.000	18.000	1.000	2.150	4.300	0.805
4	0.020	0.590	0.020	0.370	18.000	1.000	2.370	5.480	0.899
5	0.040	0.590	0.090	0.280	17.999	1.000	2.330	5.220	0.887
6	0.040	0.640	0.180	0.140	17.999	1.000	2.280	4.840	0.864
7	0.040	0.670	0.240	0.050	18.000	1.000	2.250	4.600	0.850
8	0.050	0.520	0.000	0.430	18.001	1.000	2.380	5.620	0.913
9	0.050	0.640	0.270	0.040	18.000	1.000	2.260	4.600	0.855
10	0.080	0.480	0.020	0.420	17.999	1.000	2.360	5.560	0.913
11	0.080	0.550	0.220	0.150	18.000	1.000	2.290	4.880	0.878
12	0.090	0.460	0.010	0.440	18.000	1.000	2.360	5.600	0.916
13	0.090	0.500	0.110	0.300	18.000	1.000	2.320	5.240	0.897
14	0.090	0.510	0.140	0.260	18.001	1.000	2.310	5.140	0.891
15	0.100	0.500	0.170	0.230	18.000	1.000	2.300	5.060	0.889
16	0.100	0.510	0.330	0.060	18.001	1.000	2.290	4.700	0.879
17	0.120	0.440	0.440	0.000	17.999	1.000	2.320	4.640	0.892
18	0.130	0.430	0.090	0.350	18.000	1.000	2.310	5.320	0.905
19	0.140	0.440	0.210	0.210	18.000	1.000	2.280	4.980	0.891
20	0.160	0.360	0.000	0.480	18.000	1.000	2.320	5.600	0.921
21	0.220	0.310	0.080	0.390	18.000	1.000	2.250	5.280	0.908
22	0.220	0.320	0.130	0.330	18.000	1.000	2.240	5.140	0.902
23	0.220	0.330	0.220	0.230	18.001	1.000	2.230	4.920	0.895
24	0.220	0.330	0.280	0.170	18.000	1.000	2.230	4.800	0.892
25	0.230	0.310	0.350	0.110	18.000	1.000	2.230	4.680	0.892
26	0.240	0.280	0.050	0.430	18.001	1.000	2.240	5.340	0.911
27	0.250	0.290	0.170	0.290	18.000	1.000	2.210	5.000	0.898
28	0.280	0.250	0.360	0.110	17.999	1.000	2.190	4.600	0.891
29	0.300	0.190	0.500	0.010	17.999	1.000	2.210	4.440	0.899
30	0.300	0.210	0.440	0.050	18.000	1.000	2.190	4.480	0.894
31	0.310	0.180	0.500	0.010	18.000	1.000	2.200	4.420	0.898
32	0.320	0.170	0.500	0.010	17.999	1.000	2.190	4.400	0.897
33	0.340	0.190	0.140	0.330	18.001	1.000	2.130	4.920	0.894

34	0.350	0.180	0.140	0.330	18.000	1.000	2.120	4.900	0.893
35	0.360	0.150	0.440	0.050	18.001	1.000	2.130	4.360	0.888
36	0.430	0.090	0.420	0.060	18.001	1.000	2.050	4.220	0.877
37	0.440	0.060	0.500	0.000	18.000	1.000	2.060	4.120	0.881
38	0.440	0.100	0.220	0.240	18.001	1.000	2.020	4.520	0.877
39	0.450	0.080	0.090	0.380	18.000	1.000	2.020	4.800	0.885
40	0.510	0.010	0.470	0.010	18.000	1.000	1.970	3.960	0.867

