ANALYSIS OF LAMINAR HEAT TRANSFER ON THERMAL VISCOUS DISSIPATIVE FLOW OF A NEWTONIAN FLUID BETWEEN PARALLEL PLATES GEOMETRIES WITH CONSTANT HEAT FLUX BOUNDARY CONDITIONS

BY

UWAEZUOKE, MARTIN UCHE
B.Sc (Hons) UNIUYO, MATHEMATICS
M.SC (UNIZIK) HYDRODYNAMIC STABILITY

PG/Ph.D/14/68265

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE AWARD OF Ph.D. IN APPLIED MATHEMATICS

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DECEMBER, 2019

DECLARATION

I, UWAEZUOKE, MARTIN UCHE. Registration number PG/Ph.D/14/68265, a post graduate student of Doctor of Philosophy at University of Nigeria, Nsukka, do hereby solemnly declare that the thesis entitled " Analysis of Laminar Heat Transfer on Thermal Viscous Dissipative Flow of a Newtonian fluids between Parallel Plates Geometries with Constant Heat Flux Boundary Conditions". Submitted by me in partial fulfillment of the requirements for Doctor of Philosophy degree in applied Mathematics is my original work and has not been submitted and shall not, in future, be submitted by me for obtaining any degree from this or any other University or institution. I hereby declare that all information in this thesis has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that as required by these rules and conduct, I have fully referenced all materials that are not original to this work.

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UWAEZUOKE, MARTIN UCHE DATE

PG/Ph.D/ 14/68265

CERTIFICATION

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The board of examiners certifies as follows, that to the best of our knowledge, this is the original work of the candidate. That the thesis is accepted in partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy (Ph.D) in applied mathematics

Prof. M.O. Oyesanya ……………………….. …………………
(Supervisor) Signature Date

Dr. A. Odio ……………………….. …………………..
(Ag. Head of Department) Signature Date

Prof. Lawrence Ezeanyika ……………………….. …………………..
(Dean Graduate Studies) Signature Date

……………………….. …………………..
(Chairman, Board of Examiners) Signature Date

Prof. S.S. Okoya ……………………….. …………………..
(External Examiner) Signature Date

DEDICATION

To:

My parents, who have always been a source of inspiration, zeal and strength for me.

Memories of my late father,

All members of my family.

ACKNOWLEDGEMENTS

Praise is to Almighty God, creator of the heavens and earth and Lord of Lords, who gave me the potential and ability to complete this thesis. All of my respect goes to Jesus (peace upon him) who emphasized the significance of knowledge and research.

Academic research involves not only the efforts of the researcher but it also requires generous and sincere cooperation of many other accessories like different departments, research institutions, libraries and consulting personalities. The present Ph.D Thesis is likewise the result of valuable input from various quarters.

I would like to acknowledge the following people for their guidance and support during my doctoral study and writing of this thesis at the University of Nigerisa, Nsukka.

First of all, I would like to pay tribute and thanks to Prof. M.O. Oyesanya, who accepted to supervise this thesis. His teaching, patience, trust and encouragement throughout my doctoral study were invaluable. Many thanks should be ascribed to Prof. G.C.E. Mbah with whom I had many useful discussions in the area of my doctoral study.

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Worthy of acknowledgement are members of staff in the Department of Mathematics and Statistics and the entire community of Imo State University, Owerri for the Support given to me throughout the study.

Last but not the least, I would like to thank my Family: my dear wife Chidiebere Chituru Uwaezuoke, my parents Mrs. Justina Uwaezuoke and to my brothers and sisters whose constant, unending and countless prayers buoyed me up.

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ABSTRACT

In the thesis, an investigation has been made to analyze the effects of viscous dissipation on the analytical solutions of temperature distributions and heat transfer rates for both hydro dynamically and thermally fully developed laminar plane Couette flow and flow between two fixed infinitely long parallel plates. On the basis of some routine assumptions made in the literature, a closed form analytical expressions of Nusselt numbers for the flow of Newtonian fluid with constant properties has been developed for three different cases of constant heat flux boundary conditions. The significant effect of the viscous dissipation as compared to other terms in the energy equation is manifested by the Brinkman number. In order to have a generalized idea about the viscous heating effect on the heat transfer analysis, different definitions of the Brinkman number have been used in the present study. For different values of the relative velocity for both flows, the effect of the modified Brinkman number and Brinkman number on the temperature distributions and the Nusselt numbers are discussed. Comparison of the present analytical solutions for the simple constant heat flux boundary conditions with those available in the literature indicates that some of the reported results were different from what we have obtained independently and also suggest a new way of expressing the Nusselt numbers.

NOMENCLATURE

A- PLANE COUETTE FLOW

$a_1 - a_3$	constant
Br	Brinkman number
Br_m	Modified Brinkman number
C_p	Specific heat at constant pressure (J / gK)
h_1	Heat transfer coefficient at upper plate (W/ m^3-k)
H	Channel height (m)
K	Thermal conductivity (W / mK)
Nu_H	Nusselt number at the upper plate
q_1	Upper wall heat flux (W / m^2)
q_2	Lower wall heat flux (W / m^2)
T	Temperature (K)
T_1	Upper wall temperature (K)
T_2	Lower wall temperature (K)
u	Velocity (m /s)
U	Dimensionless velocity
U_p	Velocity of the moving plate (m /s)
x	Axial coordinate direction (m)
y	Vertical coordinate direction (m)
Y	Dimensionless vertical coordinate

Greek symbols

θ Dimensionless temperature

θ_m Dimensionless bulk mean temperature

μ Dynamic viscosity (Kg / m- s)

ρ Density (Kg / m^3)

Subscripts

f Uniform fluid

m mean

w wall

B - FLOW BETWEEN FIXED PARALLEL PLATES CHANNEL

a_1 Parameter defined in Eqs. (3.091a), (3.091b) and (3.091c)

a_2 Parameter defined in Eqs. (3.124a), (3.124b) and (3.124c)

a_3 Parameter defined in Eqs. (3.110a), (3.110b) and (3.110c)

Br Brinkman number

Brq_1 Modified Brinkman number

C_p Specific heat at constant pressure (J /g K)

h Convective heat transfer coefficient

K Thermal conductivity (W / m^2)

L Width of plate (m)

Nu Nusselt number

P	Pressure (Pa)
q_1	Upper wall heat – flux (W /m K)
q_2	Lower wall heat – flux (W /m K)
T	Temperature (K)
T_o	Wall temperature when both walls are kept at constant heat flux (K)
T_1	Upper wall temperature (K)
T_2	Lower wall temperature (K)
ΔT	General temperature difference (K)
u	Velocity (m/ s)
U	Dimensionless velocity
w	Half – channel height (m)
W	Channel height (= 2w) (m)
x	Coordinate in the axial direction (m)
y	Coordinate in the vertical direction (m)
Y	Dimensionless vertical coordinate

Greek symbols

α	Thermal diffusivity (m^2 / s)
θ	Dimensionless temperature
θ_m	Mean dimensionless temperature
μ	Dynamic viscosity (Kg / m- s)
ρ	Density (Kg / m^3)

Subscripts

c	Centerline
m	Mean

LIST OF FIGURES

A Fractional Mathematical Model on Transmission and Control of the Spread of Myiasis in Human System Using Insect Repellents

Eze, Hyginus Ejike

November, 2019

Key; Mathematical Model, Transmission, Human System, Insect Repellents

In this project, we proposed a fractional Susceptible, Expose, Infected, Treated and Recovered order model to study the transmission dynamics of Myiasis. We showed the existence of the equilibrium states. The basic reproduction number of the model was evaluated in terms of parameters in the model using the next generation matrix approach. We provided the conditions for the stability of the diseasefree and the endemic equilibrium points. A detailed stability analysis of the model was carried out. Numerical simulations of the model were carried out using Adams-type predictor-corrector method which showed in detail the population dynamics of the disease. From the simulations, the level of impacts of the parameters of the model were demonstrated.

Mathematical Model on Middle East Respiratory Syndrome and Control

Ezema,Godwin Izuchukwu

November, 2019

Key; Mathematical Model, Respiratory Syndrome, disease infection, Ezema,Godwin Izuchukwu

In this work,we considered the general overview of Middle East Respiratory Syndrome(MERS-CoV). A model for the transmission dynamics of the disease infection with control was formulated using a set of ordinary differential equations. The solution to the model equations were obtained using Homotopy Perturbation method, and ways for controlling or preventing the disease presented. The effective reproduction number ,R0 of the system was obtained and it was shown that the disease will spread only if R0 > 1 and would die off with time, if R0 < 1. Numerical simulations carried out on the model showed that with the judicious use of Modified Virus Ankara(MVA) in combination with MERS-CoV inoculation, the disease will phase out rapidly in the population.

Mathematical Modeling Relating Psychosocial Therapy as a Veritable Panacea for Bipolar Ii Disorder Patient

Michael, Uchenna Emmanuel

November, 2019

Key; Mathematical Modeling, Psychosocial Therapy, Veritable Panacea, Bipolar Ii Disorder Patient

This study applies mathematical models to emotional variation in patients with bipolar II disorder using dynamical oscillator. Three models of the mood swing are studied and solved using multiple scale perturbation and some numerical simulation. We first incorporated the psychosocial therapy in the forcing function which reveals that if Family and Friends Therapy (FFT) is constructively administered the patient recuperate speedily. The second model studies the effect of HPA axis secretions to the emotional variation of bipolar II disorder patient; it is observed that the increase or decrease of the HPA hormone from the basal level in the body system affects the mood variation of bipolar II disorder patients but should not be used as an etiology maker. Finally, Fractional – ordered oscillator is used to model the mood variation of bipolar II disorder patient to absorb the continuous stressors in the environment that triggers the illness; it is observed that the fractional model brings out a better explaination of the medication parameter (pharmacotherapy and psychotherapy) effect on the amplitude (magnitude) of the mood of the patient with the stressors variation. Generally, it is observed that FFT is of great importance for the recuperation of the patient when combined with administered psychotic drugs both for in the onset of the disorder and the diagnosed patient.

Analysis of a Fractional Order Model for Influenza A

Nwachukwu, Anthonia Uchenna

August, 2018

Key; Influenza A, Transmission Dyanmics, Mathematical Model, Nwachukwu, Anthonia Uchenna

In this project work, a fractional order mathematical model that describe the transmission dynamics of Influenza A Virus was given and control mechanism for the spread of the Virus was given.

Qualitative dynamics of the model is determined by the basic reproduction number,R_0 using the next generation matrix. Through the analysis of the model, the stability conditions of the equilibria were derived using Jacobian transformation method. To illustrate our theoretical analysis, some numerical simulations were carried out using Adams-type predictor-corrector method to show the behavior of the solutions of the proposed fractional order system.

The simulation results provide a theoretical basis to control the spread of influenza A and also show that the fractional model is better than the classical model.

Analysis of Laminar Heat Transfer on Thermal Viscous Dissipative Flow of a Newtonian Fluid between Parallel Plates Geometries with Constant Heat Flux Boundary Conditions

Uwaezuoke, Martin Uche

December, 2019

Key; Laminar Heat Transfer, Newtonian Fluid, Parallel Plates Geometries, Heat Flux

In the thesis, an investigation has been made to analyze the effects of viscous dissipation on the analytical solutions of temperature distributions and heat transfer rates for both hydro dynamically and thermally fully developed laminar plane Couette flow and flow between two fixed infinitely long parallel plates. On the basis of some routine assumptions made in the literature, a closed form analytical expressions of Nusselt numbers for the flow of Newtonian fluid with constant properties has been developed for three different cases of constant heat flux boundary conditions. The significant effect of the viscous dissipation as compared to other terms in the energy equation is manifested by the Brinkman number. In order to have a generalized idea about the viscous heating effect on the heat transfer analysis, different definitions of the Brinkman number have been used in the present study. For different values of the relative velocity for both flows, the effect of the modified Brinkman number and Brinkman number on the temperature distributions and the Nusselt numbers are discussed. Comparison of the present analytical solutions for the simple constant heat flux boundary conditions with those available in the literature indicates that some of the reported results were different from what we have obtained independently and also suggest a new way of expressing the Nusselt numbers.

CHAPTER 1

INTRODUCTION

Interest in plate heat exchanger surfaces with a small ratio of heat transfer area to core volume is increasing at an accelerated pace. The primary reasons for the use of these less compact surfaces is that a larger, lighter weight and lower cost exchange is the result. These gains are brought about by both the direct geometric advantage of small "area density" and because forced convection heat transfer in large dimension passages generally results in small heat transfer coefficients (heat transfer power per unit area and temperature difference) for a specified flow friction power per unit area.

The flow passage for these small area density surfaces have a large hydraulic diameter. Consequently, with liquid flows particularly, the plate heat exchanger design range for Reynolds number usually falls well within the laminar flow regime. It follows then that the theory derived laminar flow solutions for friction and heat transfer in channels of various flow across-sections geometries become important and these solutions are the subject matter of this analysis A direct application of these results may be in the development of new surfaces with improved characteristics. A critical examination of the theory solutions may prove to be fruitful because there is a wide range for the heat transfer coefficient, at a given friction power for different cross-section geometries.

It has been realized that laminar flow heat transfer is dependent on the channel geometry, flow inlet velocity profile and the wall temperature and/or heat flux boundary conditions. These conditions are difficult to control in the laboratory, nevertheless there is a substantial ongoing experimental research effort devoted to this task. A theory base is needed in order to interpret the experimental results and to extrapolate these results for the task of designing practical heat exchanger systems.

However, it is recognized that this theory is founded on idealizations of geometry and boundary conditions that are not necessarily well duplicated either in application or even in the laboratory. The development of this theory base has been a fertile field of applied mathematics since the early days of the science of heat transfer. Today, by the application of modern computer technology, analysis to some degree as exceeded experimental verification.

1.1 Overview of the Study

The many applications of fluid mechanics makes it one of the most vital and fundamental of all applied scientific studies. The flow of fluids in pipes and channels is of importance to civil engineers. The study of fluid machinery such as pumps, compressors, heat exchanger, jet and rocket engines and the like, makes fluid mechanics of importance to mechanical engineers. The flow of air over objects, aerodynamics, is of fundamental interest to aeronautical and space engineers in the design of aircraft, missile, and rocket. In meteorology, hydrology and oceanography the study of fluids is basic since the atmosphere and the oceans are fluids. Today in modern sciences many new disciplines combine fluid mechanics with classical disciplines. For example, fluid mechanics and electromagnetic theory are studied together as magneto hydrodynamics, fluid mechanics and heat transfer theory are studied together as thermo-fluidic transport.

1.2 Scope of Study

The thesis presents an introduction to the science of laminar flow heat transfer in channel geometry with a small ratio of heat transfer area to core volume. To lay the foundation for the treatment of macro channel convection, theoretical framework are presented. Analytic solutions to plane Couette flow and flow between fixed parallel plate channel and heat transfer will be detailed. Attention will be focused on convection of Newtonian fluid in macro channels. The treatment will be limited to single phase shear driven flow between parallel plates (Couette flow) and pressure driven flow through rectangular channel.

Although extensive research on fluid flow and heat transfer in macro channel has been carried out during the past two decades much remains unresolved. Due to the complex nature of the phenomena, the role of various factors such as channel size, Reynolds number, surface roughness, dissipation, axial conduction and thermo physical properties is not fully understood.

As with all new research areas, discrepancies on findings and conclusions are not uncommon. Conflicting findings are attributed to the difficulty in making accurate measurements of channel size, surface roughness, pressure distribution as well as uncertainties in entrance effects and the determination of thermo physical properties.

1.3 Aims of Research

The aims of the present study are:

(i) To develop a mathematical model which is capable of accurately predicting temperature and velocity profiled, heat transfer rates for viscous Newtonian fluid by analytical means of the simultaneous momentum, energy and continuity partial differential equations during heat transfer in laminar flow channels with constant wall heat fluxes.

The variable separable method was used. The expressions of the Nusselt numbers is to yield certain design information such as heat transfer rates. Two important geometric cases are considered: Plane Couette flow, and a channel flow between two fixed, parallel plates. Since Newtonian fluids are usually viscous, the inlet velocity is assumed to be fully developed, but the inlet temperature is taken as uniform (The program actually developed is easily modified and is suitable for the solution of the combined hydrodynamic thermal entry length problem) The temperature heat flux boundary conditions is left sufficiently general so as not to allow consideration of any arbitrary specified condition in the direction of flow. In both channels the two plates may have different temperature or heat flux conditions. The thermal conductivity, heat capacity and density of the fluid are assumed to be uniform. Buoyany affects, natural convection effects and conduction of heat in the axial direction are neglected, but the facility for handling viscous dissipation is included.

(ii) To carry out detailed and rigorous study to explore analytically the effect of viscous dissipation on the heat transfer rates for a fully developed plane Couette flow and flow between fixed parallel plates of a Newtonian fluid between two parallel plate subjected to the constant heat flux boundary conditions.

(iii) To compare the results with other work available in the literature, especially that of Aydin and Avci (2006) for plane Poiseuille flow.

In the realm of macro flows, there are many practical applications where heat and fluid transport involve fixed and the movement of boundaries e.g. extrusion, hot rolling, drawing, and continuous casting and so on. The thermo-fluid transport through those systems, especially transfer of heat either from the boundaries to the surrounding fluid or the other way round, is of immense importance. However, the moving boundary deforms the fluid velocity profile, and shears the fluid layer near the boundary, and thus results in local changes in velocity gradient. Hence the viscous dissipation effects cannot be neglected in heat transfer analysis of a system associated with moving boundaries. In order to obtain the actual heat transfer rate in the application of fixed or moving boundaries for proper design of the system, it is important to take into account the effects of viscous dissipations by using accurate velocity distribution.

The thesis is devoted to problems involving fluid flow and heat transfer in parallel-plate channel with fixed and a moving boundary of solid body or liquid that can be found in many manufacturing processes such as polymer coatings of wires(or tubes) for corrosion protection. In such processes, the fixed and moving body continuously exchanges heat with the surrounding environment. For such cases, the fluid involved may be Newtonian or non-Newtonian and the flow situations encountered can be either laminar or turbulent.

The viscous fluid flow induced by a shearing motion of a wall in parallel-plate channel has been the subject of extensive research because of its numerous applications in many branches of science and technology. The velocity field for such flow usually serves as a starting estimate for the velocity of more complex flows induced by convection and various boundary conditions. One of the fundamental fluid flow situations in a non-porous medium is the Couette flow. Investigation of heat transfer in such flow has many important industrial applications.

The mathematical model describing such flow is the Navier Stokes equations and the reason for studying this flow is that it is a simple example of shear flow and the steady analytical solution is known.

1.4 Theoretical Frame Work

1.4.1 Navier-Stokes Equation

The Navier-Stokes equations are derived from conservation principles of mass, momentum and energy.

1.4.1.1 Mass Conservation

The continuity equation expresses the mass, conservation for a moving fluid particle as

$$\frac{d\rho}{dt} + \rho div\vec{u} = 0 \tag{1.0}$$

For the applications presented in this thesis, the density ρ may be considered as constant so that the continuity equation reduces to:

$$div\vec{u} = 0 \tag{1.1}$$

1.4.1.2 Momentum Conservation

The Navier Stokes equations expresses the budget of momentum for a particle without loss of generality, we can write:

$$\rho\frac{d\vec{u}}{dt} = \rho\vec{F} + div\overline{\overline{\sigma}} \tag{1.2}$$

Where $\vec{F}$ represents the body force vector per unit mass (the most usual example is that of gravity, with $\vec{F} = \vec{g}$). $\overline{\overline{\sigma}}$ is the stress tensor, expressed with index notations for a Newtonian fluid by:

$$\overline{\overline{\sigma}} = -p\delta ij - \frac{2\mu}{3}\left(div\vec{u}\right)\delta ij + 2\mu dij \tag{1.3}$$

In equation (1.3), δij is the Kronceker symbol ($\delta ij = 1$ if $i = j$, $\delta ij = 0$ if $i \neq j$) and dij is the pure strain tensor $\left(dij = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)\right)$

The Naiver-Stokes equations are then obtained for an incompressible flow of a fluid with constant dynamic viscosity μ . They are expressed in vector notations are:

$$\rho\frac{d\vec{u}}{dt} = \rho\vec{F} - \overline{grad}\rho + \mu\Delta\vec{u} \tag{1.4}$$

1.4.1.3 Energy Equation

Inside a flow, a fluid particle exchanges heat by conduction with the neighbouring particles during its motion. It also exchanges heat by radiation with the environment but this mode of transfer is not covered in this work. The conductive heat transfer is governed by Fourier's law:

$$\vec{q}'' = -k\,\overline{grad}T \tag{1.5}$$

where $\vec{q}''$ is the heat flux vector in W/m^2. The heat transfer rate flowing through a surface element ds of normal $\vec{n}$ is $\vec{q}''\vec{n}ds$. Combining the first principle of thermodynamics, the kinetic energy equation, Fourier's law, and introducing some fluid physical properties the energy equation is obtained without loss of generality as:

$$\rho C_p \frac{dT}{dt} = div\left(k\,\overline{grad}T\right) + q''' + \beta T \frac{dp}{dt} + Dv \tag{1.6}$$

This equation shows that the temperature variations of a moving fluid particle are due to:

- Conductive heat exchange with the neighbouring particles (first term of right-handside), - internal heat generation (q''' :Joule effect, radioactivity etc).
- Mechanical power of the pressure forces during the particle fluid compression or dilatation (third term of the right hand side).
- Specific viscous dissipation (Dv, power of the friction forces inside the fluid).

It is worth noting that the left hand side represents the transport (advection) of enthalpy by the fluid motion. All the terms of equation (1.6) are expressed in N/m. The specific viscous dissipation is calculated for a Newtonian fluid by:

$$Dv = 2\mu dijdij - 2\mu/3\left(div\bar{u}\right)^2 \tag{1.7}$$

For flows with negligible dissipation or for gas flows at moderate velocity (Dv and dp/dt are assumed to be negligible), the energy equation reduces to;

$$\frac{dT}{dt} = \alpha \Delta T \tag{1.8}$$

Using Cartesian coordinates, the terms of equation (1.6) are expressed in the following form:

$$\frac{dT}{dt} = u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}$$

$$div\left(k\overline{grad}T\right) = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)$$

$$D_v = 2\mu\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + D_{v2}$$

$$D_{v2} = \mu\left[\left(\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial z}\right)^2\right] - \frac{2\mu}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2$$

Equation (1.8) reads:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \tag{1.9}$$

1.5 Exact Solutions of the Navier-Stokes Equation

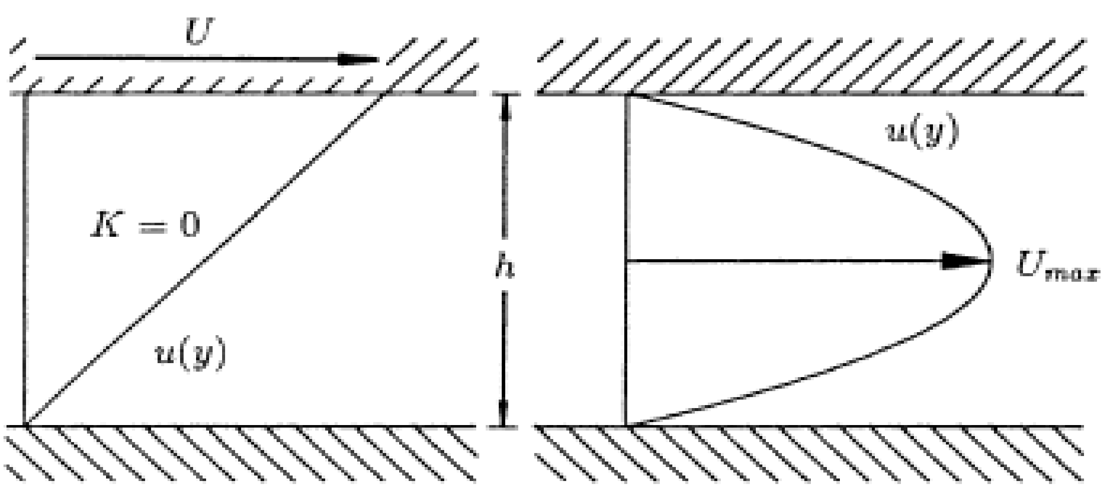
Because of the great complexity of the full compressible Navier-Stokes equations, no known general analytical solution exists. Hence, it is necessary to simplify the equations either by making assumptions about the fluid, about the flow or about the geometry of the problem in order to obtain analytical solutions. Typical assumptions are that the flow is laminar steady two-dimensional, the fluid is incompressible with constant properties and the flow is between parallel plates. By so doing it is possible to obtain analytical and exact solutions to the Navier-Stokes equations. Finding analytical solutions of the Navier-Stokes equations even in the uncoupled case, presents almost insurmountable mathematical difficulties due to the nonlinear character of the equations. However, it is possible to find analytical solutions in certain particular cases, generally when the nonlinear convective terms vanish naturally.

1.6 Wall Bounded Shear Flow

In shear flows, the fluid motion is dominated by sheets of fluid moving in different velocities parallel to each other. Although seemingly a substantial simplification, shear flows are present in more complicated flows when considering the flow sufficiently close to an object. In this thesis, the study describes two classical wall bounded shear flows which have been studied extensively; the main reason for this achieved popularity being that they all are analytical solutions of the Navier-Stokes equations.

All two flows concern the flow in simple geometries. For fixed parallel plates, the flow is driven by a constant non-zero pressure gradient in the stream wise direction. The other classical wall bounded shear flow is the plane Couette flow which concern the flow between two infinite parallel plates. In planer Couette flow, the upper plate is moving at a constant speed relative to the lower plate.

The steady solutions of these flows are parallel flows, where the velocity only depends on the distance from the wall; for plane Couette flow, the solution is a velocity profile which varies linearly between the velocities of the two plates and for flow between fixed parallel plate, the solution is a parabolic velocity profile in the direction of the negative pressure gradient see Figure 1.00



(a) Plane Couette flow for K = 0 (b) Fixed parallel plate channel flow for K ≠ 0

Figure 1.00

The length scale used in the definition of the Reynolds number for fixed parallel plate channel flow is half the distance between the plates. The velocity scale used is not half the velocity difference between the plates in plane Couette flow but half the maximum and mean velocity in fixed parallel plate channel flow. For the fixed parallel plate channel flow, the coordinate is chosen such that x is the streamwise direction, y the direction normal to the plates. The plates are located at y = ±h,

The steady solutions are now given by

$$u = \begin{cases} (1-y^2) & \text{for fixed parallel plate channel flow} \\ y & \text{for plane Couette flow} \end{cases}$$

Plane Couette flow and fixed parallel plate channel flow have been extensively studied by applied mathematicians throughout years, mainly because they are some of the simplest examples of flows available. However, despite their simplicity much is still unknown about the effect of viscous dissipation on heat transfer of the flows. A better understanding of the important mechanisms in these flows could have implications for others, more complicated flows.

1.7 Heat Transfer in Parallel Plate Channels

We consider internal flow through channels as ducts, tubes and parallel plates. The following factors are noted in analyzing heat transfer in internal flow.

1) Laminar vs turbulent flow, Transition from laminar to turbulent flow takes place, when the Reynolds number reaches the transition value. For flow through tubes the experimentally determined transition Reynolds number Re_{Dt} is

$$\mathrm{Re}_{Dt} = \frac{\bar{u}D}{\upsilon} \approx 2300 \tag{1.10}$$

where

D - channel diameter

$\bar{u}$ - mean velocity

υ - kinematic viscosity

2) Entrance vs. fully developed region. Based on velocity and temperature distribution, two regions are identified.
   i) Entrance region
   ii) Fully developed region
3) Surface boundary conditions. Two common thermal boundary conditions
4) will be considered.
   i) Uniform surface temperature
   ii) Uniform surface heat flux

A common problem involves a fluid entering a channel with uniform velocity and

temperature. The objective of analyzing internal flow heat transfer depends on the thermal boundary condition.

1.7.1 Hydro dynamic and Thermal Regions: General Features

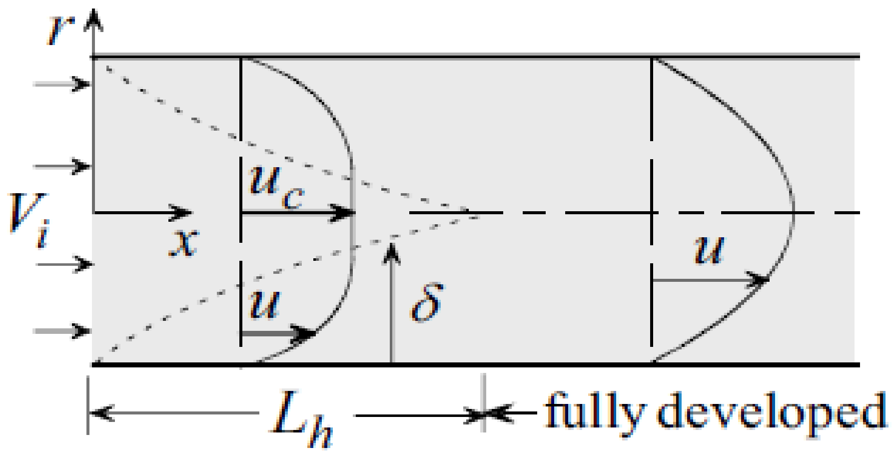
We consider fluid entering a channel with uniform velocity V_i and temperature T_i. Velocity and temperature boundary layers form on the inside surface of the channel. The two boundary layers grow as the distance x from the entrance is increased. Two regions are identified for each of the flow (hydrodynamic) and

temperature (thermal) fields.

1) Entrance region. This is also referred to as the developing region. It extends from the inlet to the section where boundary layer thickness reaches the channel centre.
2) Fully developed region: This zone follows the entrance region.

It is noted in general that the length of the velocity and temperature entrance regions are not identical. The general features of velocity and temperature fields in the two regions are examined in detail.

1.7.2 Flow Field



1) Entrance Region: (Developing flow, $0 \le x \le L_h$)

Fig (1.01) shows the developing velocity boundary layer in the entrance region of a channel. This region is called the hydrodynamic entrance region. Its length L_h, is referred to as the hydrodynamic entrance length. This region is characterized by the following features.

- Streamlines are not paralleled. Thus transverse velocity component does not vanish $(v \neq 0)$
- Core velocity u_c increases with axial distance $x(u_c \neq \text{constant})$
- Pressure decreases with axial distance $(d\rho/dx < 0)$
- Velocity boundary layer thickness is within channel radius $(\delta < D/2)$

2) Fully Developed Flow Region. At $x \ge L_h$ the flow is described as fully developed. It is characterized by the following features.

* Streamlines are parallel $(v = 0)$

* For two-dimensional incompressible flow the axial velocity u is invariant with axial distance x .

That is $\partial u/\partial x = 0$.

1.7.3 Temperature Field

1) Entrance Region (Developing Temperature, $0 \le x \le L_t$)

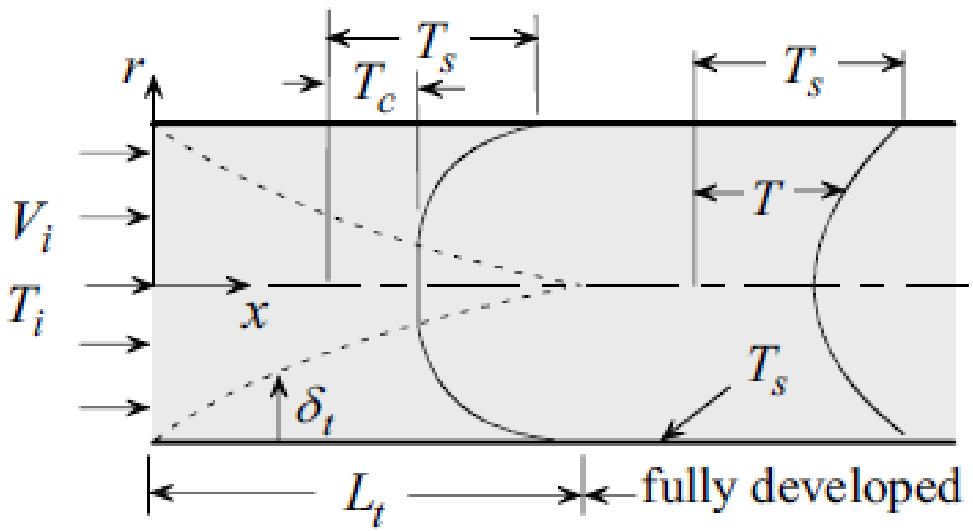


Fig (1.02) shows fluid entering a channel with uniform velocity v_i and temperature T_i. The surface is at uniform temperature T_s. The region in which the temperature boundary layer forms and grows is referred to as the thermal entrance region. The length of this region, L_t is called the thermal entrance length. At $x = L_t$ the thermal boundary layer thickness δ_t reaches the channels centre. This region is characterized by the following features.

- Core temperature T_c is uniform equal to inlet temperature $(T_c = T_i)$
- Temperature boundary layer thickness is within the channels radius $(\delta_t < D/2)$

2) Fully Developed Temperature Region: The region $x \ge L_t$ is characterized by the following features.

* Fluid temperature varies transversely and axially. Thus $\partial T/\partial x \ne 0$

* A dimensionless temperature θ (to be defined later) is invariant with axial distance x. That is $\partial \theta/\partial x = 0$.

1.8 Thermal Considerations in Parallel Plate Channel Flows

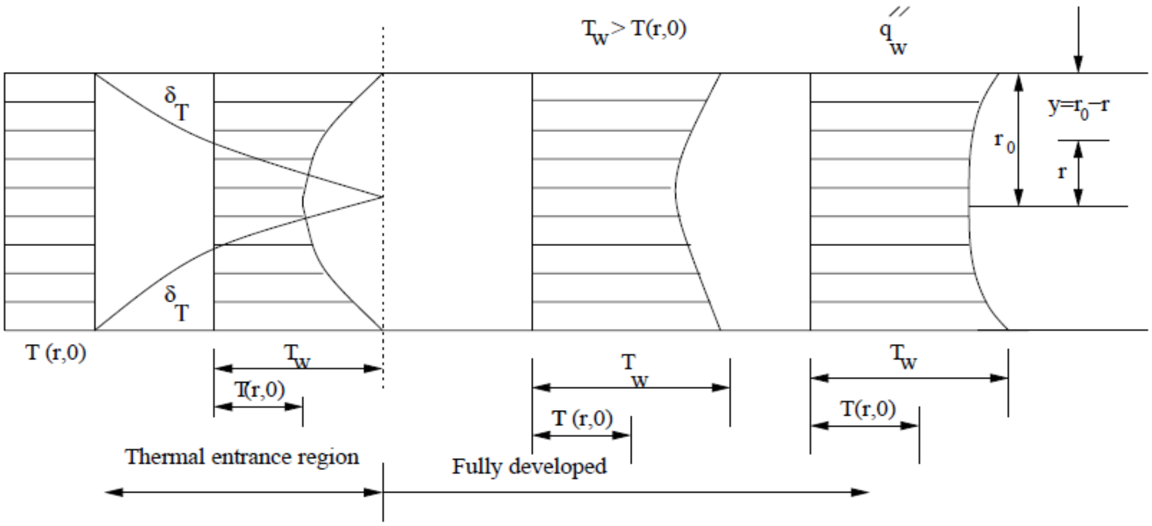


Figure (1.03): Thermally developing channel flow.

Channel surface condition: either a uniform wall temperature (UWT) condition or a uniform wall heat flux condition (UWH).

A fully developed temperature profile differs according to the surface conditions. For both surface conditions, however, the amount by which the fluid temperature exceeds the entrance temperature increases with increasing x (Fig 1.03). In order to study the development of thermal boundary layer, a hydro-dynamically fully developed flow may be considered (Fig 1.04)

1. For laminar flow, the thermal entrance length ($x_{e,t}$) may be expressed as

$$\left(\frac{x_{e,t}}{D}\right)_{lam} \approx \begin{cases} 0.033Re_D \ Pr & \text{for UWH} \\ 0.043Re_D \ Pr & \text{for UWT} \end{cases} \tag{1.11}$$

We can say, ($x_{e,t}$ / D) = 0.05 R_{eD} P_r.

Comparing this with hydro- dynamic boundary layer, it can be said that if Pr > 1, hydro-dynamic boundary layer grows more rapidly.

Pr > 1, $x_{e,h} < x_{e,t}$ => $\delta > \delta_T$ at any section. Therefore, for Pr > 1, δ has to be > δ_T.

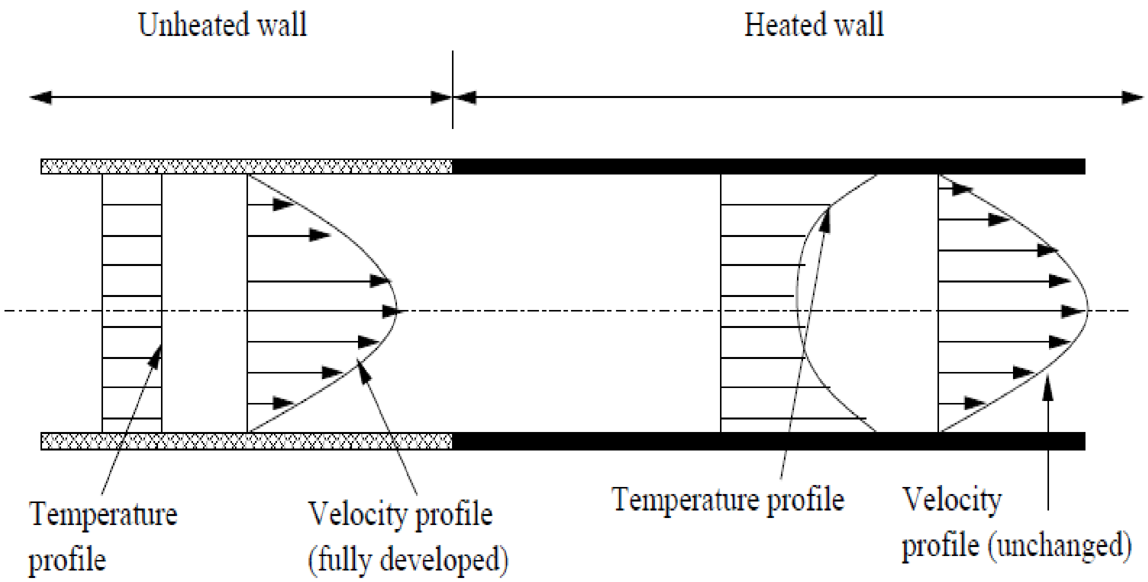


Figure (1.04) Developing velocity and temperature fields.

1. Inverse is true for Pr < 1, and consequently δ_T is greater than δ. However, for externally high Prandtl number fluids (such as oil), Pr ≥ 100, $x_{e,h} << x_{e,t}$. Throughout the thermal entrance region, hydro-dynamically fully developed velocity profile can be assumed (Figure 1.05)

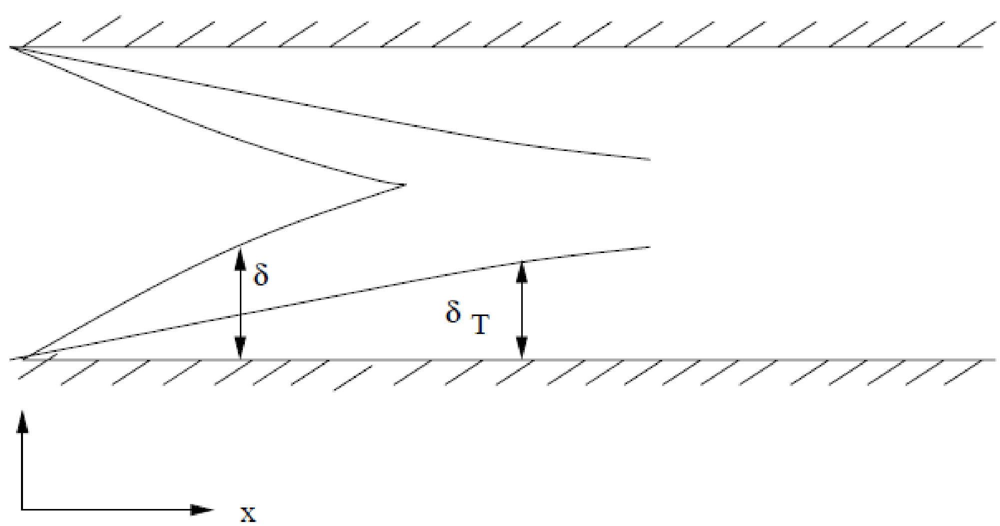


Figure (1.05) Hydro-dynamic and thermal boundary layer thickness for Pr >> 1.

For turbulent flow, conditions are nearly independent of Prandtl number. It is fair enough to accept $\left(\frac{x_{e,t}}{D}\right)_{tur} = 10$.

Bulk mean temperature at any section (x) is given by

$$T_m = \frac{\int \rho u T dA}{\int \rho u dA} \tag{1.12}$$

1.8.1 Newton's Law of Cooling

$$q''_w = h(T_w - T_m) \tag{1.13}$$

The bulk mean temperature T_m is a convenient reference temperature for internal flows. T_m behaves like T_∞ for external flows. Essential difference is T_m varies in the flow direction. T_m is increased in the flow direction if heat transfer is from surface to fluid.

1.8.2 Thermally Fully Developed Conditions in Parallel Plate Channels Flows

In terms of heat transfer, the fully developed thermal condition may be expressed in the following manner in terms of the non-dimensional temperature,

$$\theta_1 = \left(\frac{T_w - T}{T_w - T_m}\right)_1 = \left(\frac{T_w - T}{T_w - T_m}\right)_2 = \theta_2 \tag{1.14}$$

where the subscripts 1, 2 and W refer to an upstream location, downstream location, and wall respectively. The mean temperature of the fluid T_m is given by

$$T_m = \frac{\int \rho u T dA}{\int \rho u dA} \tag{1.15}$$

it still remains that the variation of the mean temperature with axial position be found. Unlike the fluid flow problem, where the mean velocity remains constant, the value of T_m changes in the downstream direction. Otherwise, if T_m was constant, then no heat transfer would occur. As a result, zero stream-wise derivatives of θ are assumed for thermally fully developed conditions, rather than zero stream-wise derivatives of T_m. Also, the convective heat transfer coefficient, h, remains constant for fully developed thermal conditions.

Since the existence of convective heat transfer between the surface and the fluid dictates that the fluid temperature must continue to change with x, one might question whether fully developed thermal conditions ever can be reached. The situation is certainly different from the hydro-dynamic case, for which $(\partial u/\partial x)=0$ in the fully developed region. In contrast, if there is heat transfer, (dT_m/dx) as well as $(\partial T/\partial x)$ at any distance y is not zero. Introducing a dimensionless temperature $(T_w - T)/(T_w - T_m)$, condition for which this ratio becomes independent of x are known to exist. Although the temperature profile $T(y)$ continues to change with x, the relative shape of the profile does not change and the flow is said to be fully developed. Instead of $(dT_m/dx)=0$ or $(\partial T/\partial x)=0$, the condition is

$$\frac{d}{dx}\left[\frac{T_w(x)-T(r,x)}{T_w(x)-T_m(x)}\right]=0 \tag{1.16}$$

i.e, $(T_w - T)$ changes in the same way as $(T_w - T_m)$.