The Maximum value of strength predictable by this model is 27.15071 N/sq.mm

The Desired Strength is 19

Count	**x1**	**x2**	**x3**	**x4**	**y**	**z1**	**z2**	**z3**	**z4**
1	0.040	0.700	0.020	0.240	19.001	1.000	2.220	4.920	0.847
2	0.050	0.850	0.080	0.020	19.000	1.000	2.050	4.140	0.781
3	0.060	0.750	0.100	0.090	19.001	1.000	2.130	4.440	0.814
4	0.070	0.680	0.090	0.160	19.000	1.000	2.180	4.680	0.837
5	0.100	0.640	0.160	0.100	18.999	1.000	2.160	4.520	0.835
6	0.100	0.680	0.210	0.010	19.000	1.000	2.120	4.260	0.817
7	0.110	0.650	0.230	0.010	19.000	1.000	2.130	4.280	0.823
8	0.140	0.470	0.010	0.380	19.001	1.000	2.250	5.260	0.888
9	0.140	0.480	0.030	0.350	19.001	1.000	2.240	5.180	0.883
10	0.140	0.570	0.280	0.010	19.001	1.000	2.150	4.320	0.838
11	0.160	0.520	0.250	0.070	19.001	1.000	2.160	4.460	0.849
12	0.180	0.480	0.240	0.100	19.000	1.000	2.160	4.520	0.855
13	0.190	0.430	0.100	0.280	18.999	1.000	2.190	4.940	0.875
14	0.190	0.450	0.170	0.190	19.000	1.000	2.170	4.720	0.865
15	0.200	0.440	0.200	0.160	19.000	1.000	2.160	4.640	0.862
16	0.210	0.370	0.010	0.410	19.001	1.000	2.210	5.240	0.892
17	0.230	0.400	0.260	0.110	19.000	1.000	2.140	4.500	0.861
18	0.230	0.400	0.300	0.070	19.001	1.000	2.140	4.420	0.859
19	0.250	0.320	0.010	0.420	18.999	1.000	2.180	5.200	0.892
20	0.250	0.330	0.040	0.380	19.001	1.000	2.170	5.100	0.887
21	0.280	0.330	0.310	0.080	19.000	1.000	2.110	4.380	0.862
22	0.290	0.310	0.370	0.030	19.001	1.000	2.110	4.280	0.862
23	0.320	0.280	0.210	0.190	19.000	1.000	2.080	4.540	0.866
24	0.330	0.260	0.140	0.270	18.999	1.000	2.080	4.700	0.871
25	0.370	0.210	0.410	0.010	19.000	1.000	2.050	4.120	0.860
26	0.400	0.190	0.340	0.070	18.999	1.000	2.010	4.160	0.856
27	0.440	0.150	0.340	0.070	19.000	1.000	1.970	4.080	0.852
28	0.460	0.110	0.040	0.390	18.999	1.000	1.970	4.720	0.870
29	0.480	0.100	0.420	0.000	19.000	1.000	1.940	3.880	0.849

30	0.520	0.050	0.010	0.420	19.000	1.000	1.910	4.660	0.865
31	0.520	0.070	0.120	0.290	19.001	1.000	1.890	4.360	0.853
32	0.600	0.000	0.400	0.000	19.000	1.000	1.800	3.600	0.830

The Maximum value of strength predictable by this model is 27.15071 N/sq.mm

The Desired strength is 20

Count	x1	x2	x3	x4	y	z1	z2	z3	z4
1	0.080	0.770	0.000	0.150	20.001	1.000	2.070	4.440	0.801
2	0.150	0.690	0.150	0.010	19.999	1.000	2.010	4.040	0.791
3	0.260	0.440	0.190	0.110	19.999	1.000	2.040	4.300	0.833
4	0.260	0.450	0.240	0.050	20.000	1.000	2.030	4.160	0.828
5	0.270	0.430	0.220	0.080	20.000	1.000	2.030	4.220	0.831
6	0.280	0.420	0.270	0.030	20.000	1.000	2.020	4.100	0.828
7	0.300	0.390	0.300	0.010	20.001	1.000	2.010	4.040	0.829
8	0.310	0.370	0.220	0.100	20.000	1.000	2.010	4.220	0.835
9	0.330	0.310	0.070	0.290	20.001	1.000	2.030	4.640	0.855
10	0.330	0.340	0.200	0.130	20.000	1.000	2.000	4.260	0.838
11	0.360	0.280	0.090	0.270	20.000	1.000	2.000	4.540	0.851
12	0.380	0.280	0.220	0.120	19.999	1.000	1.960	4.160	0.836
13	0.420	0.230	0.180	0.170	20.000	1.000	1.930	4.200	0.837
14	0.500	0.120	0.030	0.350	20.000	1.000	1.880	4.460	0.847
15	0.520	0.120	0.120	0.240	20.000	1.000	1.840	4.160	0.834
16	0.540	0.090	0.060	0.310	19.999	1.000	1.830	4.280	0.838
17	0.610	0.050	0.230	0.110	20.000	1.000	1.730	3.680	0.812
18	0.610	0.050	0.320	0.020	20.000	1.000	1.730	3.500	0.809
19	0.630	0.030	0.190	0.150	19.999	1.000	1.710	3.720	0.812
20	0.670	0.000	0.270	0.060	20.000	1.000	1.660	3.440	0.801
21	0.670	0.000	0.280	0.050	20.000	1.000	1.660	3.420	0.801