Now, we can write

$$\frac{(T_w-T_m)\left(\frac{dT_w}{dx}-\frac{dT}{dx}\right)-(T_w-T)\left(\frac{dT_w}{dx}-\frac{dT_m}{dx}\right)}{(T_w-T_m)^2}=0$$

or

$$\frac{1}{(T_w-T_m)}\frac{dT_w}{dx}-\frac{dT}{dx}\frac{1}{(T_w-T_m)}-\frac{T_w-T}{(T_w-T_m)^2}\frac{dT_w}{dx}+\frac{(T_w-T)}{(T_w-T_m)^2}\frac{dT_m}{dx}=0$$

or,

$$\frac{dT_w}{dx}-\frac{dT}{dx}-\frac{T_w-T}{T_w-T_m}\frac{dT_m}{dx}+\frac{(T_w-T)}{(T_w-T_m)}\frac{dT_m}{dx}=0$$

or,

$$\frac{dT}{dx}=\frac{dT_w}{dx}-\frac{T_w-T}{T_w-T_m}\frac{dT_w}{dx}+\frac{(T_w-T)}{(T_w-T_m)}\frac{dT_m}{dx} \tag{1.17}$$

Two surface conditions are available. Uniform (q''_w = constant) and uniform surface temperature, $(dT_w/dx=0)$

For q''_w = constant if the channel wall were heated electrically or if the outer surface were uniformly irradiated.

For $(dT_w/dx)=0$ if a phase change were occurring at the outer surface.

However, for constant heat flux, we get

$$\frac{dT_w}{dx}=\frac{dT_m}{dx} \tag{1.18}$$

Substituting this condition in the above equation for dT/dx, we get

$$\left|\frac{dT}{dx}\right|_{\text{fully developed}}=\left|\frac{dT_m}{dx}\right|_{\text{fully developed}} \tag{1.19}$$

For constant wall temperature, $\frac{dT_w}{dx}=0$

$$\left.\frac{dT}{dx}\right|_{fd.t}=\frac{(T_w-T)}{(T_w-T_m)}\left.\frac{dT_m}{dx}\right|_{fd.t} \tag{1.20}$$

Several important features of thermally developed flow may be inferred from equation (1.17). since non-dimensional temperature profile is independent of x, derivative of the profile with respect to y (gradient) must also be independent of x. evaluating this derivative at the channel surface, we obtain

$$\frac{\partial}{\partial r}\left[\frac{(T_w-T)}{(T_w-T_m)}\right]_{y=y_0}=\frac{-\partial T/\partial y|_{y=y_0}}{(T_w-T_m)}\neq f(x) \tag{1.21}$$

Again we know,

$$q''_w = K\frac{\partial T}{\partial y}\bigg|_{y=y_0} = h(T_w - T_m)$$

or,

$$\frac{-\dfrac{\partial T}{\partial y}\bigg|_{y=y_0}}{(T_w - T_m)} = \frac{h}{k} \tag{1.22}$$

From (1.21) and (1.22), we get, $\frac{h}{k} \neq f(x)$ for fully developed (Figure 1.06). Here k is assumed to be constat, hence h is independent of x.

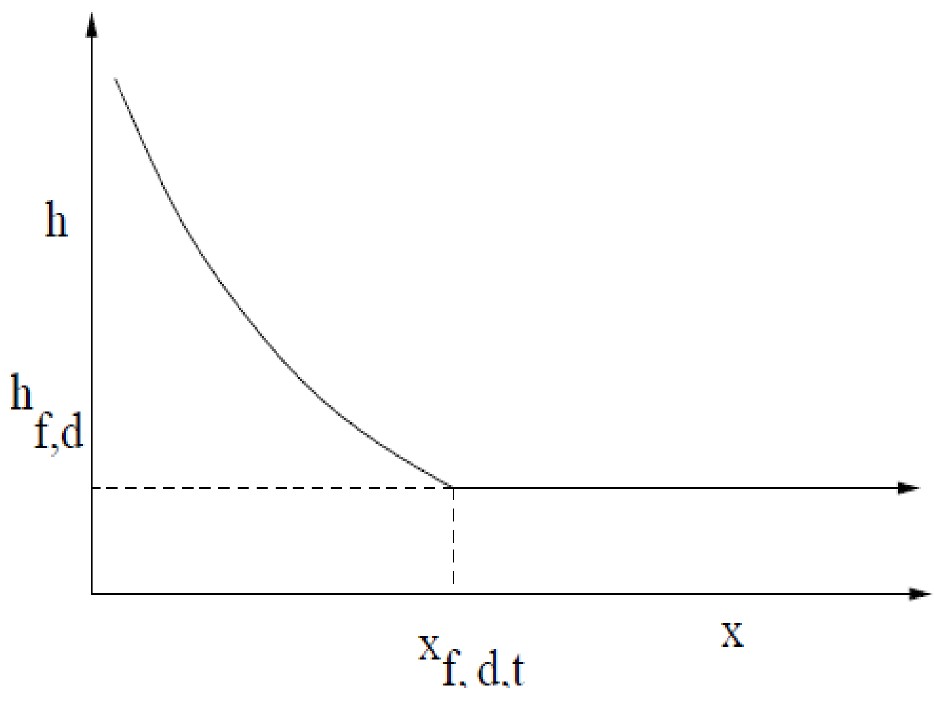


Figure 1.06: Heat transfer coefficient for thermally fully developed flow.

1.9 Energy Balance in Parallel Plate Channels

Because the flow in a channel is completely enclosed, an energy balance may be applied to determine how the mean temperature (T_m) varies with position along the channel and how the total convective heat transfer is related to the difference in temperatures at the channel inlet outlet.

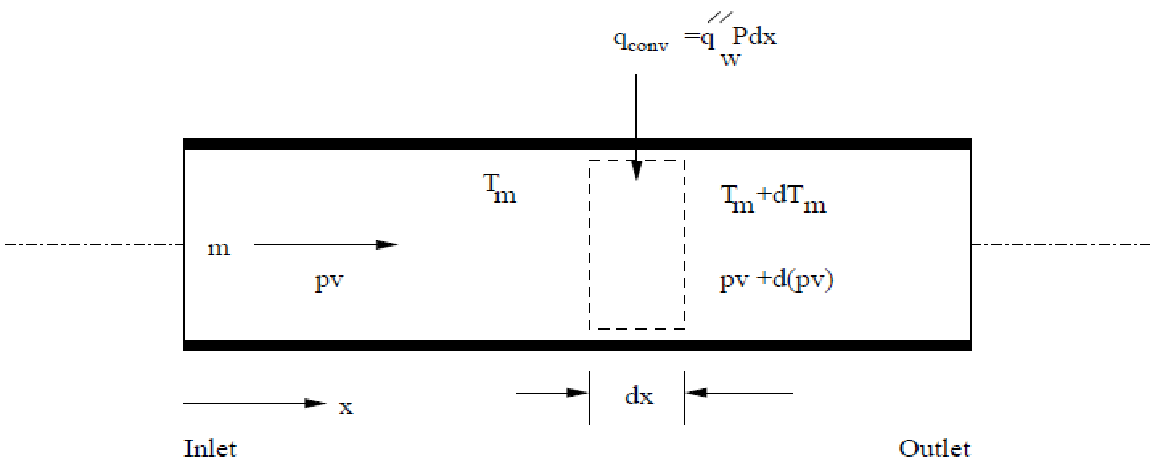


Figure (1.07) : Energy balance in a parallel plate channel flow

Consider an energy balance over a control volume within the channel (disk of thickness dx; perimeter of P around the channel). The change of energy within the control volume balances the rate of heat addition or removal (q_s) from the boundary so that

$$dq + \dot{m}(c_v T_m + pv) - \left[\dot{m}(c_v T_m + pv) + \dot{m}\frac{d(c_v T_m + pv)}{dx}dx \right] = 0$$

or

$$dq_{conv} = \dot{m}d(c_v T_m + pv) \tag{1.23}$$

For compressible flow:

$$dq_{conv} = \dot{m}d(c_v T_m + pv)$$

or,

$$dq_{conv} = \dot{m}d(c_v T_m + RT_m)$$

or,

$$dq_{conv} = \dot{m}d[c_v T_m + (c_p - c_v)T_m]$$

or,

$$dq_{conv} = \dot{m}c_p dT_m \tag{1.24}$$

For incompressible flow:

$$dq_{conv} = \dot{m}c_p dT_m \;\; dq_{conv} = \dot{m}c_p dT_m \text{ [since } d\left(pv\right) = 0 \text{ and } c_p = c_v \text{]}$$

Therefore, whether the flow is compressible or incompressible, we can write

$$q_{conv} = \dot{m}c_p (T_{m,o} - T_{m,i}) \tag{1.25}$$

We can also write

$$dq_{conv} = \dot{m}c_p dT_m$$

or,

$$q''_w p dx = \dot{m}c_p dT_m \text{ [} P \text{ is the surface perimeter } = \pi D \text{]}$$

or,

$$\frac{dT_m}{dx} = \frac{q''_w p}{\dot{m}c_p} = \frac{Ph(T_w - T_m)}{\dot{m}c_p} \tag{1.26}$$

So long $T_w > T_m$, heat is transferred to fluid and T_m increases with x. The manner in which equation (1.26) varies should be noted.

Constant Heat Flux

q''_w is independent of x . Total heat transfer rate $q_{conv}^{total} = q''_w PL$; again, from equation

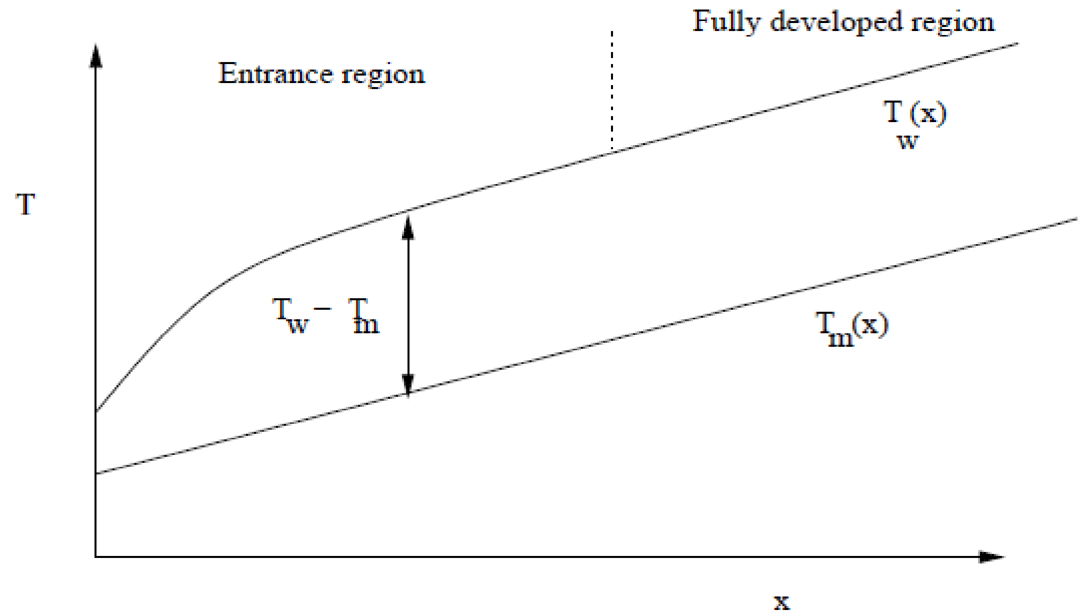


Figure (1.08): variation of bulk – mean temperature in a channel (constant wall heat flux)

(1.23-1.25), $q''_w PL = \dot{m} c_p \left(T_{m,o} - T_{m,i} \right)$. So if q''_w, $\dot{m}$, c_p and the geometry are known, it is possible to calculate the fluid temperature rise $\left(T_{m,o} - T_{m,i} \right)$. Otherwise,

$$\frac{dT_m}{dx} = \frac{q''_w p}{\dot{m} c_p} \text{ or } T_m = \frac{q''_w P}{\dot{m} c_p} x + c_1 \qquad (1.27)$$

Where $\dot{m}$ refers to the mass flow rate. This energy balance suggest that dT_m/dx is constant and so T_m varies linearly throughout the channel. At the inlet section $T_m = T_{m,i}$ and so the following linear profile of mean temperature is obtained:

At $x = 0$, $T_m = T_{m,i}$ therefore $C_1 = T_{m,i}$

$$T_m(x) = T_{m,i} + \frac{q''_w p}{\dot{m} c_p} x \qquad (1.28)$$

From this result and Newton's law of cooling, it can be shown that the difference between the surface (wall) temperature and the mean temperature remain constant, or alternatively, q''_w/h remains constant in the fully developed region.

We can see the variation $T_m\left(x\right)$ with x for constant wall heat flux (Figure 1.08), let us now find out the mathematical relation for variation of $T_m\left(x\right)$ with x for constant wall temperature case.

Constant Surface Temperature:

P may vary with x, but it is constant for pipe. $P/\dot{m}c_p$ is constant. For fully developed flow, h is constant, although it varies with x in the entrance region. Finally T_w is constant. So, T_m will vary with x except for the trivial case $\left(T_w = T_m\right)$ of no heat transfer (Figure 1.10)

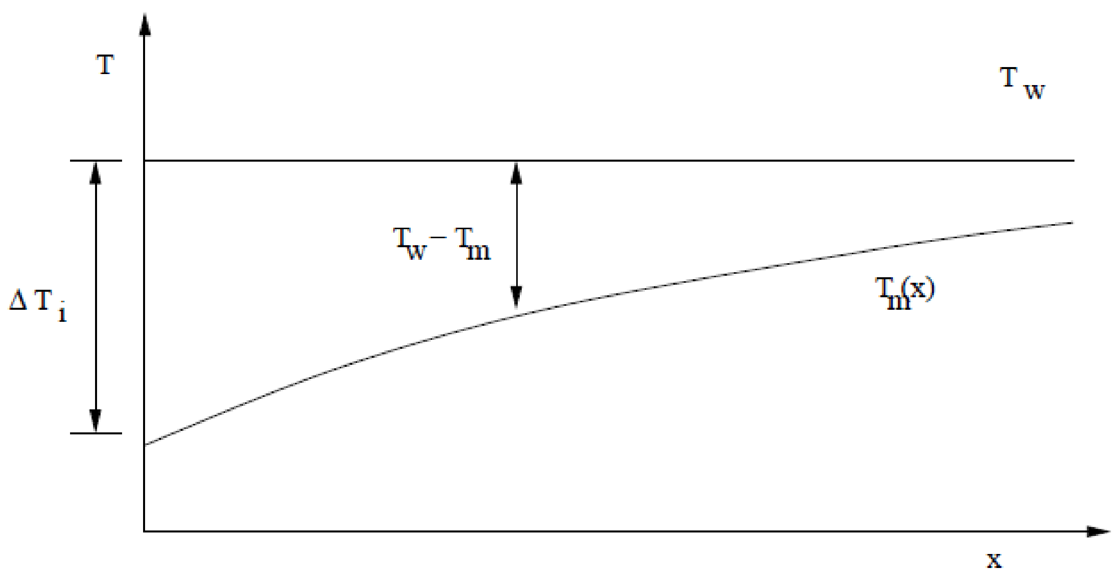


Figure (1.09): variation of bulk – mean temperature in a channel (constant wall temperature)

Mathematical Relation For Variation Of $T_m(x)$ For Constant Wall Temperature

P may vary with x, but it is constant for pipe. $p/\dot{m}c_p$ is constant. For fully developed flow, h is constant, although it varies with x in the entrance region. Finally T_w is constant. So, T_m will vary with x except for the trivial case $(T_w = T_m)$ of no heat transfer.

From equation (1.26),

$$\frac{dT_m}{dx} = \frac{q_w'' p}{\dot{m}c_p} = \frac{Ph(T_w - T_m)}{\dot{m}c_p}, \quad \text{defining}$$

$\Delta T = T_w - T_m =$ difference between surface temperature and mean temperature.

$$\frac{dT_m}{dx} = -\frac{d(\Delta T)}{dx} = \frac{Ph(\Delta T)}{\dot{m}c_p} \tag{1.29}$$

or,

$$\int_0^L \frac{d(\Delta T)}{(\Delta T)} = -\frac{PL}{c_p}\int_0^L hdx$$

or,

$$In\frac{(\Delta T_0)}{(\Delta T_i)} = -\frac{PL}{\dot{m}c_p}.\frac{1}{L}\int_0^L hdx$$

or,

$$In\frac{\Delta T_0}{\Delta T_i} = -\frac{PL}{\dot{m}c_p}\bar{h}_L$$

$$\frac{\Delta T_0}{\Delta T_i} = \frac{T_w - T_{m,o}}{T_w - T_{m,i}} = \exp\left(\frac{-PL}{\dot{m}c_p}\bar{h}_L\right) \tag{1.30}$$

Had we integrated from the channel inlet to some axial position x within the channel, we would have obtained the similar, but more general expression.

$$\frac{T_w - T_m(x)}{T_w - T_{m,i}} = \frac{T_w - T_m(x)}{T_w - T_{m,i}} = \exp\left(-\frac{px}{\dot{m}c_p}\bar{h}\right) \tag{1.31}$$

Where $\bar{h}$ is now the average value of h from the inlet to the distance x. This result suggests that the difference $\left(T_w - T_m\right)$ delays exponentially (also clear from the figure shown earlier). We can also write

$$q_{conv} = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right)$$

or,

$$q_{conv} = \dot{m}c_p\left[\left(T_w - T_{m,i}\right) - \left(T_w - T_{m,0}\right)\right]$$

or

$$q_{conv} = \dot{m}c_p\left(\Delta T_i - \Delta T_0\right) \tag{1.32}$$

On the other hand, we have seen that

$$In\frac{\Delta T_0}{\Delta T_i} = \frac{-PL}{\dot{m}c_p}\bar{h}L \tag{1.33}$$

Substituting for $\dot{m}c_p$ in (1.32) from (1.33) we get

$$q_{conv} = \frac{\bar{h}_L PL(\Delta T_0 - T_{i)}}{In(\Delta T_0 / \Delta T_i)} = \bar{h}_{av} A_m \Delta T_{l,m.} \tag{1.34}$$

This is a form of Newton's law of cooling for the entire channel. $\Delta T_{l,m.}$ is the appropriate average of temperature difference for the entire channel. The log nature of this average temperature difference [in contrast to an arithmetic mean temperature difference of the form $\Delta T_m = \frac{\Delta T_i + \Delta T_o}{2}$] is due to exponential nature of the curve.

The problem of laminar flow between parallel plates geometries has been treated theoretically and the results has been derived to expressed convection coefficients. We wish to analyze heat transfer under hydro-dynamically and thermally fully developed condition, suggesting a quantitative relation between the different performance index parameters of heat transfer, including viscous dissipation, in a comprehensive way. Rigorous study is carried out to investigate the effect of viscous dissipation on the laminar forced convection for fully developed flows between two parallel plates subjected to the constant heat flux boundary conditions. In the analysis a rectangular coordinates system (x, y) is used to model the flows where x and y denote the horizontal and vertical coordinates. Also in the analysis, two definitions of the Brinkman number, i.e. temperature based and heat flux based, have been used along with the intermediate steps, to whet interest amongst the researchers

1.10 Organization of the Thesis

The applicable continuity, momentum and energy equations with appropriate boundary conditions are outlined in chapter III to describe the flow characteristics and heat transfer through the geometries. The different methods used in the heat transfer literature to solve the problems formulated in chapter III are presented in chapter II. Chapter IV discusses the graphical solutions obtained for various geometries. Comparisons and discussion of analytical solutions and thermal boundary conditions are also presented. Conclusions and summary of these solutions are presented in chapter V. Recommendations for future studies is also made.

CHAPTER 2

LITERATURE REVIEW

2.1 Review of Some Previous Work

Parallel to the technological development in macro engineering especially in macro machining and construction, scientific research in the macro scale flow and heat transfer area has increased in the last two decades. Thus, numerous studies have been conducted to understand the fundamentals: such as flow characteristics and heat transfer rates in simple geometries as well as specific effects like rare faction, viscous dissipation, axial heat conduction, thermal entrance, and published in the literature. However, in this chapter, studies on single phase flow and convective heat transfer in macro channels and micro channels are reviewed. In particular, studies dealing with vicious dissipation effects are taken into consideration.

Effect of viscous plays an important role in the study of heat transfer characteristics and this effect is frequently encountered in many applications such as viscous fluid flow, high speed flows and fluid flow through micro scale channels. For Newtonian fluids, there have been abundant studies investigating, either theoretically or experimentally, the viscous dissipation affect on the forced convection in ducts and channels have been investigated by a number of researchers Shah and London (1978), Irvine and Karni (1987), Nield and Bejan (2006), Aydin and Avci (2006), Kays et al.(2005), Bird et al. (1977). As pointed out in several recent investigations by Nield and Kuznetsov (2003), Haji-Sheikh et al., (2005), Hung and Tso (2008), Hung and Tso (2009), Chen and Tso (2011), the importance of the effects of viscous dissipation in the analysis of forced convection in porous media is also in disputable. According to Bird et al., (1977), this effect is comparatively well known in polymeric fluids which are non Newtonian in nature. Understanding the viscous dissipation effect in the flow of polymeric fluid is of paramount importance in connection with plastic material processing, application of paints and lubricants, processing of food stuff, as well as movement of biological fluids. As the notion of polymeric fluids cannot be described by the Navier-Strokes equations, the study of such fluids is becoming a big challenge for the past few decades. Many important practical problems of engineering interest involve steady state shear flows and to obtain better physical insights into the elucidation of some of the interesting phenomena, relatively simple generalized Newtonian models such as power law fluids and Bingham plastics have been employed to obtain approximate solutions to the heat and fluid flow problems. For unsteady flow phenomena Bird et al., (1977), the linear visco elastic fluid model is used to describe the elastic response of the polymeric fluids while the retarded motion expansion is valid for slowly varying flows.

Many important industrial problems involves non-Newtonian flows and numerous studies of viscous dissipation effect on convection heat transfer in ducts and channels can be found in the literature. Different rheological models have been employed for solving problems in different flow regimes, such as the generalized Newtonian fluid models, the linear visco elastic fluid model and the retarded motion expansion model Bird et al., (1977). Previous studies of viscous dissipation effect of forced convection involving non-Newtonian flows primarily focused on the generalized Newtonian fluids such as power law fluid Cheng (1976), Etemand et al.(1994), Barletta (1997), Jambal et al., (2006), Valko (2005), Chuba et al., (2008), Tso et al., (2010), KolitaWong et al., (2011). Bingham fluids Min et al., (1997) and Herschel Bulkley fluids Quaraesma and Marcedo (1998). The effect of viscous dissipation has been investigated for third grade fluids, Massondi and Christie (1995) as well as the visco elastic fluids obeying the oldroyd –B model Olagunju (2003), the Phan-Thien-Tanner model Pinho and Oliveira (2000), Coelho et al., (2003), and the FENE-P model Olivera et al., (2004). In the ease of fluid flow past an unsteady stretching sheet, the viscous dissipation effect has been considered for power-law analysis, the effect of viscous dissipation imposes pronounced influence on the entropy generation rate which is strongly related to the flow and temperature fields of the non-Newtonian fluids Saceldi and Aiboud (2007), Hung (2008).

The Couette-Poiseuille non-Newtonian flow describes a macro-molecular fluid confined between two horizontal plates with either one moving in the longitudinal direction with a constant speed. The motion of the moving plate induces complexity on the solution method of the momentum equation due to the rheological properties of non-Newtonian fluids. Theoretical studies with different hydrodynamic and thermal conditions have been conducted.

For Newtonian fluids, analytical solutions for heat transfer with effect of viscous dissipations were derived for the Couette-Poiseuille flow between parallel plates which are kept at constant heat flux and one of the plates is insulated Aydin and Avci, (2006). Various problems of non-Newtonian Couette-Poiseuille flow such as investigations of elastic effect Cruz and Pinho,

(2004) and slip effect Chen and Zhu, (2008) on the hydrodynamic behavior as well as stability problem with pressure dependent viscosity Sulslov and Tran, (2008), Tran and Suslov, (2009), have been conducted. On the other hand, limited studies on Couette-Poiseuille non-Newtonian flow taking into account the viscous dissipation effect are available. The plane Couette-Poiseuille flow of power law fluids with viscous dissipation was exactly solved for the case of constant heat flux at one wall with the other insulated Davan et al., (2000). By considering Couette-Poiseuille flow, with the stationary plate subjected to constant heat flux and the other plate moving with constant velocity, but insulated, analytical solutions with effect of viscous dissipation were obtained for visco elastic fluids Hashemabadi et al., (2004). Numerical study was carried out investigating viscous dissipation effect for Poiseuille Couette non-Newtonian fluid flow with the boundary conditions of constant heat flux at one wall and the other plate insulated Davaa et al., (2004). Heat transfer on Couette-Poiseuille flow for visco elastic non-Newtonian fluid with simplified Phan-Thien-Tanner constitutional equation was analysed, where the stationary plate was maintained at constant temperature and the moving plate is insulated Hashemabadi et al., (2003).

Different thermal boundary conditions need to be considered in the design of thermal equipment. One of the earliest theoretical works has dealt with the forced convective heat transfer in the entry length through an asymmetrically heated parallel channel Halton and Turton, (1962). Following a numerical analysis, the influence of viscous dissipation for the flow of a Newtonian fluid in a parallel plate channel has been investigated in detail Etemand et al., (1994). In another study, the combined forced and free convective heat transfer, including the effect of viscous dissipation, in the fully developed region of a parallel plate, vertical channel, has been analyzed Barletta, (1998). The study has considered constant wall temperatures, and reviewed the interest effects of squeezing and extrusions parameters on the velocity, the temperature profile and the heat transfer characteristics Lahjomri et al., (2002) have used a functional analysis method to investigate the influence of viscous dissipation on the heat transfer of a thermally developing laminar Hartman flow though a parallel plate channel with the aid of a magnetic field. In a study of thermal development of forced convection in a parallel plate channel filled with porous medium, Nield et al., (2003) have presented the effects of viscous dissipation with the thermal boundary condition of uniform wall temperature, including axial conduction effects. The effect of viscous heating on the stability of Taylor-Couette flow has been investigated experimentally

White and Muller, (2000). The study has considered Newtonian and visco elastic fluid for experimental exploration of the critical Reynolds number from the onset of the transition to a new state.

Many authors have explored the effects of viscous dissipation in a forced convective heat transfer phenomenon in a parallel plate channels and annular ducts, which significantly alter the thermo-fluidic transport and hence, cannot be neglected even for the flow of an ordinary fluid through macro channel or pipes. The analysis of laminar forced convection in a pipe for Newtonian fluid of constant properties has been made by Aydin, (2005) considering the effects of viscous dissipation. In part-1, both hydro dynamically and thermally fully developed convection have been studied; part 2 of the study considers the hydro dynamically developed but thermally developing case. In both cases, two different types of thermal boundary conditions, e.g. constant heat flux (CHE) and constant wall temperature (CWT) , have been considered. The variations of the dimensionless radial temperature and Nusselt number have been observed for different values of the Brinkman number. The influence of viscous dissipation on heat transfer has been found to be strong for higher values of Brinkman number $(Br > 1)$, while the influence has been negligible for lower values of Brinkman number. In the thermally developing case, comparing the temperature distribution with that of the same obtained by neglecting the viscous dissipation, it has been observed that the temperature distribution increases in the axial duration which is attributed to the effects of viscous dissipation. In their study, an important role of the Brinkman number (For the CWT case) and the modified Brinkman number (for the CHE case) has been investigated on the development of the Nusselt number. The analytical work done by Aydin and Avci, (2006) has dealt with a convective heat transfer problem for a plane Poiseuille flow and an emphasis on the viscous dissipation effect. The energy equation has been solved for thermally developed and developing cases separately with the boundary condition of CWT and CHF, respectively. In both cases, the flow has been considered to be hydro-dynamically developed. It has been found from the study that with increasing intensity of viscous dissipation (i.e., increase in the Brinkman number), the heat transfer decreases up to a critical value, and that is attributed to the internal heat generation due to the viscous dissipation effect. In another work Aydin and Avci, (2006) have studied the laminar forced convective heat transfer problem in a Couette-Poiseuille flow with an emphasis on the viscous dissipation effect. The effect of viscous dissipation on the forced convective heat transfer in fully developed flows of Newtonian fluid through circular ducts has been analyzed in an analytical framework Avci and Aydin, (2006). The study has considered two different cases of thermal boundary conditions and has shown the influence of viscous dissipation on the temperature distribution and Nusselt number.

Several approaches have been presented in the literature to obtain various analytical expression of the Nusselt number as a function of the Brinkman number. The steady state laminar heat transfer to a plane Poiseuille-Couette flow of a Newtonian fluid with simultaneous pressure gradient and axial movement of one of the plates has been investigated by several authors Hudson and Bankoff, (1965), Sestak and Reigar (1969), Bruin, (1972). In all these cases, the energy equation containing viscous dissipation term has been solved numerically to obtain the effects of viscous dissipation on the heat transfer characteristics for the two alternate causes of thermal boundary condition, e.g. temperature specified at both the plate or a specified temperature at the stationery plate with the moving plate insulated. All works reported above have discussed about the effect of viscous dissipation on the hydro-thermal characteristics in macro-scale flows. However, in all cases, it is important to observe the point of singularities on the variation of the Nusselt number with the Brinkman number or the modified Brinkman number. The appearance of this point singularities has been highlighted considering energy balance between wall surface heat and the heat generation due to viscous dissipation during the thermo fluidic transport.

Following a numerical analysis Lin, (1977) has studied the laminar heat transfer in a non-Newtonian Couette flow with pressure gradient, using the power-law model Davara et al., (2006) have performed a numerical study to investigate the influence of viscous dissipation on fully developed laminar heat transfer in a non-Newtonian fluid flowing between two parallel plate while one of the plates moving axially. An investigation has been made using an analytical method to probe the effect of the viscous dissipation on forced convective heat transfer in a power-law fluid within fixed parallel plates Kolitawong et al., (2011). Considering the constant heat flux boundary condition in the analysis, the study has shown that the heat transfer characteristics depend on the power-law index, and the Brinkman number, The study has also revealed that pseudo-plastic and dilatants fluids exhibit different heat transfer characteristics in a viscous dissipative environment. The effect of viscous dissipation on a fully developed forced convection Couette flow through parallel plate channel, partially filled with porous medium, has also been analyzed Ghazian et al., (2011). The study has shown that the dimensionless temperature and consequently, the Nusselt number decreases with increasing Brinkman number. Lately, through a numerical analysis, Ramiar and Ranjbar, (2011) have investigated the effect of viscous dissipation on the forced convection heat transfer of Al_2O_3 - water nano fluid in micro-channel. The study has explored that the viscous dissipation decreases the Nusselt number. In another work published, an effort has been for analyzing the effects of viscous dissipation on the limiting Nusselt number for a hydro-dynamically fully developed laminar shear-driven flow through an asymmetrically heated annulus of two infinitely long concentric cylinders Mondal and Mukherjee, (2012). When a thin slab was symmetrically heated on both sides, the hyperbolic heat conduction equation was solved analytically Lewandoewska and Malinowski, (2008). Considering the effects of viscous dissipation and pressure stress work of the fluid, the steady laminar boundary layer flow along a vertical stationary isothermal plate was studied. The variation of wall heated transfer and wall shear-stress along the plate was discussed Pantokratoras, (2003). The Bigham fluid was assumed to be flowing between two porous parallel plates. With the slip effect at the porous walls, the analytical solutions were obtained for the Couette-Poiseuille flow Suslov and Tran, (2008). Numerical evaluation for developing temperature profiles by a finite difference method were carried out for non-Newtonian fluid through parallel plates and circular ducts. The effect of viscous dissipation and axial conduction were taken into account. Graphical representation of Nusselt numbers were noted for various parameters Jambal et al., (2005). A numerical analysis was carried out taking viscous dissipation into account for pseudo-plastic non-Newtonian fluids aligned with a semi-infinite plate Li et al., (2001). A common geometry in heat exchangers is the concentric annular or duct which has been the topic of countless heat transfer and fluid mechanics research, as summarized by Sah and London, (1978) and Kays et al., (2005) amongst others. The work of Brinkman, (1951) appears to be the first theoretical work dealing with viscous dissipation. The temperature distribution in the entrance region of a circular pipe at the wall of which was maintained at either a constant temperature as that of the entering fluid or constant heat flux was examined. The highest temperature were, not surprisingly discovered to be localized in the wall region. Tyagi, (1996) performed a wide study on the effect of viscous dissipation on the fully developed Laminar forced convection in cylindrical tubes with an arbitrary convection and uniform wall temperature. Ou and Cheng, (1973) employed the separation of variable method to study the Graetz problem with viscous dissipation. They obtained the solution in the form of eigen values and eigen functions of which satisfy the Sturm-Liouville system. The solution techniques follow the same approach as that applicable to the classical Graetz problems and therefore surfers from that the effect of viscous dissipation was very relevant in the fully developed region if convective boundary conditions were considered. With these boundary conditions and if viscous dissipation was taken into account, the fully developed value the of Nusselt number was $48/5$ for every value of the Biot number and of the other parameters. On the other hand, it is well known that, if a forced convection model with no viscous dissipation is employed , the fully developed value of the Nusselt number for convective boundary conditions depends on the value of the Biot number. Basu and Roy, (1985) analyzed the Graetz problem by taking account of viscous dissipation but neglecting the effect of axial conduction. They showed that the effect of viscous dissipation could not be neglected when the wall temperature was uniform. For the thermal condition, Liou and Wang, (1990) using the uniform wall heat flux, Bernardic et al., (1995) using the convection with an external isothermal fluid, Lawal and Mujumdar, (1992) and Dang, (1983) using the uniform wall temperature studied the effect of viscous dissipation in the thermal entrance region in a pipe. The effect of viscous dissipation in the thermal entrance region of slug flow forced convection in a circular duct was studied by Barletta and Zanchini, (1997). The temperature field and the local Nusselt number were determined analytically for any prescribed axial distribution of wall heat flux including uniform, linearly varying and exponentially varying heat fluxes. Zanchini, (1997) analyzed the asymptotic behavior of Laminar forced convection in a circular tube, for a Newtonian fluid at constant properties by taking into account the viscous dissipation effects. It was disclosed that particularly for the boundary condition of uniform wall temperature and of heat transfer by convection to an external fluid yielded the same asymptotic behavior of the Nusselt number, namely, $Nu_\infty = 48/5$. And therefore, he obviously stated that for these boundary conditions when the wall but flux $q_w(x)$ tend to zero, i.e., the local Brinkman $Br(x)$ tend to infinity, it is completely wrong to neglect the effect of viscous dissipation on the asymptotic behavior of the forced convection problem. Barletta, (1997) studied the asymptotic behavior of the temperature field for the laminar and hydro dynamically developed forced convection of a power-law fluid which flows in a circular duct taking the viscous dissipation into account. The asymptotic Nusselt number and the asymptotic temperature distribution were evaluated analytically in the cases of either the uniform wall temperature or convection with an external isothermal fluid. Barletta and Rossidischin, (1997) investigated the laminar duct in the case of sinusoidal axial variation of the wall heat flux. Morini and Spigal, (1977) analytically determined the steady temperature distribution and the Nusselt numbers for a Newtonian incompressible fluid in a rectangular duct, in fully developed laminar flow with viscous dissipation, for any combination of heated and adiabatic sides of the duct. Pinho and Oliveira (2000), investigated the forced convection of Phan-Thien-Tanner fluid to laminar pipe and channel flows including the effects of viscous dissipation. It was shown that the beneficial effects of fluid elasticity were enhanced by viscous dissipation. The effect of viscous dissipation on the laminar forced convection in a circular duct for a Bingham fluid under different thermal boundary conditions was studied by Vradis et al., (1993), Min et al.(1997) and Khahyr et al.(2003), The asymptotic temperature profile and the asymptotic Nusslet number were determined form axial distributions of wall lheat flux which yielded a thermally developed region. Payvar (1978) investigated the effects of viscous dissipation on power-law fluid, Bingham plastic fluid, and Ellis fluid for a thermally fully developed forced convection with constant wall heat flux.

In another study by Davaa et al.(2004), the modified power-law model defined by Irvine and Karni, (1987) in the governing momentum and energy equations was used to improve the accuracy of the velocity field in the region of lower shear rates. The governing equations were solved numerically for fully developed flows subjected to constant heat flux. Hashemabadi et al (2004) included viscous dissipation effect in a Couette-Poiseuille flow between parallel plates for visco elastic flow by adopting the simplified Phan-Thein-Tanner model. Analytical solutions were obtained by Pinho and Coelho (2006) for heat transfer in concentric annular flows of visco elastic fluids modeled by the simplified Phan-Thien-Tanner constitutive equation. Solutions for Thermal and dynamic fully developed flow are presented for both imposed constant wall heat fluxes and imposed constant wall temperature always taking into account viscous dissipation. Khatryr et al., (2010) give analytical solution in a circular ducts for a Herschel – Bulkley fluid in a horizontal duct heated uniformly, and with variable axial distribution of wall heat flux for which polynomial and logarithmic functions were considered as examples. Heat transfer with the effect of viscous dissipation for steady, laminar, both hydrodynamiclly and thermally fully developed pseudo-plastic fluid through a channel of Couette-Poiseuille flow, where both plates are kept at specified but different constant heat flux ratios being considered as thermal boundary conditions was studied by Sheela et al.(2012),Analytical study was done by Rashidi and Erfami

(2002) for the thermal-diffusion (Soret effect), and diffusion transfer of a steady MHD (magneto hydrodynamic) convective and slip flow due to a rotating disk with viscous dissipation and ohmic heating. They presented the influence of the slip parameter and the magnetic field parameter and of Eckert, Dufor and Soret numbers on the profiles of the dimensionless velocity, temperature and concentration distribution, Rabha et al.(2016) studied the laminar and hydro-dynamically developed forced convection of a Herschel-Bulkey fluid flowing in a circular tube with a prescribed axial distribution of wall heat flux. The effect of viscous dissipation has been taken into account, while the axial heat conduction in the fluid has been considered s negligible. In their investigation comparison, between theoretical results and those published in the literature for the Newtonian fluid case and the non-Newtonian fluid case (power-law fluid) shows very close agreement. Hisham and Tamimi (2013) performed analytical study on the forced convection heat transfer problem with viscous dissipation between two plates subjected to constant heat flux, which is a type of the Graetz problem. Complete analytical solutions for the fluid temperature and local Nusselt number (Nu) have been derived. The effects of the Brinkman number (Br) and rheological properties (power-law index) on the distribution of the local Nusselt number have been shown through numerical calculations. Mohan and Mandapati (2016) examined the effect of viscous dissipation and axial conduction on heat transfer for laminar forced conduction flow through a circular pipe with constant wall temperature boundary condition imposed on the circular pipe. Caldas and Coelho (2017) obtained analytical expressions for the Nusselt number in a laminar flow of a power.-law fluid between parallel plates duct. The results are valid for a fully developed flow with constant and different heat fluxes at the walls in the presence of viscous dissipation. In these analytical solutions the “mathematical” and “classical approaches were used, i.e. both Nusselt numbers are as usual, based in the same bulk temperature calculated for the entire duct cross section. Kundu (2015) studied the influence of viscous dissipation on the limiting temperature profile for an unsteady shear-driven and Poiseuille flow of Newtonian fluid between two asymmetrically heated parallel plates. Semi- analytical formalism was presented in an exhaustive way to solve the governing transport equation of energy for the case of unequal but constant temperature thermal boundary conditions. Hammoudar et al.(2018) have performed a numerical study of viscous dissipation effects on incompressible forced convection flow of nano-fluids over a two dimensional horizontal backward facing step(BFS) placed in a porous channel .In the flow modeling the

Brinkman-Forchheimer extended Darcy model(DBLF) is incorporated in the momentum equation and local thermal equilibrium(LTE) is assumed for study. Chandhuri and Das(2018) have examined the hydro dynamically and thermally fully developed flow of a Sisko fluid through a cylindrical tube considering the effect of viscous dissipation. The effect of the convective term in the energy equation has been taken into account which can significantly affect the temperature distribution if the radius of the tube is relatively large. The equations governing the flow and heat transfer are solved by the least square method(LSM) for both heating and cooling of the fluid. Bela et al.(2014) using the direct simulation monte carlo (DSMC) method, convection heat transfer of argon gas flow through a micro/nano channel with uniform heat flux wall boundary condition was investigated. The study considered the influence of rarefaction and viscous dissipation on the Nusselt number. They used the GHS collision model to calculate thermal conductivity and viscosity as a function of temperature accurately. Liu et al (2013) used the separation of variables to obtain the solution of the energy equation of circular micro channels which considers axial heat conduction, velocity slip, temperature jump, viscous dissipation and thermal entrance effect. The effect of Brinkman number, Peclet number, temperature jump and velocity slip on local Nusselt number are analyzed by numerical simulation using Mat lab. Chan et al.(2015) derived an analytical temperature profile for a thermally and hydro dynamically fully developed, laminar Couette-Poiseuille flow, for a power-law fluid subjected to asymmetric heating at the boundaries. Interest has been focused on the effect of viscous dissipation by introducing a modified Brinkman number. Heat transfer in the parallel plates of a Couette-Poiseuille flow is affected by the moving plate velocity and viscous dissipation, which in turn depend on the power- law index .The Nusselt number has a higher value when $u_p^* > 0$, with increasing Brinkman number, unlike $u_p^* = 0$ which shows an opposite trend, Shiming (2015) investigated the convection heat transfer of power-law fluid in the flat plate channel filled with porous media based on Darcy-Brinkman-Forchheimer flow model. It was observed that with increasing Br or more intense viscous dissipation, the dimensionless temperature of power-law fluid reduced sharply, resulting in the reduction of the convection heat transfer rate. The results suggests that viscous dissipation greatly affect the temperature distribution and should not be neglected. The temperature distribution of the fluid was also subjected to the influences of dimensionless parameters, such as Darcy number, integrated inertial parameter, and power-law index. Umavathi et al,(2010) presented an analytical study of

Couette-Poiseuille flow in an inclined channel in a composite porous medium. The flow is modeled using the Darcy-Lapwood-Brinkman equation. The viscous and Darcy dissipation terms are included in the energy equation. The coupled non-linear equations are solved analytically using the regular perturbation method ans numerically using the finite difference method. The effect of various parameters such as the porous parameter, inclination angle, Grashof number, ratios of height, viscosities and thermal conductivities were discussed. Botong et al.,(2011) complete a numerical analysis for hear transfer in pseudo-plastic non Newtonian fluids aligned with a semi-infinite plate by extending the study in two directions. On one hand, viscous dissipation are included in the energy equation, and on the other hand, unlike most classical works, the effects of power-law viscosity on temperature field are taken into account by assuming that the thermal diffusivity varies as a function of velocity gradient.The solution presented numerically by using the similarity transformation and shooting technique. Mohammed Abd El-Aziz(2010) performed an analysis to study the effect of variable viscosity and variable thermal conductivity on the flow and heat transfer of a thin visco elastic liquid(obeying Walters liquid B model) film on a horizontal unsteady stretching sheet taking into account the effect of viscous dissipation. The fluid viscosity is assumed to decrease exponentially with temperature but the thermal conductivity is assumed to vary as a linear function of temperature .Numerical solutions are obtained for some representative values of the viscosity and thermal conductivity variation parameters, unsteadiness parameter, and Eckert number. Rajesh and Pankaj(2012) analyzed the numerical simulation of laminar flow in a parallel plate channel for different entrance velocity using the finite element method through solving partial differential equations of the fluid flow. The fluid flow is expressed by partial differential (Poisson’s equation). While heat transfer is analyzed using the energy equation .The Navier-Stokes equations along with the energy equation have been solved by using simple technique that corresponds to a total number of nodes 2842.The channel has a constant heat flux at the two walls and the three dimensional numerical simulations. Numerical solutions were obtained using commercial software Ansys Fluent. The working fluid was air(Pr = 0.7). The local Nusselt numbers are obtained, which can be used in estimation of flow and heat transfer performance in a channel.