The Maximum value of strength predictable by this model is 27.15071 N/sq.mm

The Desired strength is 21

Count	x1	x2	x3	x4	y	z1	z2	z3	z4
1	0.150	0.720	0.000	0.130	21.000	1.000	1.980	4.220	0.786
2	0.190	0.720	0.090	0.000	21.001	1.000	1.900	3.800	0.763
3	0.230	0.550	0.060	0.160	20.999	1.000	1.990	4.300	0.810
4	0.310	0.400	0.040	0.250	21.000	1.000	1.980	4.460	0.831
5	0.320	0.440	0.180	0.060	20.999	1.000	1.920	3.960	0.804
6	0.340	0.350	0.020	0.290	21.000	1.000	1.970	4.520	0.836
7	0.350	0.390	0.170	0.090	21.000	1.000	1.910	4.000	0.810
8	0.360	0.370	0.150	0.120	21.000	1.000	1.910	4.060	0.813

9	0.380	0.300	0.020	0.300	20.999	1.000	1.940	4.480	0.836
10	0.380	0.340	0.140	0.140	21.000	1.000	1.900	4.080	0.816
11	0.380	0.350	0.180	0.090	21.001	1.000	1.890	3.960	0.810
12	0.420	0.300	0.190	0.090	21.000	1.000	1.860	3.900	0.810
13	0.430	0.270	0.110	0.190	20.999	1.000	1.870	4.120	0.820
14	0.440	0.280	0.220	0.060	20.999	1.000	1.840	3.800	0.806
15	0.580	0.090	0.010	0.320	21.001	1.000	1.750	4.140	0.820
16	0.590	0.110	0.140	0.160	21.000	1.000	1.710	3.740	0.802
17	0.650	0.040	0.060	0.250	21.000	1.000	1.660	3.820	0.804
18	0.680	0.000	0.000	0.320	21.001	1.000	1.640	3.920	0.807

The Maximum value of strength predictable by this model is 27.15071 N/sq.mm

The Desired strength is 22

Count	x1	x2	x3	x4	y	z1	z2	z3	z4
1	0.220	0.740	0.020	0.020	21.999	1.000	1.820	3.680	0.743
2	0.250	0.600	0.010	0.140	22.001	1.000	1.900	4.080	0.783
3	0.280	0.620	0.100	0.000	22.000	1.000	1.820	3.640	0.757
4	0.290	0.570	0.080	0.060	22.000	1.000	1.850	3.820	0.772
5	0.300	0.530	0.060	0.110	22.001	1.000	1.870	3.960	0.784
6	0.380	0.440	0.170	0.010	21.999	1.000	1.800	3.620	0.775
7	0.400	0.350	0.030	0.220	22.000	1.000	1.850	4.140	0.806
8	0.400	0.400	0.150	0.050	21.999	1.000	1.800	3.700	0.782
9	0.460	0.310	0.130	0.100	22.001	1.000	1.770	3.740	0.789
10	0.590	0.140	0.040	0.230	21.999	1.000	1.680	3.820	0.795
11	0.630	0.090	0.000	0.280	22.000	1.000	1.650	3.860	0.796
12	0.690	0.080	0.210	0.020	22.001	1.000	1.540	3.120	0.762
13	0.730	0.010	0.030	0.230	22.001	1.000	1.530	3.520	0.777
14	0.740	0.030	0.170	0.060	22.001	1.000	1.490	3.100	0.759