Most of the above mentioned studies have concentrated on either Poiseuille flow or combined Couette-Poiseuille flow and circular ducts. Laminar forced convection for a steady shear-driven flow between parallel plate channels, suggesting a quantitative relation between the different performance index parameters of heat transfer, including viscous dissipation in a comprehensive and exhaustive way, have not been reported in the literature. Hence the study follows suite.

CHAPTER 3

RESEARCH METHODOLOGY

Mathematical modeling design was employed in this study. The study made use of the governing equations for dynamic and thermally fully developed steady, laminar flow of Newtonian fluids between two parallel plates with temperature-independent properties and internal viscous heating of the fluids which are continuity equation, momentum equation and the energy equation. The fluid dynamic problem was decoupled from the thermal problem. The momentum equation was first solved for velocity under the no-slip condition. The resulting velocity profile was then substituted into the energy equation. Analytical heat transfer solutions were obtained under imposed constant wall heat fluxes as well as under imposed constant wall temperatures using variable separable method.

3.1 Laminar Heat Transfer Between Parallel Plates For Hydro dynamically and Thermally Developed Couette Flow With Constant Heat Flux Boundary Conditions

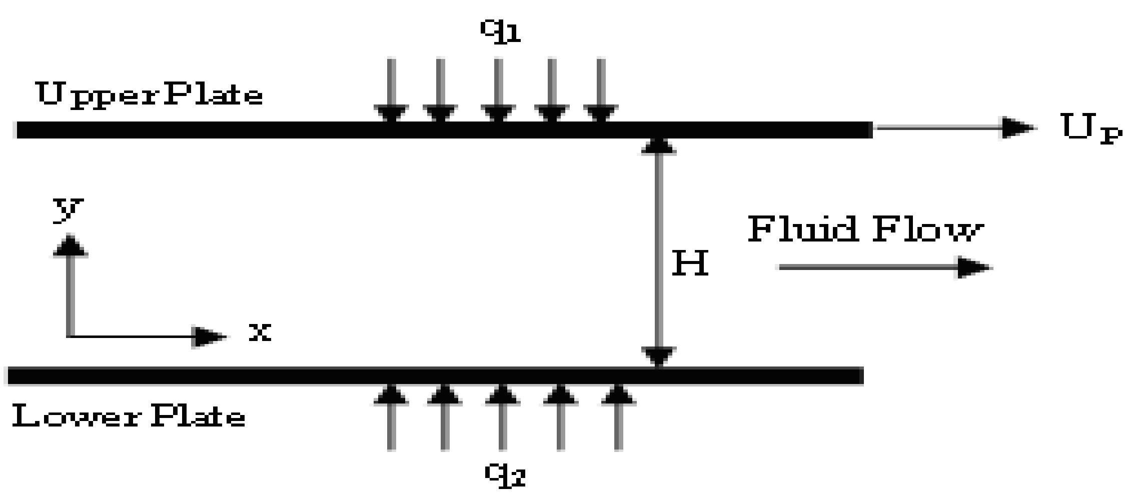


Fig (3.00) Schematic diagram describing the problem.

3.1.1 PHYSICAL CONSIDERATIONS

Let us consider the steady laminar flow of a viscous incompressible fluid between two parallel plates at a distance H, the upper plate moving with a uniform constant velocity U_p. Let u,v be the components of velocity in the direction of x,y; μ and ρ being the coefficient of dynamic viscosity and the density of the fluid respectively.

If we take x in the direction of the flow and y along the direction perpendicular to the stationary plate, then for a two-dimensional motion we have

$$v = 0 \quad \text{and} \qquad u = \frac{U}{H}y \tag{3.000}$$

The flow field is identical in all planes (z =const.). The property common in this flow, is that the only non vanishing velocity component (in this case u) only varies perpendicular to the flow direction, is a consequence of the continuity equation.

$$\nabla \cdot u = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.001}$$

From this, because $v = 0$, we obtain

$$\frac{\partial u}{\partial x} = 0 \qquad \text{or} \qquad u = f(y) \tag{3.002}$$

of which (3.000) is a special case. The x-component of the Navier-Stokes equations reads

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{3.003}$$

Because of (3.000), all the convective (nonlinear) terms on the left-hand side vanish.

Since in this special case of Couette flow u is a linear function of y, all the terms in the brackets on the right-hand side of (3.003) vanish and we are led to the equation.

$$\frac{\partial p}{\partial x} = 0 \quad \text{or} \quad p = f(y) \tag{3.004}$$

The component of the Navier-Stokes equations in the y-direction

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{3.005}$$

directly leads us to

$$\frac{\partial p}{\partial y} = 0 \tag{3.006}$$

which together with (3.004) furnishes the final result.

$$p = const \tag{3.007}$$

Because Newtonian fluids adhere to the walls, for an impermeable wall this means that both the tangential and the normal velocities of the fluid and of the wall must correspond at every point on the wall. The velocity vector $\vec{u}$ of the fluid at the wall must be equal to the vector of the wall velocity $\vec{u}_w$.

$$\vec{u} = \vec{u}_w \text{ (at the wall)} \tag{3.008}$$

The boundary condition when the wall is at rest $(\vec{u}_w = 0)$ is

$$\vec{u} = 0 \text{ at the wall.}$$

Therefore, this gives the most simple nontrivial exact solution of the Navier-Stokes equations.

By equation (3.002); we consider the velocity field

$$u = f(y),\ v = 0 \tag{3.009}$$

The x-component of the Navier-Stokes equation then reduces to

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2} \tag{3.010}$$

and the y-component read

$$0 = -\frac{1}{\rho}\frac{\partial \rho}{\partial y} \tag{3.011}$$

A consequence of the last equation is that p can only be a function of x. However since by assumption the right-hand side of (3.010) is not a function of x, neither is the left-hand side, i.e. $\partial p/\partial x$ is not a function of x. Therefore $\partial p/\partial x$ is a constant which we shall call $-k$. From (3.010) we then extract a differential equation of the second order for the desired function $u(y)$.

$$\mu \frac{\partial^2 u}{\partial y^2} = -k \tag{3.012}$$

Integrating (3.012) twice leads to the general solution

$$u(y) = -\frac{k}{2\mu} y^2 + c_1 y + c_2 \tag{3.013}$$

We specialize the general solution to flow through a plane channel whose upper wall moves with velocity U in the positive x-direction. The function we are looking for, $u(y)$, must by (3.008), satisfy the two boundary conditions

$$u(0) = 0 \tag{3.014a}$$

and

$$u(H) = U \tag{3.014b}$$

so that we determine the constants of integration as

$$C_1 = \frac{U}{H} + \frac{k}{2\mu}H, \quad C_2 = 0 \tag{3.015}$$

Thus the solution of the boundary value problem is

$$\frac{u(y)}{U} = \frac{y}{H} + \frac{kH^2}{2\mu U}\left[1 - \frac{y}{H}\right]\frac{y}{H} \tag{3.016}$$

For $k = 0$ we get the simple shearing flow again, for $U = 0$ and $k \neq 0$ we obtain a parabolic velocity distribution (two-dimensional Poiseuille flow); the general case $(U \neq 0, k \neq 0)$ yields the Couette-Poiseuille flow (eg)

3.1.2 MATHEMATICAL FORMULATIONS

The equations governing fluid flow and heat transfer between two parallel plates with constant heat fluxes are the continuity, momentum and energy equations. To obtain the velocity and temperature distributions between two plates, the governing equations have been derived based on the assumptions.

Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.017}$$

From the assumptions, there is no velocity in the y-direction, i.e., $v = 0$, which gives

$$\frac{\partial u}{\partial x}=0 \tag{3.018}$$

Eq (3.018) implies that the velocity in the x-direction is a function of y only.

X - momentum Equation:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{\partial u}{\partial t}\right)-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{3.019}$$

where p is the pressure. Using continuity equation, one can write the x-momentum equation as follows:

$$\mu\left(\frac{\partial^2 u}{\partial y^2}\right)=\frac{\partial p}{\partial x}+\rho\frac{\partial u}{\partial t} \tag{3.020}$$

From y-momentum equation using the above assumptions, it can be shown that

$$\frac{\partial p}{\partial y}=0 \tag{3.021}$$

3.1.3 STEADY ANALYSIS FOR THE MOVABLE UPPER PLATE WITH A UNIFORM VELOCITY

The fluid flow is assumed to be due to dragging of the upper plate only. Therefore, Eq (3.020) reduces to

$$\mu\left(\frac{d^2 u}{dy^2}\right)=C \tag{3.022}$$

Where, C is a constant and this is equal to zero for shear driven flow. No slip condition is assumed at the plates, and thus the boundary conditions are as follows;

at $y=0,\ u=0$ (3.023a)

and

$$y=H,\ u=U_p \tag{3.023b}$$

Solving Eq (3.022) with above boundary conditions, the velocity profile is obtained as:

$$u = \frac{U_p}{H} y \tag{3.024}$$

However, defining the non-dimensional quantities

$$U = \frac{u}{U_p} \text{ and } Y = \frac{y}{H} \tag{3.025}$$

the above velocity profile reduces to

$$U = Y \tag{3.026}$$

Energy Equation:

$$\rho C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \frac{\partial T}{\partial t} \right) = K \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu \phi \tag{3.027}$$

where ϕ is the viscous dissipation term that contains only $\left(\frac{\partial u}{\partial y} \right)^2$ from (1.7). Based on the above assumptions, the energy equation reduces to

$$\rho C_p \left(u \frac{\partial T}{\partial x} + \frac{\partial T}{\partial t} \right) = K \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 \tag{3.028}$$

The energy Eq (3.028) is written under steady condition as follows:

$$\rho C_p u \frac{\partial T}{\partial x} = K \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 \tag{3.029}$$

Now, to analyze the energy equation with uniformly heated boundary walls, the following case of unequal constant heat fluxes, equal constant heat fluxes and insulated lower plate are considered while neglecting the longitudinal conduction term in the energy equation.

3.1.4 BOTH PLATES AT DIFFERENT CONSTANT HEAT FLUXES

Consider the case where the upper plate is at constant heat flux q_1 and lower plate is at constant heat flux q_2 as shown in Fig ( ), the non- dimensional quantities used as

$$Y = \frac{y}{H},\ U = \frac{u}{U_p},\ \text{and defining the dimensionless temperature } \theta = \frac{T - T_1}{\frac{q_1 H}{k}} \tag{3.030}$$

Again the term $\left(\frac{\partial u}{\partial y}\right)^2$ is equal to $\left(\frac{U_p}{H}\right)^2$. Thus the governing equation for different constant heat fluxes reduces to

$$\frac{q_1}{H}\frac{d^2\theta}{dY^2} + \frac{\mu U_p^2}{H^2} = \frac{\rho C_p U_p y}{H}\frac{\partial T}{\partial x} \tag{3.031}$$

In accordance to the assumption of a thermally fully developed flow with uniformly heated boundary walls, the longitudinal conduction term is neglected in the energy equation. Following this, the temperature gradient along the axial direction is independent of the transverse direction. From Eq. (3.031)

$\frac{\partial T}{\partial x} = \frac{dT_1}{dx}$ and $Br_m = \frac{\mu U_p^2}{q_1 H}$ is the modified Brinkman number based on the upper plate heat flux q_1.

By defining $\left(\frac{\rho C_p U_p H}{q_1}\right)\frac{dT_1}{dx} = a_1$,

Eq. (3.031) can finally be written as

$$\frac{d^2\theta}{dY^2} = a_1 Y - Br_m \tag{3.032}$$

Eq. (3.032) is subjected to the following the thermal boundary conditions

$$Y=1,\ \begin{cases} \theta = 0 \text{ or } T = T_1 \text{ at } y=1 \\ \dfrac{d\theta}{dy} = 1 \text{ or } k\dfrac{\partial T}{\partial y} = q_1 \text{ at } y=1 \end{cases} \tag{3.033a}$$

To evaluate a_1 in equation (3.032) a third boundary condition is required

$$Y=0 \quad \frac{d\theta}{dy} = -\frac{q_2}{q_1} \text{ or -}k\frac{\partial T}{\partial y} = q_2 \text{ at } y=0 \tag{3.033b}$$

The dimensionless temperature profile is obtained by solving Eq. (3.032) with the above boundary conditions [Eqs. (3.0033a,b)]. Therefore, solving Eq. (3.032) with the above boundary conditions of plates at different constant heat fluxes, the dimensionless temperature profile in convection case is obtained as:

$$\theta(Y) = \left(1+\frac{q_2}{q_1}\right)\frac{Y^3}{3} + Br_m\left(\frac{Y^3}{3} - \frac{Y^2}{2} + \frac{1}{6}\right) + \frac{q_2}{q_1}\left(\frac{2}{3} - Y\right) - \frac{1}{3} \tag{3.034}$$

Where q_1 is the upper plate heat flux and q_2 is a lower plate heat flux.

However, to obtain the expression of Nusselt number, it is usual to define the mean temperature, T_m rather than the centreline temperature in a case of fully developed flow. The mean temperature is given by

$$T_m = \frac{\int_{y=0}^{y=1} uT dy}{\int_{y=0}^{y=1} u dy} \tag{3.035}$$

Heat transfer at the lower plate is expressed as:

$$q_1 = h_1\left(T_1 - T_m\right) = K\frac{\partial T}{\partial y}\Big|_{y=0} \tag{3.17}$$

The non-dimensional mean temperature is given by:

$$\theta_m = \frac{T_m - T_1}{\frac{q_1 H}{k}} = 2\left(\frac{q_2}{15q_1} + \frac{Br_m}{40} - \frac{1}{10}\right) \quad (3.036)$$

The convective heat transfer coefficient at the lower plate can be calculated using the equation

$$q_1 = -k\left.\frac{\partial T}{\partial y}\right|_{y=o} = h_1\left(T_1 - T_m\right) \quad (3.037)$$

Establishing the non-dimensional quantity, Nusselt number is expressed as

$$Nu_H = \frac{h_1 H}{k} = \frac{Hq_1}{\left(T_1 - T_m\right)} = -\frac{1}{\theta_m} \quad (3.038)$$

However, using the Eqs. (3.036), (3.037) and (3.038) the expression for Nusselt number is given by

$$Nu_H = -\frac{60}{8\frac{q_2}{q_1} + 3Br_m - 12} \quad (3.039)$$

Based on the expression of Nusselt number obtained at the unequal constant heat flux condition as mentioned above, the expression of the Nusselt number can now be derived for some limiting cases to understand the heat transfer characteristics in a viscous dissipative environment. Some of these cases are discussed in the next sub-sections.

3.1.5 UPPER PLATE AT CONSTANT HEAT FLUX q_1 AND LOWER PLATE INSULATED.

The Nusselt number in this condition from Eq.(3.039) for $q_2 = 0$ is given below:

$$Nu_H = \frac{20}{\left(4 - Br_m\right)} \quad (3.040)$$

The above equation suggests a new way of expressing the Nusselt number as compared to what is available in the literature.

3.1.6 BOTH PLATES AT EQUAL CONSTANT HEAT FLUX q_1

In this case, one can express the Nusselt number from equation (3.039) for $q_1 = q_2$, as given below:

$$Nu_H = \frac{60}{\left(4 - 3Br_m\right)} \tag{3.041}$$

The above equation also expresses the Nusselt number in a different manner as compared to what is available at present in the literature.

3.1.7 SOLUTION USING A BRINKMAN NUMBER BASED ON TEMPERATURE DIFFERENCE

Here, a different kind of analytical method is adopted, and the Brinkman number is defined such as to obtained closed-form solution of the temperature difference, and, subsequently the expression of Nusselt number

3.1.7.1 UPPER PLATE AT CONSTANT HEAT FLUX AND LOWER PLATE INSULATED.

In this section, a case is considered where the upper plate is at constant heat flux q_1 and the lower plate is insulated, which resembles Fig ( ) with $q_2 = 0$.

Moreover, it is assumed that the temperature of the upper and lower plate are T_1 and T_2 respectively, when both temperature varies along $x-$direction. However, for this case, following non-dimensional quantities are defined to obtain the thermal energy equation in a non-dimensional framework.

$$Y = \frac{y}{H}, \qquad \theta = \frac{\left(T - T_1\right)}{\left(T_1 - T_2\right)} \tag{3.042}$$

With the aid of the above non-dimensional quantities, the energy equation, (3.029) is obtained as follows

$$k\frac{\Delta T}{H^2}\frac{d^2\theta}{dY^2}+\frac{\mu U_p^2}{H^2}+\rho C_p U_p Y\frac{dT_1}{dx} \quad (3.043)$$

Eq. (3.043) can be simplified as

$$\frac{d^2\theta}{dY^2}=a_2Y-Br \quad (3.044)$$

where $a_2 = U_\rho \frac{H^2}{\alpha\Delta T}\frac{dT_1}{dx}$, $\Delta T = (T_1 - T_2)$

and the Brinkman number $Br=\frac{\mu U_p^2}{k\Delta T}$ (3.045)

However, Eq. (3.044) is subjected to the boundary condition below

$$Y=0,\ \begin{cases}\frac{d\theta}{dy}=0 \quad \text{or } k\frac{\partial T}{\partial y}=0 \quad \text{at} \quad Y=0\\ \theta=-1 \quad \text{or } T=\mathrm{T}_2 \quad \text{at} \quad Y=0\end{cases}, \quad (3.046a)$$

To evaluate a_2 in equation (3.044) a third boundary condition is required

$$Y=1\ , \quad \theta = 0 \ \text{ or } T=\mathrm{T}_1 \quad \text{at} \quad Y=1 \quad (3.046b)$$

Solving Eq. (3.044) with the above set of boundary conditions, the temperature profile is obtained as:

$$\theta(Y)=Y^3+\frac{Br}{2}\left(Y^3-Y^2\right)-1 \quad (3.047)$$

However, using Eq. (3.046) the expression of dimensionless mean temperature is obtained as:

$$\theta_m=\frac{T_m-T_1}{T_1-T_2}=-\left(\frac{Br}{20}+\frac{3}{5}\right) \quad (3.048)$$

From the heat flux at the upper plate, the expression of Nusselt number comes out to be

$$Nu_H = \frac{h_1 H}{k} = \frac{(T_1 - T_2)}{(T_1 - T_m)}\left[\frac{\partial\theta}{\partial Y}\right]_{y=1} = \frac{\left(3 + \frac{Br}{2}\right)}{\left(\frac{Br}{20} + \frac{3}{5}\right)} \tag{3.049}$$

However, it is interesting to note from the above expression of the Nusselt number that when $Br = 0,\ Nu_H = 5$, this is identical to the result obtained under heat flux condition when $Br_m = 0$ i.e., Eq. (3.040)

3.1.7.2 BOTH PLATES AT EQUAL CONSTANT HEAT FLUX

In this section, a case is considered where both plates are maintained at the same constant heat flux q_1 (Fig with $q_2 = q_1$). Considering symmetry of the problem, the temperature of both plates is assumed to be T_w varying along $x-$ direction. However, for this, the following non-dimensional quantities are defined to make the thermal energy equation, Eq. (3.029) dimensionless

$$Y = \frac{y}{H} \quad , \quad \theta = \frac{(T - T_w)}{(T_f - T_w)} \tag{3.050}$$

where T_f is the uniform fluid temperature at the centreline. With the aid of the above non-dimensional quantities, the energy equation (3.029) is obtained as:

$$K\frac{\Delta T}{H^2}\frac{d^2\theta}{dY^2} + \mu\frac{U_p^2}{H^2} = \rho C_p U_p Y \frac{dT_w}{dx} \tag{3.051}$$

The above equation (3.051) can be written as:

$$\frac{d^2\theta}{dY^2} = a_3 Y - Br \tag{3.052}$$

where

$$a_3 = U_p \frac{H^2}{\alpha\Delta T}\frac{dT_w}{dx} \quad , \quad \Delta T = (T_f - T_w)$$

and the Brinkman number $Br = \frac{\mu U_p^2}{k\Delta T}$ (3.053)

However, Eq. (3.052) is subjected to the boundary conditions as below:

$$Y = \frac{1}{2}, \begin{cases} \frac{d\theta}{dy} = 0 \quad \text{or } k\frac{\partial T}{\partial y} = 0 \quad \text{at} \quad Y = \frac{1}{2} \\ \theta = 1 \quad \text{or } T = T_f \quad \text{at} \quad Y = \frac{1}{2} \end{cases}, \tag{3.054a}$$

To evaluate a_3 in equation(3.052), a third boundary condition is required

$$Y = 0, \; \theta = 0 \quad \text{or } T = T_w \quad \text{at} \quad Y = 0 \tag{3.054b}$$

The solution of Eq (3.052) subjected to the above set of boundary conditions is:

$$\theta(Y) = -2Y^3 + \frac{3}{2}Y + \frac{Br}{2}\left(Y^3 - Y^2 + \frac{1}{4}Y\right) \tag{3.055}$$

However, the expression of the dimensionless mean temperature is found to be

$$\theta_m = \frac{T_m - T_w}{T_f - T_w} = \left(\frac{Br}{30} + \frac{1}{5}\right) \tag{3.056}$$

For the heat flux at the upper plate, the expression of the Nusselt number reduces to:

$$Nu_H = \frac{h_1 H}{k} = \frac{\left(T_f - T_w\right)}{\left(T_w - T_m\right)}\left[\frac{\partial\theta}{\partial y}\right]_{y=1} = \frac{\left(\frac{9}{2} - \frac{5}{8}Br\right)}{\left(\frac{Br}{30} + \frac{1}{5}\right)} \tag{3.057}$$

In this result, when $Br = 0$, the Nusselt number also reduces to $Nu_H = \frac{45}{2}$.

However, comparing our results with the work of Aydin and Avci (2006) for plane Poiseuille flow, it is noted that the results Eq. (3.049 and Eq. (3.057) are new and not mentioned.

It is very important to note that the present analytical method can be applied to heat transfer in a channel not only with moving wall but also with fixed plates because there is no restriction on the velocity distribution form of the fluid.

3.2 LAMINAR HEAT TRANSFER IN HYDRO DYNAMICALY AND THERMALLY DEVELOPED FLOW BETYWEEN FIXED PARALLEL PLATES CHANNEL WITH CONSTANT HEAT FLUX BOUNDARY CONDITIONS

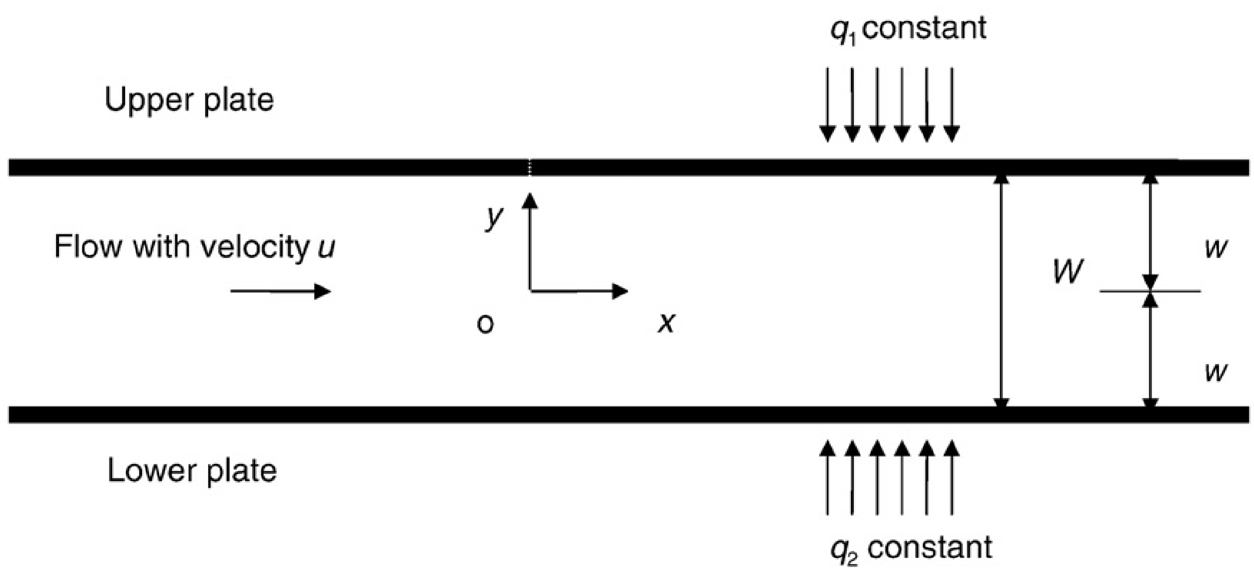


Figure (3.01) : Schematic diagram of the flow domain

3.2.1 PHYSICAL CONSIDERATIONS

We demonstrate our application considering the case of a flow between parallel plates. We remain stick to the other conditions unlike the previous case except the movement of the upper plate. We assume that the fluid is bounded by two parallel plates fixed at $y = -w$ and $y = w$ as shown in Fig (3.01). We also consider that the fluid is Newtonian and an axial applied pressure gradient set the fluid in motion suddenly along the $x-$direction.

The flow is considered parallel since only one component of the velocity is different from zero. In order to illustrate this concept, consider two-dimensional steady flow in a channel with straight parallel sides. This flow is two-dimensional since the velocity is in $x-$direction and its variation is in the $y-$direction which is a consequence of the continuity equation

$$\nabla \cdot u = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.058}$$

The $x-$component of the Navier-Strokes equation reads

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{3.059}$$

and $y-$component of the Navier-Strokes equation is given as

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{3.060}$$

Since the flow is constrained by the flat parallel walls of the channel, no component of the velocity in the $y-$direction is possible, that is, $v = 0$. This implies that $v = 0$ everywhere. Hence the gradient in v are also equal to zero i.e.

$$\frac{\partial v}{\partial y} = \frac{\partial v}{\partial x} = \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial x^2} = 0 \tag{3.061}$$

From the continuity equation, we have

$$\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y} = 0 \tag{3.062}$$

which implies that $\frac{\partial^2 u}{\partial x^2} = 0$, the momentum equations are thus reduced to

$$\frac{dp}{dx} = \mu\frac{d^2 u}{dy^2} \tag{3.063}$$

and

$$\frac{\partial p}{\partial y} = 0 \tag{3.064}$$

Since u is not a function of x the distance along the axis, there is no physical mechanism to provide a change in the pressure gradient. Thus, the pressure gradient is constant, i.e.

$dp/dx = \text{constant}$. Note that under these conditions all the nonlinear convective terms in the momentum equation are eliminated. Eq. (3.063) is a linear second order ordinary differential equation. It is easily integrated twice to yield.

$$\mu(y) = \frac{1}{2}\frac{1}{\mu}\frac{dp}{dx}y^2 + Ay + B \tag{3.065}$$

Where A and B are integration constants. In order to evaluate the integration constants, we apply no-slip boundary conditions at the channel walls. The boundary conditions at the walls are

$$y = \pm\, w, \quad u = 0 \tag{3.066}$$

Using this condition to evaluate the integration constants, we have

$$A = 0 \tag{3.067}$$

$$B = -\frac{1}{2}\frac{h^2}{\mu}\frac{dp}{dx} \tag{3.068}$$

Substitution into Eq. (3.065)

$$u(y) = -\frac{1}{2}\frac{h^2}{\mu}\frac{dp}{dx}\left[1 - \left(\frac{y}{w}\right)^2\right] \tag{3.069}$$

Here we see that the velocity distribution in the channel is parabolic and symmetrical about the axis. The maximum velocity, which occur at the centre of the channel is given by

$$u_m = -\frac{1}{2}\frac{h^2}{\mu}\frac{dp}{dx} \tag{3.070}$$

Putting Eq. (3.070) in Eq. (3.069), we obtain the parabolic velocity profile as

$$u(y) = u_m\left[1 - \left(\frac{y}{w}\right)^2\right] \tag{3.071}$$

3.2.2 MATHEMATICAL FORMULATION

The applicable continuity, momentum and energy equations with appropriate boundary conditions are given to describe the flow characteristics and heat transfer through the channel.

Continuity Equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3.072}$$

From the assumption, there is no velocity in the $y-$direction, i.e., $v=0$, which gives

$$\frac{\partial u}{\partial x}=0 \tag{3.073}$$

Again, Eq (3.073) shows that the velocity in the x-direction is a function of y only.

X -Momentum Equation

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{\partial u}{\partial t}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{3.074}$$

where ρ is the fluid density, p is the pressure. Using continuity equation , we write the x-momentum equation as:

$$\mu\left(\frac{\partial^2 u}{\partial y^2}\right)=\frac{\partial p}{\partial x}+\rho\frac{\partial u}{\partial t} \tag{3.075}$$

From y-momentum equation using the above assumption, it can be shown that

$$\frac{\partial p}{\partial y}=0 \tag{3.076}$$

3.2.3 STEADY ANALYSIS FOR FIXED PLATES

The fluid flow is bounded by two parallel plates fixed at $y=w$ and $y=-w$ and an axial pressure gradient set the fluid in motion along the x–direction. Thus, Eq (3.075) becomes

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial u} \tag{3.077}$$

Integrating Eq. (3.077) twice yield

$$u(y) = \frac{1}{2}\frac{1}{\mu}\frac{dp}{dx}y^2 + Ay + B \tag{3.078}$$

where A and B are integration constants. Applying the no-slip conditions at the channel walls in order to evaluate the integration constants, the boundary conditions at the walls are;

$$y = w, \quad u = o \qquad (3.079a)$$

$$y = -w, \quad u = o \qquad (3.079a)$$

Solving Eq. (3.077) with the above boundary conditions, the velocity profile is obtained as

$$u(y) = -\frac{1}{2}\frac{h^2}{\mu}\frac{dp}{dx}\left[1 - \left(\frac{y}{w}\right)^2\right] \tag{3.080}$$

The maximum velocity which occur at the centre $(y = 0)$ of the channel is given by

$$u_c = -\frac{1}{2}\frac{h^2}{\mu}\frac{dp}{dx} \tag{3.081}$$

From Eq. (3.081) and (3.080), the well-known parabolic velocity distribution is given by

$$\frac{u}{u_c} = 1 - \left(\frac{y}{w}\right)^2 \tag{3.082}$$

Where w is the channel height.

Energy Equation:

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + \frac{\partial T}{\partial t}\right) = K\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\phi \tag{3.083}$$

where ϕ is the viscous dissipation term that contains only $(\partial u/\partial y)^2$. Based on the assumptions, the energy equation reduces to

$$\rho C_p\left(u\frac{\partial T}{\partial x}+\frac{\partial T}{\partial t}\right)=K\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{3.084}$$

The energy Eq. (3.084) is written under steady condition as:

$$\rho C_p u\frac{\partial T}{\partial x}=K\frac{\partial T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{3.085}$$

Substituting Eq. (3.082) into Eq. (3.085) we obtain the equation

$$\rho C_p u_c\left(1-\left(\frac{y}{w}\right)^2\right)\frac{\partial T}{\partial x}=K\frac{\partial^2 T}{\partial y^2}+\mu\frac{4u_c^2 y^2}{w^4} \tag{3.086}$$

To analyze the energy equation with uniformly heated boundary walls, the following cases of different constant heat fluxes, equal constant heat fluxes, and insulated lower plate are considered while neglecting the longitudinal conduction term in the energy equation.

3.2.4 BOTH PLATES AT DIFFERENT CONSTANT HEAT FLUXES

Consider the case where the upper plate is at constant heat flux q_1 and the lower plate is at constant heat flux q_2 as shown in fig. (), the non-dimensional quantities used as $Y=\frac{y}{w}$, and defining the dimensionless temperature

$$\theta=\frac{T-T_1}{\frac{q_1 w}{K}} \tag{3.087}$$

where T_1 is the upper plate temperature and w is the channel height, k is the thermal conductivity, q_1 is the heat flux at the upper plate.

The governing equation for different constant heat flux reduces to the form

$$\frac{\partial^2\theta}{\partial Y^2}=\frac{u_c}{\alpha}\frac{wk}{q_1}\frac{dT_1}{dx}-\frac{u_c}{\alpha}\frac{4}{w^2}\frac{wk}{q_1}\frac{dT_1}{dx}w^2Y^2-\frac{u}{\alpha\rho c_p}\frac{wk}{q_1}\frac{4u_c^2 16}{w^4}w^2Y^2 \tag{3.088}$$

Taking $\alpha=\dfrac{k}{\rho c_p}$ and for the constant heat flux case

$\dfrac{\partial T}{\partial x}=\dfrac{dT_1}{dx}$ where T_1 is the upper wall temperature from Eq. (3.088)

Let $a_1=\dfrac{u_c wk}{\alpha q_1}\dfrac{dT_1}{dx}$ and modified Brinkman number

$$Brq_1=\frac{\mu u_c^2}{wq_1} \tag{3.089}$$

Eq. (3.088) becomes

$$\frac{d^2\theta}{dy^2}=a_1=\left(1-4Y^2\right)-64Brq_1Y^2 \tag{3.090}$$

Following the thermal boundary conditions imposed on the plates, a set of boundary conditions is given in order to obtain the dimensionless temperature profile.

(i) $\left(-k\dfrac{\partial T}{\partial y}\right)=-q_1$ at $y=w$ or $\dfrac{d\theta}{dY}=1$ at $Y=\dfrac{1}{2}$ (3.091a)

(ii) T = T_1 at y = w, or $\theta=0$ at $Y=\dfrac{1}{2}$ (3.091b)

(iii) $\left(-K\dfrac{\partial T}{\partial y}\right)=q_2$ at y = -w or $\dfrac{d\theta}{dY}=-\dfrac{q_2}{q_1}$ at $Y=-\dfrac{1}{2}$ (3.091c)

The solution of Eq. (3.090) under the above thermal conditions is

$$\begin{aligned}\theta(Y)=&-\frac{1}{3}Y^4\left(\frac{3}{2}\frac{q_2}{q_1}+\frac{3}{2}\right)+\frac{1}{2}\left(\frac{3}{2}\frac{q_2}{q_1}+\frac{3}{2}\right)Y^2+\left(\frac{1}{2}-\frac{1}{2}\frac{q_2}{q_1}\right)Y+\frac{3}{32}\frac{q_2}{q_1}-\frac{13}{32}\\&+Brq_1\left(-8Y^4+4Y^2-\frac{1}{2}\right)\end{aligned} \tag{3.092}$$

In fully developed flow, it is usual to utilize the mean fluid temperature, T_m, rather than the centre line temperature when defining the Nusselt number. The mean or bulk temperature is given by

$$T_m = \frac{\int \rho u T dA}{\int \rho u dA} \tag{3.093}$$

Where, $\int \rho u_c \left(1 - \frac{y^2}{w^2}\right) dA = \frac{2}{3} \rho u_c LW$ (3.094)

$$\int \rho u_c \left(1 - \frac{y^2}{w^2}\right) T dA = \rho u_c LW \left(\frac{q_1 w}{k}\right)\left(\frac{3}{35}\frac{q_2}{q_1} - \frac{26}{105}\right) + \rho Br q_1 u_c LW \left(\frac{q_1 w}{k}\right)\left(\frac{-8}{35}\right) + \rho u_c LW T_1 \left(\frac{2}{3}\right) \tag{3.095}$$

And L is the width of the plate. Therefore,

$$T_m = \left(\frac{q_1 w}{k}\right)\left(\frac{9}{70}\frac{q_2}{q_1} - \frac{13}{35}\right) - Br q_1 \left(\frac{q_1 w}{k}\right)\left(\frac{12}{35}\right) + T_1 \tag{3.096}$$

The dimensionless mean temperature is given by:

$$\theta_m = \frac{K}{q_1 w}\left(T_m - T_1\right) \quad \text{or} \tag{3.097}$$

$$\theta_m = \left(\frac{9}{70}\frac{q_2}{q_1} - \frac{13}{35}\right) - Br q_1 \left(\frac{12}{35}\right) \tag{3.098}$$

At this point, the convective heat transfer coefficient can be evaluated by the use of the defining equation

$$q_1 = h\left(T_1 - T_m\right) \tag{3.099}$$

Defining the Nusselt number to be

$$Nu = \frac{hw}{K} \tag{3.100}$$

$$= \frac{q_1}{T_1 - T_m} \frac{w}{K} = -\frac{1}{\theta_m}, \text{ using Eq. (3.097)} \tag{3.101}$$

Therefore, on using Eq. (3.098) and (3.101), we have the new result

$$Nu = \frac{70}{-9\left(\frac{q_2}{q_1}\right) + 26 + 24Brq_1} \tag{3.102}$$

Based on the expression of Nusselt number obtained at different constant heat flux condition, the expression of the Nusselt number can now be derived for some limiting cases. Some of these cases are:

3.2.5 UPPER PLATE AT CONSTANT HEAT FLUX q_1 AND LOWER PLATE INSULATED

From Eq. (3.012), when $q_2 = 0$,

$$Nu = \frac{35}{13 + 12Brq_1} \tag{3.103}$$

This also is a result not found in the literature.

3.2.6 BOTH PLATES AT EQUAL CONSTANT HEAT FLUX q_1

In this case, letting $q_1 = q_2$ in eq. (3.102)

$$Nu = \frac{70}{17 + 24Brq_1} \tag{3.104}$$

This result is identical to that derived by Aydin and Avci (2006) for Poiseuille flow. For the case of no viscous dissipation, $Brq_1 = 0$, $Nu = \frac{70}{17}$.

3.2.7 SOLUTIONS USING A BRINKMAN NUMBER BASED ON TEMPERATURE DIFFERENCE

In addition to the previous results, Aydin and Avci (2006) for Poiseuille flow, also analyzed the problem using a Brinkman number based on a temperature difference ΔT:

$$Br = \frac{\mu u_c^2}{K\Delta T} \tag{3.105}$$

The dimensionless temperature is also redefined using a temperature difference instead of heat flux.

3.2.7.1 UPPER PLATE AT CONSTANT HEAT FLUX AND LOWER PLATE INSULATED

Consider the case where the upper plate is at constant heat flux and lower plate is insulated, which is Fig. (3.01) with $q_2 = 0$.

It is assumed that upper plate is kept at temperature T_1 and lower plate is kept at temperature T_2, both T_1 and T_2 varying along the x-direction. By introducing following non-dimensional quantities,

$$Y = \frac{y}{w} \text{ and } \theta = \frac{T_1 - T}{T_1 - T_2} \tag{3.106}$$

Eq. (3.086) becomes

$$\frac{\partial^2\theta}{\partial Y^2} = \frac{u_c}{\alpha}\frac{dT_1}{dx}\frac{W^2}{\left(T_1 - T_2\right)} + 4\frac{u_c}{\alpha}\frac{dT_1}{dx}\frac{W^2}{\left(T_1 - T_2\right)}Y^2 + 64\frac{\mu}{\alpha\rho C_p}\frac{u_c^2}{\left(T_1 - T_2\right)} \tag{3.107}$$

Let

$$a_3 = \frac{u_c}{\alpha}\frac{dT_1}{dx}\frac{W^2}{\left(T_1 - T_2\right)} \tag{3.108}$$

Then Eq. (3107) becomes

$$\frac{d^2\theta}{dY^2} = a_3\left(4Y^2 - 1\right) + 64\frac{\mu}{k}\frac{u_c^2}{\left(T_1 - T_2\right)}Y^2 \tag{3.109}$$

The thermal boundary conditions here are.

(i) $k\frac{\partial T}{\partial y} = 0$ at $y = -w$, or $\frac{\partial \theta}{\partial Y} = 0$ at $y = -\frac{1}{2}$ (3.110a)

(ii) $T = T_1$ at $y = w$, or $\theta = 0$ at $y = \frac{1}{2}$ (3.110b)

(iii) $T = T_2$ at $y = -w$, or $\theta = 0$ at y = - $\frac{1}{2}$ (3.110c)

Now let $\Delta T = T_1 - T_2$. Therefore, Eq. (3.105) becomes

$$B_r = \frac{\mu}{k}\frac{u_c^2}{\left(T_1 - T_2\right)} \tag{3.111}$$

Then Eq. (3.109) becomes

$$\frac{d^2\theta}{dY^2} = a_3\left(4Y^2 - 1\right) + 64B_rY^2 \tag{3.112}$$

The solution of Eq.(3.112) under the above thermal boundary conditions is

$$\theta\left(Y\right) = \left(Y^4 - \frac{3}{2}Y^2 - Y + \frac{13}{16}\right) + B_r\left(8Y^2 - 4Y^2 + \frac{1}{2}\right) \tag{3.112}$$

From Eq.(3.106) and (3.113), the temperature distribution is

$$T = \left(T_2 - T_1\right)\left(Y^4 - \frac{3}{2}Y^2 - Y + \frac{13}{16}\right) + B_r\left(T_2 - T_1\right)\left(8Y^2 - 4Y^2 + \frac{1}{2}\right) + T_1 \tag{3.113}$$

Using Eq. (3.093), the expression for T_m is found to be

$$T_m = \left(T_2 - T_1\right)\frac{26}{35} - \left(T_2 - T_1\right)B_r\frac{12}{35} + T_1 \tag{3.114}$$

The dimensionless mean temperature is defined as

$$\theta_m = \frac{T_m - T_1}{T_2 - T_1} \tag{3.116}$$

Therefore, the expression for dimensionless mean temperature is

$$\theta_m = \frac{26}{35} - \frac{12}{35} Br \tag{3.117}$$

Following Eq. (3.101), the convective heat transfer is given by

$$h = \frac{\dfrac{K\left(T_2 - T_1\right)}{W}\left(\dfrac{\partial \theta}{\partial Y}\right)_{y=0.5}}{-\left(T_2 - T_1\right)\dfrac{26}{35} + \left(T_2 - T_1\right) Br \dfrac{12}{35}} = \frac{35\left(\dfrac{K}{W}\right)\left(-2\right)}{-26 + 12Br} \tag{3.118}$$

or,

$$Nu = \frac{35}{13 - 6Br} \tag{3.119}$$

In this result when $Br = 0$, the Nusselt number also reduces to $Nu = \frac{35}{13}$, which is identical to the result derived in section 3.2.5 when $Brq_1 = 0$.

3.2.7.2 BOTH PLATES AT EQUAL CONSTANT HEAT FLUX

Consider the case where both plates have same input heat fluxes q_1 which is Fig. ( ) with $q_1 = q_2$. By symmetry, it is assumed that the plates will be at equal temperature T_0 but which varies along the x-direction.