The Maximum value of strength predictable by this model is 27.15071 N/sq.mm

The Desired strength is 23

Count	x1	x2	x3	x4	y	z1	z2	z3	z4
1	0.310	0.650	0.040	0.000	23.000	1.000	1.730	3.460	0.733
2	0.320	0.630	0.050	0.000	23.001	1.000	1.730	3.460	0.736
3	0.360	0.490	0.010	0.140	22.999	1.000	1.790	3.860	0.772
4	0.440	0.390	0.060	0.110	23.000	1.000	1.730	3.680	0.770
5	0.450	0.360	0.030	0.160	23.000	1.000	1.740	3.800	0.778
6	0.460	0.380	0.100	0.060	23.000	1.000	1.700	3.520	0.762
7	0.480	0.320	0.030	0.170	22.999	1.000	1.720	3.780	0.779
8	0.540	0.290	0.150	0.020	23.000	1.000	1.630	3.300	0.756

9	0.750	0.050	0.070	0.130	22.999	1.000	1.450	3.160	0.750

The Maximum value of strength predictable by this model is 27.15071 N/sq.mm

The Desired strength is 24

Count	**x1**	**x2**	**x3**	**x4**	**y**	**z1**	**z2**	**z3**	**z4**
1	0.400	0.520	0.000	0.080	24.000	1.000	1.680	3.520	0.741
2	0.490	0.380	0.020	0.110	24.000	1.000	1.640	3.500	0.751
3	0.570	0.310	0.100	0.020	24.000	1.000	1.550	3.140	0.736
4	0.770	0.060	0.000	0.170	24.001	1.000	1.400	3.140	0.739

The Maximum value of strength predictable by this model is 27.15071 N/sq.mm

The Desired strength is 25

Count	**x1**	**x2**	**x3**	**x4**	**y**	**z1**	**z2**	**z3**	**z4**
1	0.470	0.510	0.010	0.010	25.000	1.000	1.550	3.120	0.710
2	0.490	0.470	0.010	0.030	25.000	1.000	1.550	3.160	0.716
3	0.540	0.400	0.030	0.030	25.001	1.000	1.520	3.100	0.718
4	0.680	0.230	0.060	0.030	25.000	1.000	1.410	2.880	0.715
5	0.710	0.180	0.020	0.090	25.001	1.000	1.400	2.980	0.721
6	0.760	0.140	0.050	0.050	25.001	1.000	1.340	2.780	0.711
7	0.780	0.120	0.050	0.050	25.001	1.000	1.320	2.740	0.709
8	0.850	0.040	0.010	0.100	24.999	1.000	1.260	2.720	0.708

The Maximum value of strength predictable by this model is 27.15071 N/sq.mm

The Desired strength is 26

Count	**x1**	**x2**	**x3**	**x4**	**y**	**z1**	**z2**	**z3**	**z4**
1	0.680	0.280	0.020	0.020	25.999	1.000	1.360	2.760	0.697
2	0.730	0.230	0.040	0.000	26.001	1.000	1.310	2.620	0.691

The Maximum value of strength predictable by this model is 27.15071 N/sq.mm

The Desired strength is 27

Count	**x1**	**x2**	**x3**	**x4**	**y**	**z1**	**z2**	**z3**	**z4**
1	1.000	0.000	0.000	0.000	27.000	1.000	1.000	2.000	0.650

The Maximum value of strength predictable by this model is 27.15071 N/sq.mm

The Desired Strength is 28

Count	**x1**	**x2**	**x3**	**x4**	**y**	**z1**	**z2**	**z3**	**z4**

Sorry! Desired Strength out of range of mode!