By introducing the following non-dimensional quantities.

$$Y = \frac{y}{w}, \quad \theta = \frac{T_0 - T}{T_0 - T_c} \tag{3.120}$$

where T_c is the centreline temperature. Eq. (3.086) becomes

$$\frac{\partial^2\theta}{\partial Y^2} = -\frac{u_c}{\alpha}\frac{dT_0}{dx}\frac{w^2}{\left(T_0 - T_c\right)} + 4\frac{u_c}{\alpha}\frac{dT_0}{dx}\frac{w^2}{\left(T_0 - T_c\right)}Y^2 + 64\frac{u}{\alpha\rho C_p}\frac{u_c^2}{\left(T_0 - T_c\right)}Y^2 \tag{3.121}$$

Let $\quad a_2 = \frac{u_c}{\alpha}\frac{dT_0}{dx}\frac{w^2}{\left(T_0 - T_c\right)}$ (3.122)

Therefore Eq. (3.121) becomes

$$\frac{d^2\theta}{dY^2} = a_2\left(4Y^2 - 1\right) + 64\frac{\mu}{\alpha\rho C_p}\frac{u_c^2}{T_0 - T_c}Y^2 \tag{3.123}$$

The thermal boundary conditions are:

(i) $\quad \frac{\partial T}{\partial y} = 0$ at $y = 0$, or $\frac{\partial \theta}{\partial Y} = 0$ at $Y = 0$ (3.124a)

(ii) $\quad T = T_c$ at $y = 0$, or $\theta = 1$ at $Y = 0$ (3.124b)

(iii) $\quad T = T_0$ at $y = w$, or $\theta = 0$ at $Y = \frac{1}{2}$ (3.124c)

Let $\Delta T = T_0 - T_c$, then Eq. (3.105) becomes

$$Br = \frac{\mu}{k}\frac{u_c^2}{\left(T_0 - T_c\right)} \tag{3.125}$$

and Eq. (3.123) becomes

$$\frac{d^2\theta}{dY^2} = a_2\left(4Y^2 - 1\right) + 64BrY^2 \tag{3.126}$$

The solution of Eq.(3.126) under the above thermal boundary conditions can be shown to be

$$\theta\left(Y\right) = \left(1 - \frac{24}{5}Y^2 + \frac{16}{5}Y^4\right) - \frac{8}{5}Br\left(Y^2 - 4Y^4\right) \tag{3.127}$$

In dimensional form, the temperature distribution is

$$T = \left(T_0 - T_c\right)\left(\frac{24}{5}Y^2 - 1 - \frac{16}{5}Y^4\right) + \frac{8}{5}Br\left(T_0 - T_c\right)\left(Y^2 - 4Y^4\right) + T_0 \tag{3.128}$$

Similar to Eq. (3.093)- (3.095), the mean temperature can be found to be

$$T_m = -\frac{136}{175}\left(T_0 - T_c\right) + \frac{8}{175}Br\left(T_0 - T_c\right) + T_0 \tag{3.129}$$

The dimensionless mean temperature is defined as

$$\theta_m = \frac{T_m - T_0}{T_c - T_0} \tag{3.130}$$

From which is obtained

$$\theta_m = \frac{136}{175} - \frac{8}{175}Br \tag{3.131}$$

upon using

$$q_1 = h\left(T_0 - T_m\right) = k\left(\frac{\partial T}{\partial y}\right)_{y=w} \tag{3.132}$$

The expression for h is

$$h = \frac{K\left(\frac{\partial T}{\partial y}\right)_{y=w}}{T_0 - T_m} \tag{3.133}$$

From Eq. (3.120),

$$\frac{\partial T}{\partial y} = \frac{\left(T_c - T_0\right)}{w}\frac{\partial \theta}{\partial Y} \tag{3.134}$$

In terms of θ_m, h is given by

$$h = \frac{\frac{K\left(T_c - T_0\right)}{w}\left(\frac{\partial \theta}{\partial Y}\right)_{y=0.5}}{T_0 + \theta_m\left(T_0 - T_c\right) - T_0} = \frac{-K\left(\frac{-16}{5} + \frac{8}{5}Br\right)}{w\left(\frac{136}{175} - \frac{8}{175}Br\right)} \tag{3.135}$$

or,

$$Nu = \frac{hw}{K} = \frac{35\left(2 - Br\right)}{17 - Br} \tag{3.136}$$

In this result, when $Br = 0$, the Nusselt number also reduces to $Nu = \frac{70}{17}$.

It is noted that in Aydin and Avci (2006) for plane Poiseuille flow, the result Eq. (3.136) is not mentioned, but temperature equation corresponding to Eq. (3.126) is given. However, we found that our results are different. In our Eq. (3.126), the second term on the right hand side has the coefficient 64, but in Aydin and Avci (2006), it is 16. In our Eq. (3.127), the coefficient in the second term is $\frac{8}{5}$, but in Aydin and Avci (2006), it is $\frac{2}{5}$.

CHAPTER 4

GRAPHICAL RESULTS AND DISCUSSIONS

Graphical results of the effects of viscous dissipation on the analytical solutions of temperature distribution and heat transfer characteristics for a plane Couette flow and channel flow are given in this chapter and findings are discussed.

In order to bring out the effect of viscous dissipation on the temperature profile and Nusselt number, three different cases of constant heat fluxes are presented.

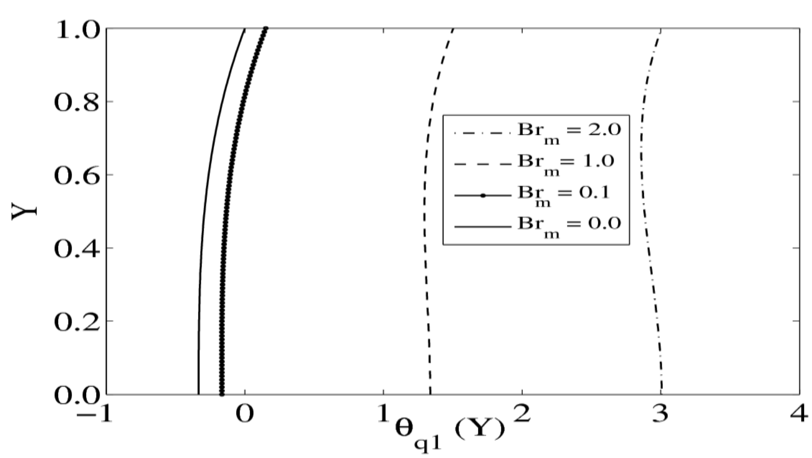
4.1 PLANE COUETTE FLOW

For the purpose of discussion on the behavior of the Couette flow, two types of graphs based on the analytical solutions are made. The temperature profile in the channel is plotted with variations of various parameters to indicate the heated region, and the Nusselt number is plotted to reveal the heat transfer characteristics of the flow.

4.1.1 PLATES AT DIFFERENT CONSTANT HEAT FLUXES q_1 AND q_2

The Brinkman number is an important parameter governing the heat transfer and fluid flow between two parallel plates. Effects of viscous dissipation in a fluid flow and heat transfer phenomenon is explained by the Brinkman number.

4.1.1.1 TEMPERATURE PROFILE FOR THE CASE OF PLATES AT DIFFERENT CONSTANT HEAT FLUXES.



Dimensionless temperature profile $\theta q_1(Y)$ for different values of Br_m

Figure (4.00) depicts the dimensionless temperature profile within the flow field at different Br_m pertaining to the case where plates are kept at different constant heat flux conditions obtained from Equation (3.034). One may observe that with increasing value of Br_m, the temperature increases as expected. Positive values of Br_m are compatible with the wall heating case, which indicates heat transfer to the fluid across the wall. Therefore,, in the case with positive Br_m, the fluid temperature increases as evident from above Figure. However, the temperature profile close to the upper plate shows an increasing trend. The increasing trend of temperature profile nearer to the upper plate is attributed to the effect of shear in the fluid layer produced by the movement of the upper plate.

4.1.1.2 NUSSELT NUMBER VARIATION

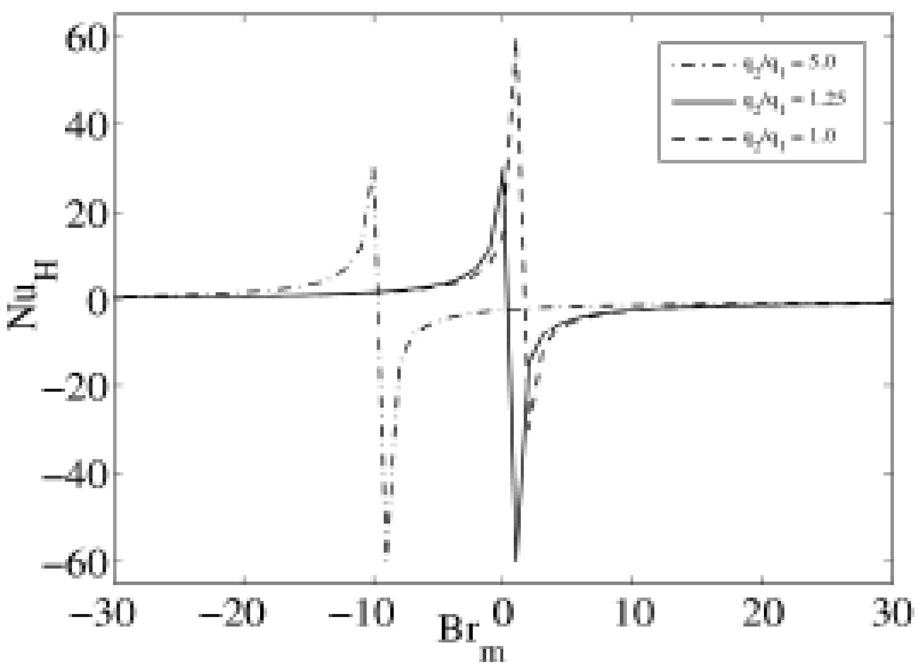


Figure (4.01a) : The influence of Br_m on the Nu_H for different q_2 / q_1

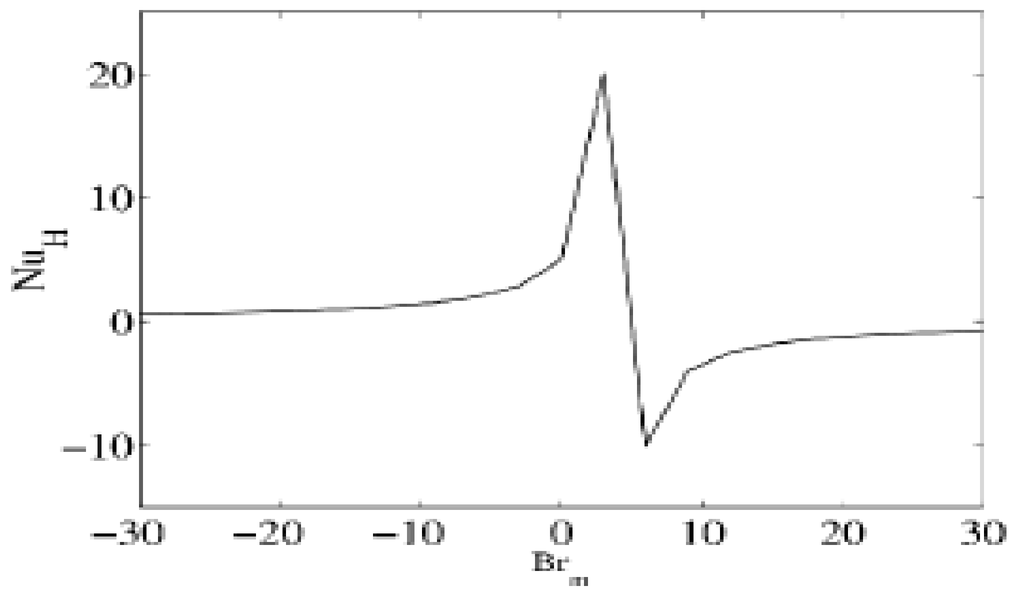


Figure (4.01b) : The influence of Br_m on the Nu_H for $q_2 / q_1 = 0$

The main physical quality of interest is the Nusselt number which represents the heat transfer rate at the wall of the plate. The variation of the Nusselt number with the Brinkman number needs to be investigated. To demonstrate the effect of viscous dissipation on the Nusselt number, Equation (3.039) is considered. However, the variation of the Nusselt number with Br_m is shown in Figure 4.01a and 4.01b for heat flux ratio $q_1/q_2 = 1, 1.25, 5$ and for $q_2/q_1 = 0$, respectively.

The choice of different heat flux ratio represents different cases. The ratio $q_2/q_1 = 1$ corresponds to the case, where both plates are at equal constant heat flux, Similarly, o corresponds to the case of an insulated lower plate. The ratio 1.25 indicates the special case occurring due to the point of singularity at the origin.

One may notice from the above figures that the variation of the Nuselt number with Br_m, is not continuous for all the cases considered in the study, rather a clear existence of the point of singularity is observed in each case at a different point at a different Br_m, as suggested by Equation (3.038). The different locations of the point of singularity are due to the different ratios of heat flux considered, and, at this point, the shear heating balances the heat supplied by the wall. However, from this point of singularity as Br_m, increases in the positive direction

$(Br_m > 0)$, the Nusselt number decreases because of the decrease in the driving potential of the heat transfer, and it finally attains different constant values asymptotically, (when $Br_m \to \infty$), for all the cases of heat flux taken into account. The negative value of the Br_m represents the wall-cooling problem and with increasing value of Br_m in the negative direction, the Nusselt number decreases and an asymptotic appears at different constant values of Nu for different cases as $Br_m \to -\infty$.

4.2 LOWER PLATE INSULATED AND UPPER PLATE AT CONSTANT HEAT FLUX

The graphical plots of the variation of the dimensionless temperature profile and the Nusselt number using the Brinkman number defined in Equation (3.045) are presented.

4.2.1 TEMPERATURE PROFILE FOR THE CASE OF INSULATED LOWER PLATE

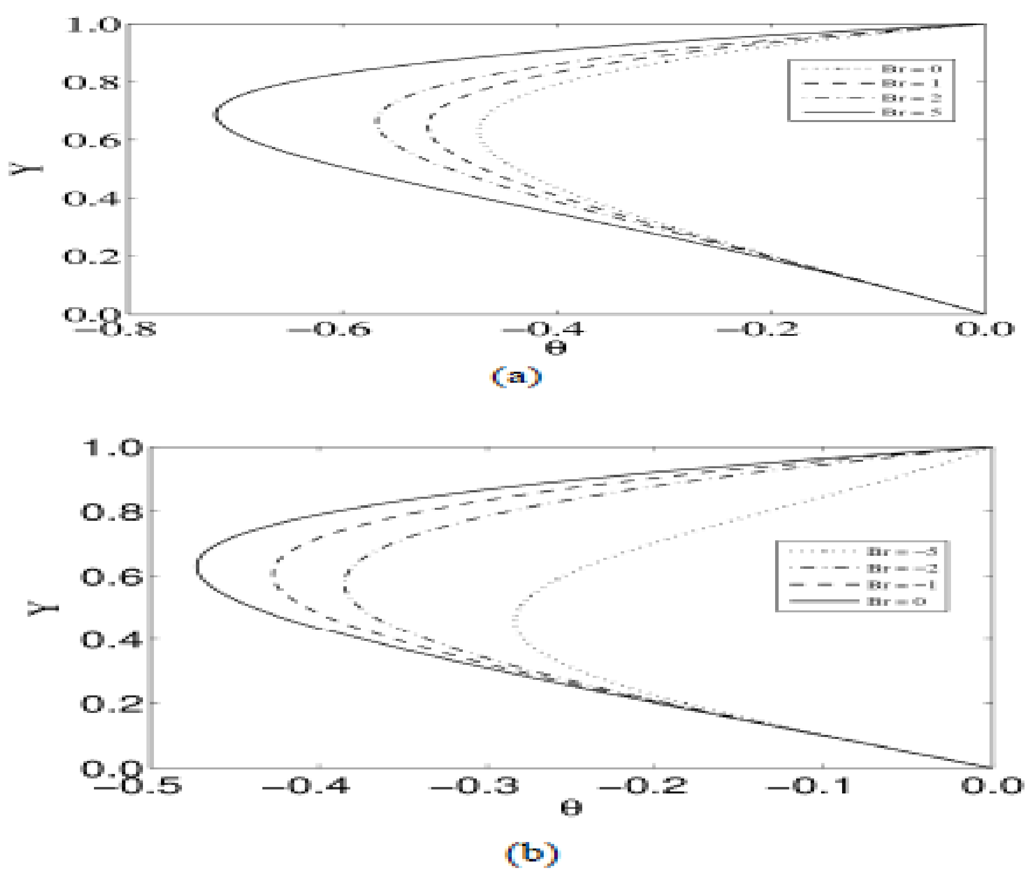


Figure (4.02): Dimensionless temperature profile $\theta(Y)$ versus Y for different values of Br for the case of insulated lower plate: (a) hot wall (b) cold wall

Figure (4.02a) corresponds to the wall-heating case and as expected, the bulk temperature of the fluid increases with increasing values of Br. This indicates that as dissipation increase, the fluid temperature increases due to the internal fluid friction. On the contrary, one may observe from Figure (4.02b) that for the wall-cooling case, with increasing Br, the bulk temperature of the fluid decreases compared to the case with a negligible Br. Actually, the wall-cooling case is applied to reduce the fluid temperature, and it is important to note from Figure (4.02b) that even of higher value of Br. The temperature of the fluid decreases, which can be attributed to the movement of the upper plate. However, this shows similarity with the viscous heating effect between parallel plates by Sestak and Rigier (1969). Interestingly, one can make an important observation from Figure 4a-b that the viscous dissipation effects become prominent in a zone, close to the upper plate due to the high shear rate over there.

4.2.2 NUSSELT NUMBER VARIATION

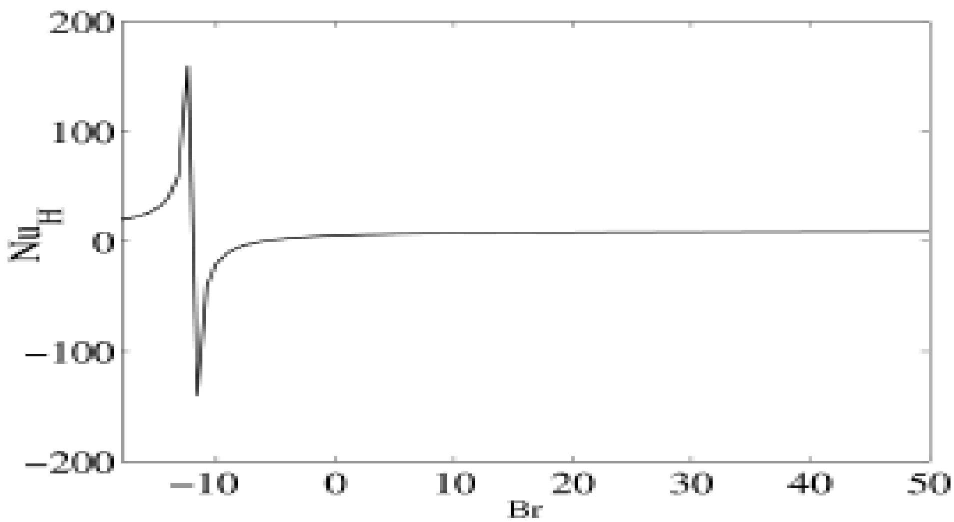


Figure (4.03): The influence of Br on the Nu_H for the case of insulated lower plate

The variation of Nusselt number as depicted in Figure 5 shows similarity with the results for parallel fixed plates. Increasing Br makes the bulk temperature of the fluid to increase and hence, the driving potential of the heat transfer is reduced, which is reflected on the variation of the Nusselt number as Br. increases in the positive direction. However, the Nusselt number decreases asymptotically, as $Br \to \infty$. As explained, the negative value of Br represents the wall-cooling problem, and with the increasing value of Br in the negative direction, the Nusselt number decreases and an asymptotic appears as $Br \to \infty$. It is important to observe the existence of the point of singularity at $Br = -12$, which is quite clear from Equation (3.049).

4.3 BOTH PLATES AT EQUAL CONSTANT HEAT FLUX

Here**,** the variation of the dimensionless temperature profile and the Nusselt number using the Brinkman number defined in Equation (3.053), is discussed through presentation of their graphical plots obtained from Equation (3.055) and (3.057)

4.3.1 TEMPERATURE PROFILE FOR THE CASES OF EQUAL CONSTANT HEAT FLUX

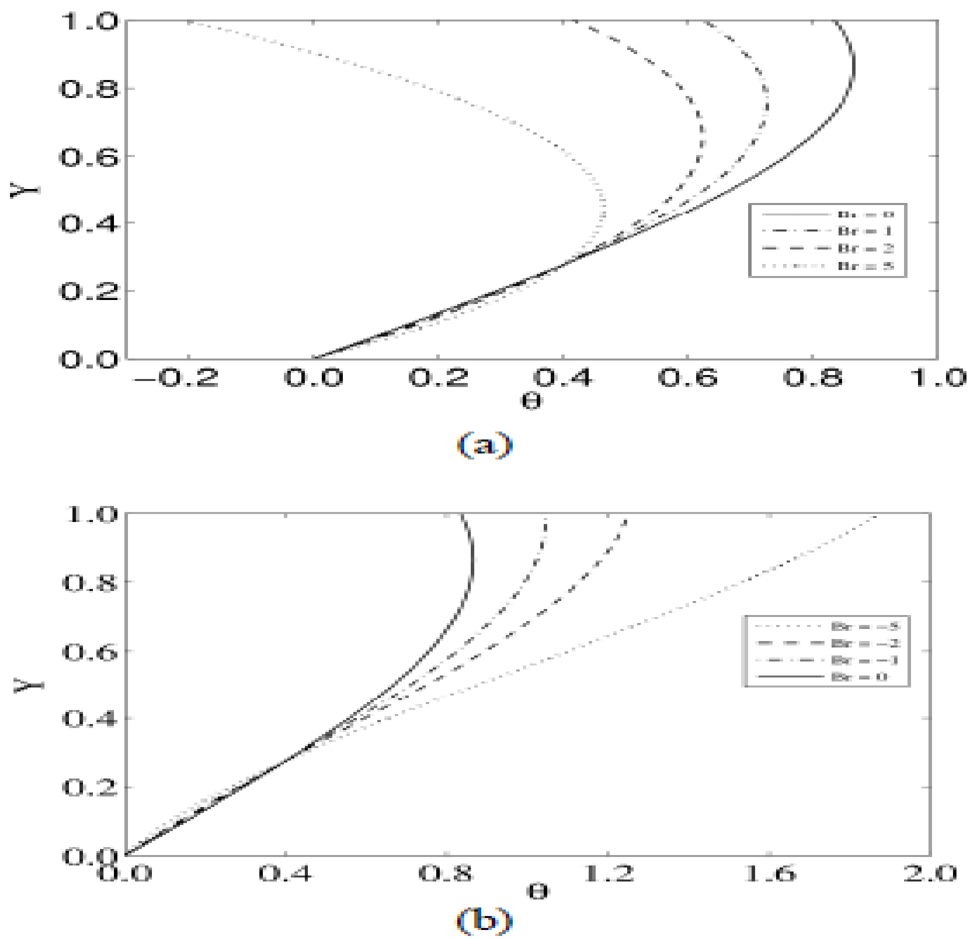


Figure (4.04): Dimensionless temperature profile $\theta(Y)$ versus Y for different values Br for the case of equal constant heat fluxes (a) hot wall (b) cold wall

The temperature variation is plotted in Figure (4.04a – b). Viscous dissipation always generates a distribution of heat source stimulating the internal energy in the fluid, and hence the temperature profile gets distorted, which is envisaged from the above Figures. Figure (4.04a) depicts the dimensionless temperature profile within the flow fluid of the wall-heating case. As explained earlier that for wall-heating case the fluid temperature increases, whereas the reverse is true for the wall-cooling case. Interestingly, one can see from above Figure that in case of equal constant heat flux, the dimensionless temperature profile exhibits usual trend of increasing temperature with positive values for Br, up to a certain distance from the lower plate at $y = 0.3$, then it is followed by a increasing trend even at positive values of Br. up to the upper plate. The increasing temperature profile of a viscous fluid squeezed and extruded between two parallel plates by Duwairi and Tashtoush (2004).

A reverse explanation holds true for negative values of the Br, which one can also observe from Figure (4.04b). This contradictory behavior of the temperature profile with Br for any particular case of wall heating as seen from the above Figures is owing to the movement of the upper plate, and the thermal boundary conditions considered in this case.

4.3.2 NUSSELT NUMBER VARIATION.

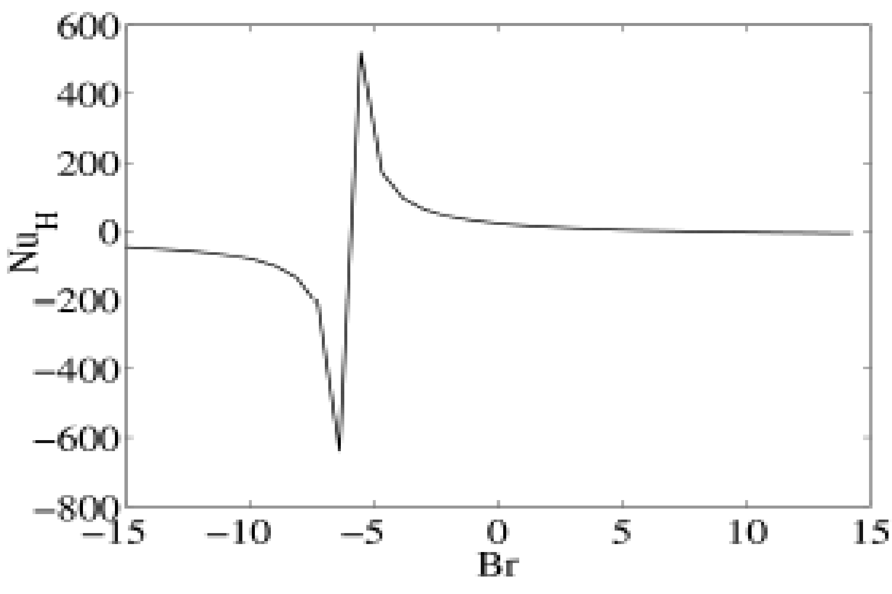


Figure (4.05): The influence of Br on the Nu_H for the case of equal constant heat fluxes.

Figure (4.05) exhibits the variation of the Nusselt number with Br. However, compared to cases with an insulated lower plate, the variation of the Nusselt number shows a distinct feature as Br changes in case of the equal constant heat flux condition. It is important to observe the existence of the point of singularity on the variation of $Br = -6$, as expected from Equation (3.057). However, from the point of singularity the Nusselt number reaches a constant value in either direction asymptotically.

4.4 FLOW BETWEEN FIXED PARALLEL PLATE CHANNEL

This section discusses briefly the graphical plots of those previous equations in section (3.2) which give the most useful results in the present study.

4.4.1 BOTH PLATES AT DIFFERENT CONSTANT HEAT FLUXES q_1 AND q_2

The effect of viscous dissipation in a fluid flow and heat transfer phenomenon is explained by the Brinkman number which governs heat transfer and fluid flow between two parallel plates channel.

4.4.1.1 TEMPERATURE PROFILE FOR THE CASE OF PLATES AT DIFFERENT CONSTANT HEAT FLUX

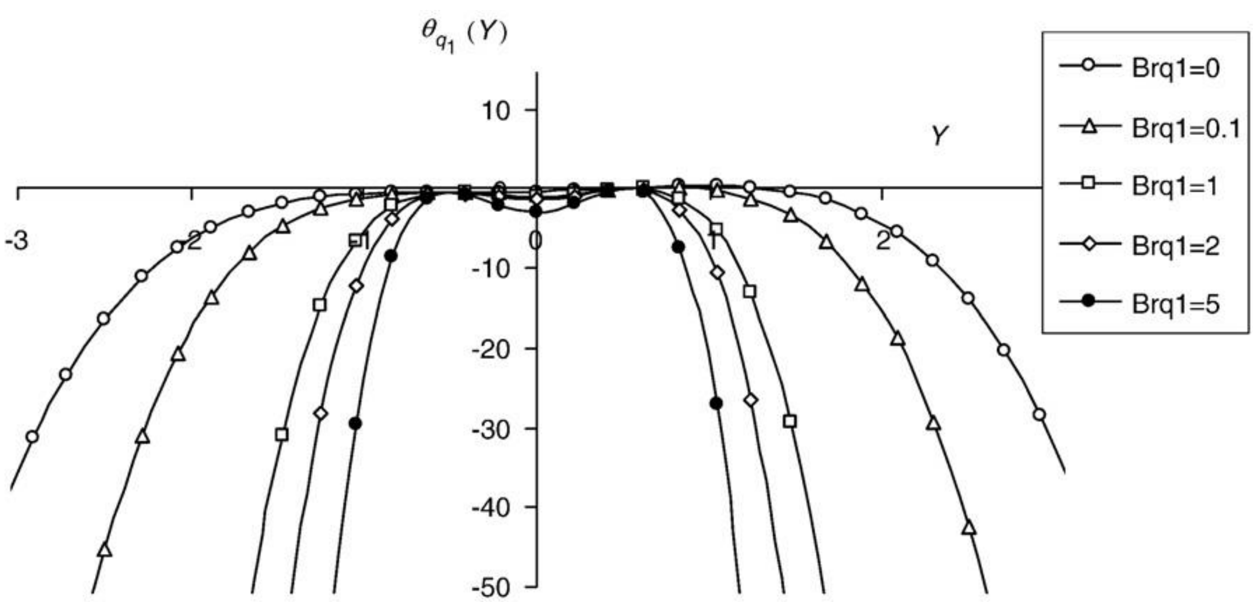


Figure (4.06): Dimensionless temperature profile $\theta(Y)$ versus Y for the case of insulated lower plate

The effect of viscous dissipation is seen in the value of modified Brinkman number (Brq_1). It is interesting to observe the behaviour of the temperature profiles for various heat flux ratios and modified Brinkman number and hence to note the effect of viscous dissipation.

The present study aims in finding out the influence of the viscous dissipation effects on the temperature profile. Figure 4.06 shows the dimensionless temperature profile within the flow field for different Brq_1 pertaining to the case where plates are kept at different constant heat flux conditions obtained from equation (3.092). It is observed that with increasing value of Brq_1 the temperature increases as expected. Positive values of Brq_1 are compatible with the wall heating case, which indicates heat transfer to the fluid across the wall. Therefore, in the case with positive Brq_1 the fluid temperature increases as evident from the above figure.

Equation (3.092) gives the temperature distribution between the two parallel plates for different constant heat flux conditions for varying Brq_1. In the case of insulated lower plate, $\frac{q_2}{q_1} = 0, \quad \theta q_1(Y) = -\frac{1}{2}Y^4 + \frac{3}{4}Y^2 + \frac{1}{2}Y - \frac{13}{12} + Br_{q_1}\left(-8Y^4 + 4Y^2 - \frac{1}{2}\right)$

Figure 4.06 is the graphical representation of $\theta q_1(Y)$ versus Y for 5 values of Brq_1 which shows that the curves converge at (0.5, 0) and are not symmetrical about the vertical axis.

4.4.1.2 NUSSELT NUMBER VARIATION

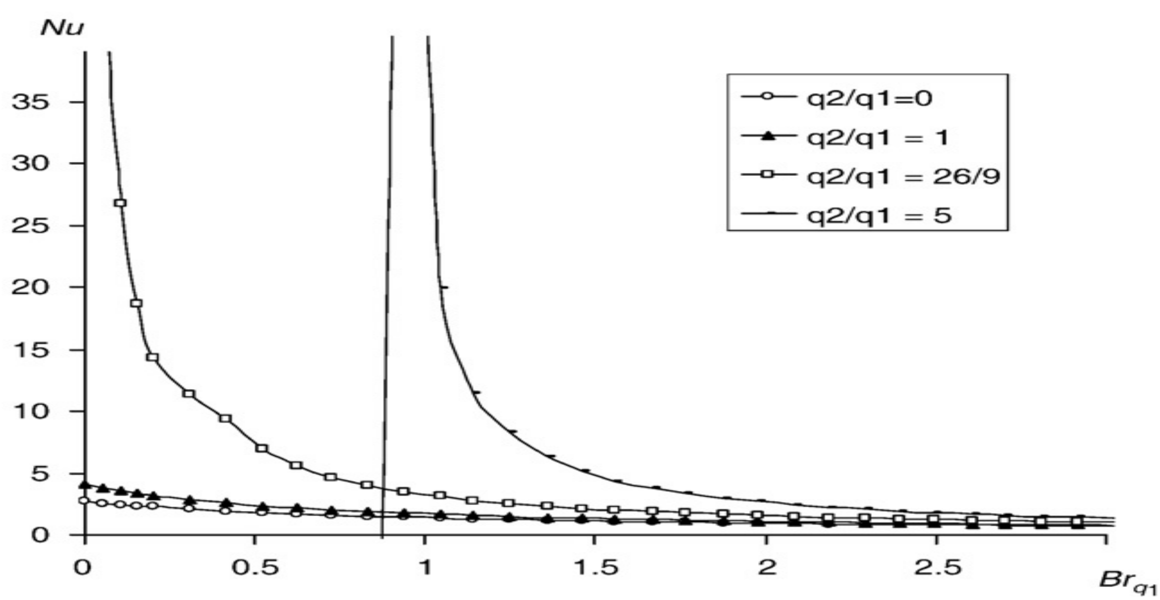


Figure (4.07): The influence of Brq_1 on Nu for different q_2 / q_1

As expected from Equation (3.102), at a given ratio of (q_1/q_2) the graph Nu Versus Brq_1 will form a rectangular hyperbola on both sides of an asymptotic of $Brq_1 = \left[\frac{3}{8}\left(\frac{q_2}{q_1}\right) - \frac{13}{12}\right]$

Four sets of curves are shown in fig 3 for heat flux ratios of $0, 1, \frac{26}{9}$ and 5. The ratio 0 corresponds to the case of insulated lower plate, the case of unity depicts the case of equal constant heat flux on both plates and the ratio of $\frac{26}{9}$ is of interest because the asymptotic lies on the vertical axis.

4.4.2 EQUAL HEAT FLUXES IN TERMS OF Br

From Figure 4.08, we know that viscous dissipation always generates a distribution of heat source, stimulating the internal energy in the fluid, and hence the temperature profile gets distorted as envisaged in the Figure. Figure 4a represents the wall heating and wall – cooling cases respectively. As expected, for wall heating case, the fluid temperature increases with increasing values of Br. in the positive direction, the wall- cooling case, the fluid temperature decreases for increasing values of Br in the negative direction.

Figure 4.09 exhibits the variation of the Nusselt number with Br. However, compared to the case with an insulated lower plate Figure 5, the variation of the Nusselt number shows a distinct feature as Br changes in the case of the equal heat flux condition. It is important to observe the existence of the point of singularity on the variation at Br = 17.

4.4.2.1 TEMPERATURE PROFILE FOR THE CASE OF EQUAL HEAT FLUXES IN TERMS OF Br

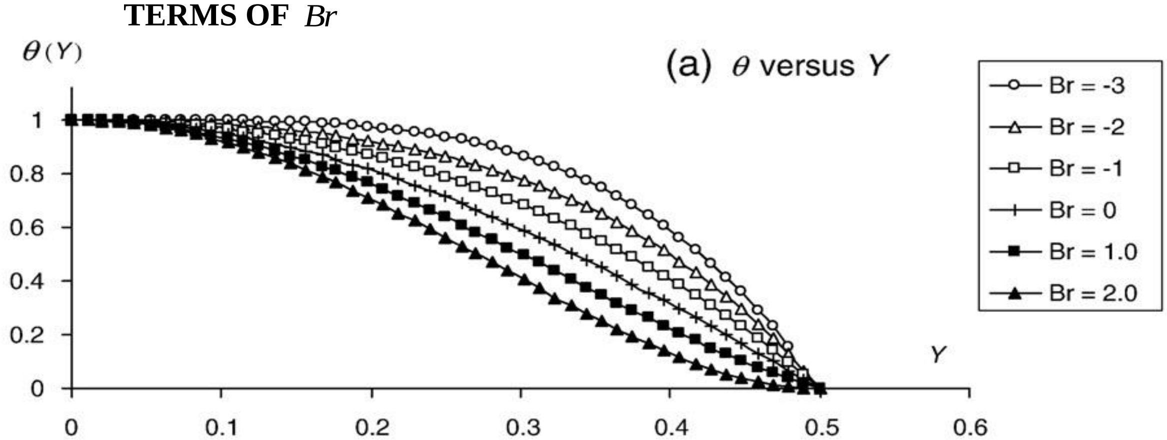


Figure (4.08): Dimensionless temperature profile $\theta(Y)$ versus Y for different values of Br for the case of equal heat fluxes.

When the Brinkman number is defined as Br, as in Equation (3.125), the corresponding non-dimensional temperature θ is defined as in Equation (3.120). The dimensionless temperature distribution for different values of Br is Equation (3.127) which is shown in Figure 4a.

4.4.2.2 NUSSELT NUMBER VARIATION

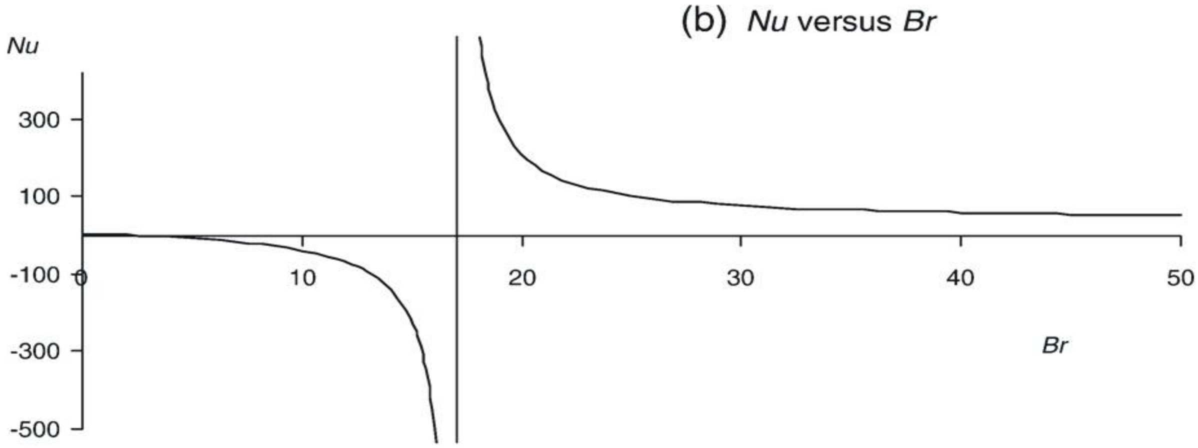


Figure (4.09): The influence of Br on Nu for equal heat fluxes

The corresponding Nusselt number for this case is Equation (3.137) as shown in Figure 4b. The expression for dimensionless mean temperature is

$\theta_m = \frac{136}{175} - \frac{8}{175} Br$ whereas, in Aydin and Avci (2006) for plane Poiseuille flow is

$\theta_m = \frac{136}{175} - \frac{2}{175} Br$. Although at $Br = 0$, the value of Nusselt number is $\frac{70}{17}$, the other values of Nu differ from those in Aydin and Avci (2006) for plane Poiseuille flow. The increase in Br values decreases the Nusselt number and the asymptote appears at $Br = 17$.

4.4.3 INSULATED LOWER PLATE IN TERMS OF Br

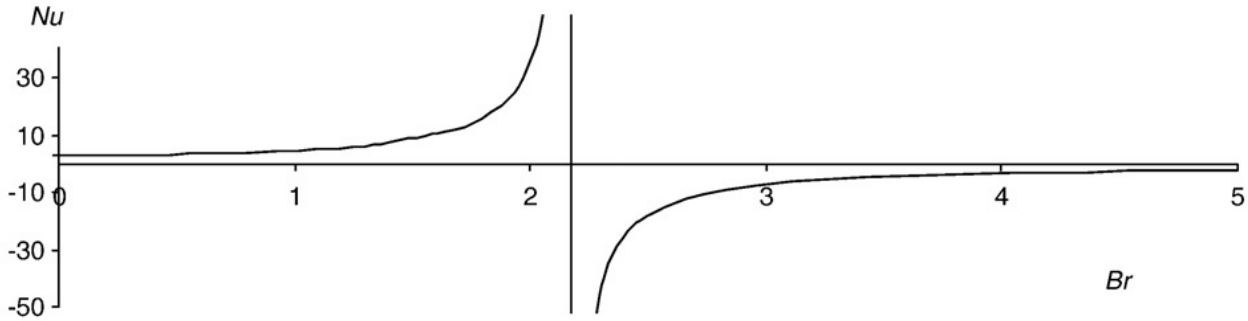


Figure (4.10): The influence of Br on Nu for insulated lower plate

In this case when the Brinkman number is defined as Br as in Equation (3.111), the corresponding non-dimensional temperature θ is defined as in equation (3.106), and the Nusselt number is as in Equation (3.119) which is graphically shown in Figure 5. The asymptote appears at $Br = \frac{13}{6}$. For no viscous dissipation, $Nu = \frac{35}{13}$.

CHAPTER 5

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

In this study the influence of the viscous dissipation on the heat transfer characteristics or the connective heat transfer in a Newtonian fluid flowing in macro channels with uniform inlet fluid velocity and constant heat flux and wall temperature boundary conditions are investigated. Here, an analytical approach is presented in an exhaustive way to suggest explicit expressions of the Nusselt number, utilizing two definitions of the Brinkman number for three different cases of constant heat flux boundary conditions. For this purpose, single phase incompressible, laminar, viscous and constant properties fluid flow between parallel plates channels at steady state and in the flow regime is considered.

Viscous dissipation effects are stimulated by high shear rate along the channel wall. Since the flow is in the flow regime, the Navier-Stokes and energy equations are solved analytically by imposing the no-slip and no-temperature jump at the boundary. Also, since the fluid is assumed to have constant thermo-physical properties, Navier- Stokes and energy equations are decoupled. Therefore, Navier-Stokes equations are first solved along the channel and then the velocities found are used in the energy equation to obtain the temperature profile and Nusselt numbers exhaustively along the channel by applying the variable separable method twice. Also, different cases of constant heat fluxes are demonstrated and expressions of the temperature profile and the Nusselt number are presented in different sub-sections. The influential role of viscous dissipation is found to be of great importance in the heat transfer analysis, hence an emphasis on the viscous dissipation is given to include the effect of the shear stress induced by axial movement of the upper plate for Couette flow in addition to the effect of the viscous dissipation due to the internal fluid heating.

Under the specified assumptions, the following general conclusions can be obtained from this study.

1) As Br increases, fully developed Nu decreases for the continuum case
2) Viscous dissipation decreases the velocity gradient at the wall which increases friction factor

3) Axial conduction plays an important role especially at low Pe and at the inlet section, and should be neglected due to low ratio of channel thickness to channel diameter for macro flows
4) In the presence of viscous heating, there is a significant increase in the temperature distributions by playing role like an energy source. Due to high shear rates at the channel wall, viscous heating should be considered in the analysis.
5) The merit of the effect of the viscous dissipation depends on whether wall is heated or cooled.

The cases of Couette flow and channel flow considered in this study showed that the effects of viscous dissipation on heat transfer is more obvious at large velocity gradient and large viscosity of fluid between parallel plates. Similar simulations should also be repeated with different channel geometries such as triangular or trapezoidal cross-section to verify this general result found for parallel plates.

It is known from the literature that different channel geometries and physical properties in such simulations yield different results. Also, there is no information on viscous dissipation effect on heat transfer in macro channel of Couette flow compared to other subjects. Thus, various simulations should be done with different geometries and different conditions to clarify viscous dissipation effect and to construct a database regarding Nusselt numbers for fully developed laminar flow in channels of differing cross-section.

It should also be noted that, such two- dimensional simulations yield accurate results for constant thermo physical properties. Thus three dimensional simulations are needed, moreover, other important factors such as; temperature dependent fluid properties, conduction at the solid boundaries, and different boundary conditions such as varying temperature profile at the solid boundaries can even be investigated together with viscous dissipation.

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